

# DYNAMIC RESPONSE OF LAMINATED COMPOSITE BEAM WITH TIP MASS AND HARMONIC BASE EXCITATION

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**Abstract:** This work is related to dynamic response of laminated composite beam. The beam is subjected to base excitation and having tip mass. The hand layup method used to manufacture the composite laminated beam. The composite rectangular type of beam analyzed for different aspect ratios and stacking sequence of the laminates. The model is simulated in ABAQUS software for 1 and 2 mm amplitude of base excitation with tip mass. We have developed the natural frequencies and mode shapes using FEA software. Also, the frequency verses displacement response is plotted.

**Keywords** – Composite laminate, tip mass, base excitation, FEA analysis.

## I. INTRODUCTION

The cantilever type of beam with a mass at the tip of beam is the basic system need to study to understand the various aspects in structural dynamics for various application in engineering. It can be utilized to model robotic arms, leaf spring, antenna masts, wings with store configurations, energy harvesting devices, vibrating beam gyroscopes, wing of aircraft and bio/chemical sensors etc. These engineering application required to be form and validate the mathematical model for execution. This model developed with theoretical study and experimental results analysis and optimization of the design for good performance. However, almost all of the industrial application given above utilizes the dynamic resonant response, sometimes implicitly nonlinear response of the system, which induce large strains which shows geometric non-linearity. This can be considered accurate modeling up to the certain approximation level or it is accurate at desired boundary condition. This dynamic resonant response has high level of dependence on the geometric and material properties of components. Any discrepancy in the stiffness, material properties associated with operational conditions such as fatigue due to cyclic loading, environmental thermal effects, and manufacturing tolerances (or defects) may lead to wrong result or mismatch between the proposed model's predicted result values and experimental results. Whereas, the material properties and hence stiffness is important in this work. Such discrepancies can be expected by performing a sensitivity analysis of the model response to small changes in the parameters. [1]

## II. THEORY AND BACKGROUND

An essential step in the design of component is no other than understanding its dynamic response. It includes natural frequencies, mode shapes, and stresses developed due dynamics. Development of mathematical model help to reduce time and effort. The cantilever beam having tip mass has been used as a comprehensive system to assess modelling.

Numerical methods are found to be accurate enough for the solution of most dynamic structural problems. In fact, some numerical methods can be exact in certain situations such as structures subjected to excitation forces described by functions of linear segments, when linear approximation is used in the method.

A laminated composite beam is made of many plies having orthographic properties. The principal material axis of a ply is considered parallel to x-axis and at mean position of beam. Considered the origin of laminated composite beam is at mid-plane of the beam as shown in figure 1.

According to first-order shear deformation (FSDT) theory and applying displacement field to laminated composite beam, gives the following expressions,

$$u(x, z, t) = u_0(x, t) + z\theta(x, t), \quad (01)$$

$$v(x, z, t) = z\psi(x, t), \quad (02)$$

$$w(x, z, t) = w_0(x, t), \quad (03)$$

where, one of the point on beam can be describe as,

$u_0$ , axial displacements at the mid plane assumed along the x-direction.  $w_0$ , axial displacements at the mid plane assumed along the z-direction. Where,  $L$ = length of the composite beam,  $h$ = thickness of the beam (height),  $b$ = width/breadth of the beam and each colored layer represent different angle of fiber direction. We have equation of motion in matrix form by assuming no damping,

$$([K^e] + [M^e])\{\ddot{\Delta}^e\} = \{F^e\} + \{Q^e\} \quad (04)$$

Where,  $[K^e]$  is  $e^{\text{th}}$  element stiffness matrix,  $[M^e]$  is mass matrix of the same element and  $\{F^e\} + \{Q^e\}$  represent

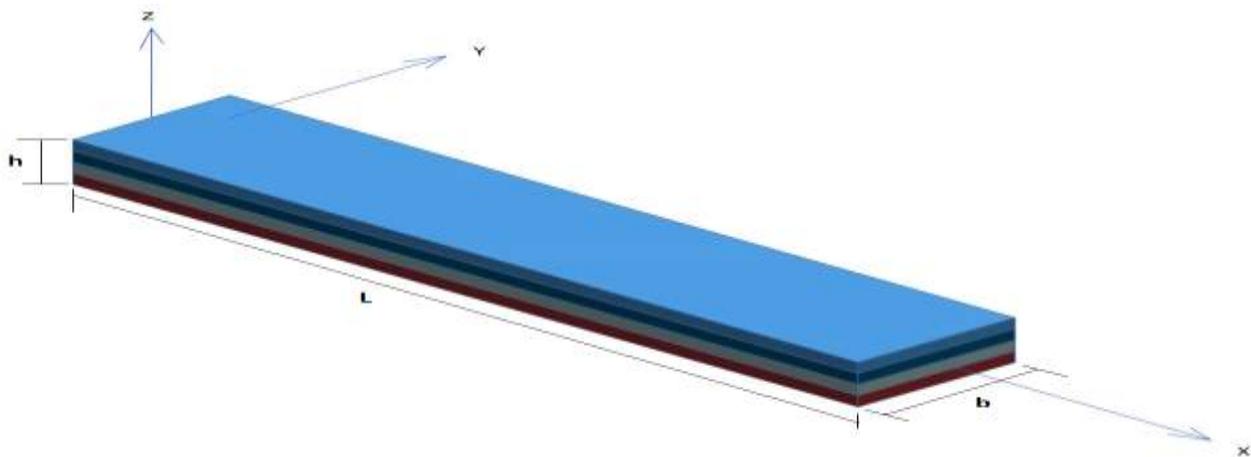


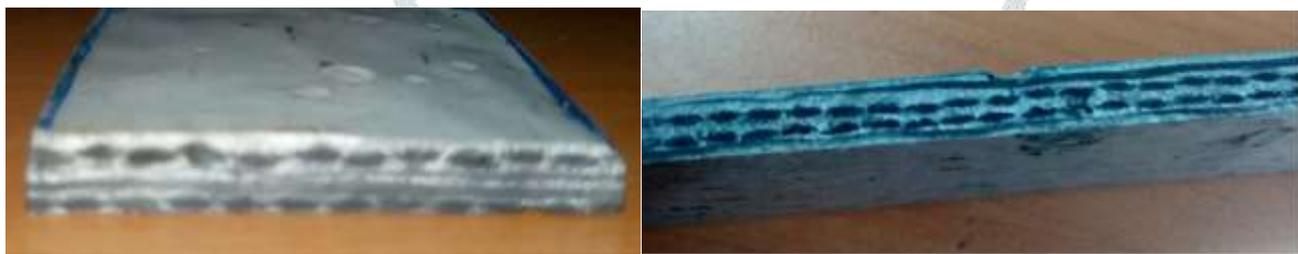
Figure 1: Laminated composite Beam

the force and boundary condition of element. To find the natural frequency of the system, we have to remove all external forces on the system hence equation (04) can be written as,

$$([k]-\omega^2 [M])\{X\}=\{0\} \quad (05)$$

### III. MANUFACTURING OF BEAM

The mold cavity generally coated with polyvinyl alcohol or a wax made of using non- silicon material which is used to release lamina easily. Coating is applied to the mold using a spray gun later on it is placed for curing at room temperature. Layer of Gel-Coat (epoxy resin) of approximately 0.3-0.5 mm thick applied to continuous strands formed like mat or fibrous such as woven knitted made from fiber is placed manually in the mold. Each lamina is sprayed with catalyzed resin. In the following figure, the rectangular cross section of beam and side view of beam are given.



a) Rectangular cross section

b) Side view

Figure 2: cross section and side view of composite laminated beam

### IV. PROPERTIES OF COMPOSITE BEAM

A unidirectional fiber-reinforced lamina can be treated as an orthotropic material. The orthotropic material also shows material symmetry planes parallel and transverse to direction of fiber. The x-axis ( $x_1$ ) is to be considered reference to the fiber direction, i.e. parallel to fiber direction. Perpendicular to x-axis in the plane of lamina considered to be transverse axis ( $x_2$ ). These material coordinates useful to decides the engineering properties of material. The orthotropic material properties of a lamina are obtained either by the theoretical approach or by laboratory testing.

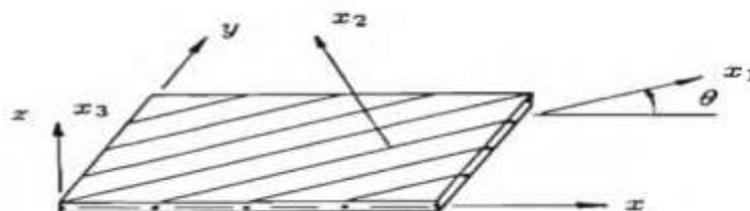


Figure 3: Global and material coordinates of a lamina

The micromechanics approach also called theoretical approach used to determine the different engineering properties, such as modulus of elasticity, modulus of rigidity many essential properties. The engineering constants will accurate for continuous fiber reinforcement, considering following assumptions,

- i Bonding between matrix and fibre is perfectly developed.
- ii Fibres are straight parallel to each other.
- iii All the fibre uniformly distributed throughout.
- iv The voids or micro cracks should not be present in the matrix.
- v The composite material should be free of initial stress.
- vi The fibre and matrix individually follows Hook's law of elasticity.
- vii The loads applied on the laminates significant only when it is applied in parallel or perpendicular direction.

Let,

$E_f$  = modulus of the fiber,  $E_m$  = modulus of the matrix,  $\nu_f$  = Poisson's ratio of the fiber,  $\nu_m$  = Poisson's ratio of the matrix,

$V_f$  = fiber volume fraction and  $V_m$  = matrix volume fraction.

Then it can be shown that the lamina's engineering constants can be derived from the following formulae,

Longitudinal modulus,  $E_1 = E_f V_f + E_m V_m$ ,

Transverse modulus,  $E_2 = E_f E_m \div (E_f V_m + E_m V_f)$

$\nu_{12}$  = Major Poisson's ratio and

Shear modulus,  $G_{12} = G_f G_m \div (G_f V_m + G_m V_f)$ ,

where,  $G_f = E_f \div 2x(1+\nu_f)$  and  $G_m = E_m \div 2x(1+\nu_m)$ .

## V. ANALYSIS OF COMPOSITE BEAMS

The composite beam is simulated using ABAQUS software. To get expected result, we have to follow the three basic step given below. The details of composite beam and the material properties given in pre-processing.

a) Pre-processing:

Beam having rectangular cross section of 0.3 m length, 0.04 m width and the beam is simulated for aspect ratio (L/h) of 60. The beam is made of 4 layers having 0/90/90/0 as stack sequence with equal thickness for each layer. The properties of beam as given below,  $E_1 = 165.22$  GPa,  $E_2 = 7.81$  GPa,  $\nu = 0.256$ ,  $G_{12} = G_{23} = G_{13} = 4.2$ . The density of the beam specimen is  $1460 \text{ Kg/m}^3$ .

b) Processing:

The model has to be meshed with required size of element. The solution should be convergence at that required size of element. As we are using planar shell element. The solution is converged at 0.01 mm size of quad element (total no. element = 153). The convergence graph is given below.

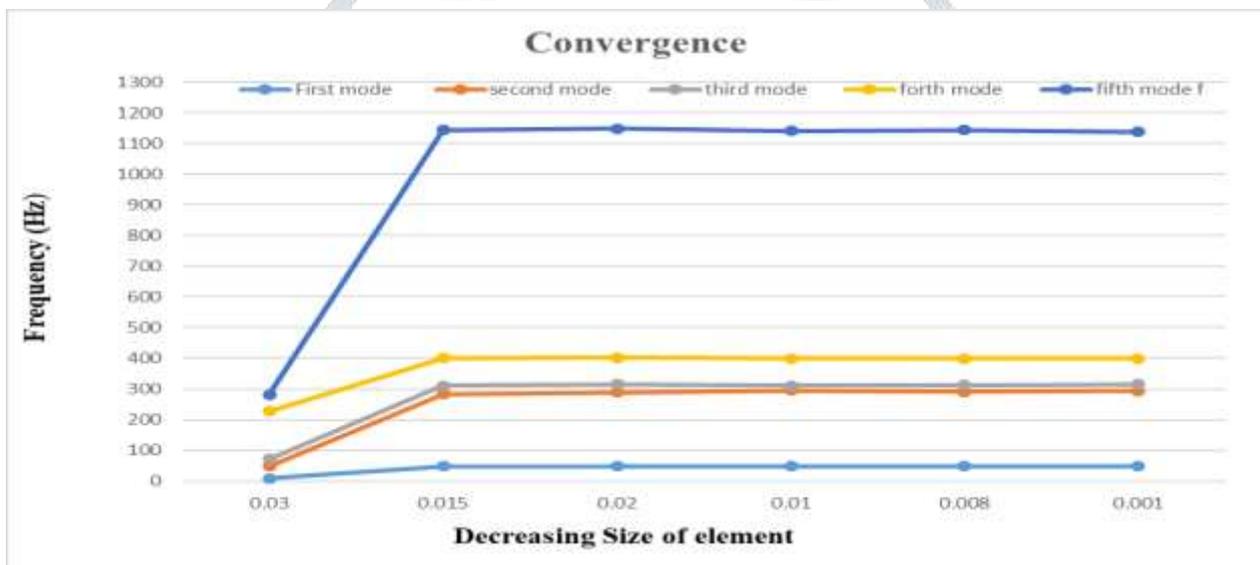


Figure 4: Convergence of solution

For modal analysis the boundary condition is to be defined. All the degrees of freedom are to be restricted of one of the side. We will have first five mode shape and the respective natural frequency. Also, for dynamic analysis of beam we need to apply proper boundary condition. Following boundary condition are applicable,  $X_1 = 0$ ,  $Z_1 = 0$ ,  $\Theta_1 = 0$ ,  $\phi_1 = 0$ ,  $\psi_1 = 0$  and  $Y_1 = Y \sin(\omega t)$  where Y is Amplitude of base excitation, the excitation amplitudes,  $Y = 1$  mm and  $2$  mm taken for analysis as automotive system vibrates in this range. The other side is free and the mass is attached  $50$  g at free end.

c) Post-processing:

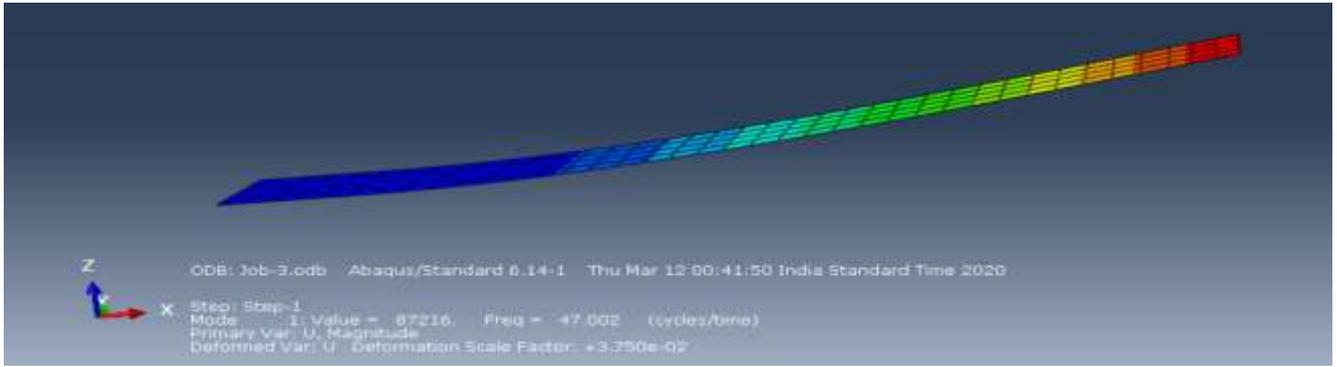
Once we solve the model, we need to extract the required data using various tools, like color contour plots, animations, deformed shape plots, and X-Y plots.

Here we have to plot frequency verses displacement graph using x-y plot. This plot should be representing the maximum displacement of the tip over the frequency of excitation.

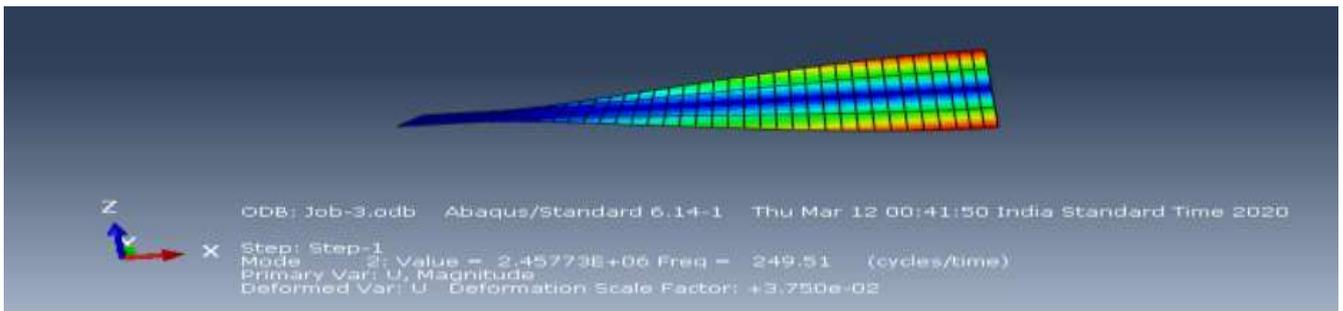
## VI. RESULTS AND DISCUSSION

The modal analysis is carried out and first five mode shape extracted and shown in figure 5. Where it is noted that fundamental frequency is  $47$  Hz.

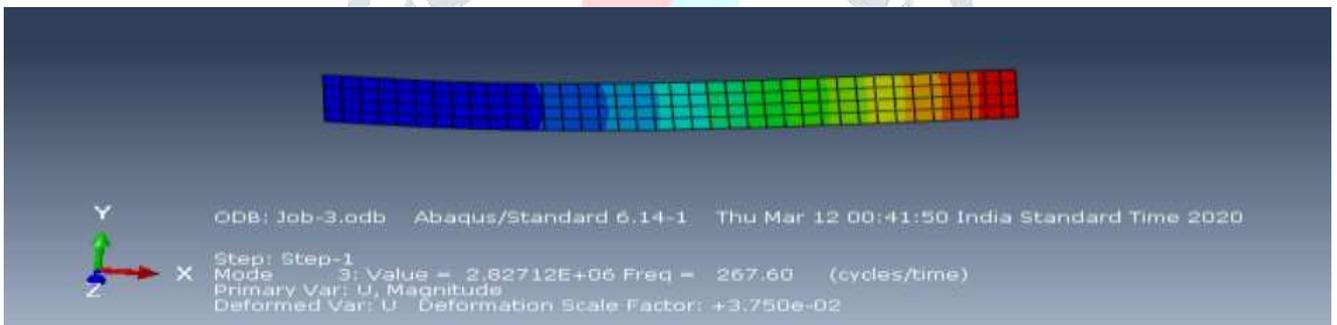
i. Mode shapes and natural frequency



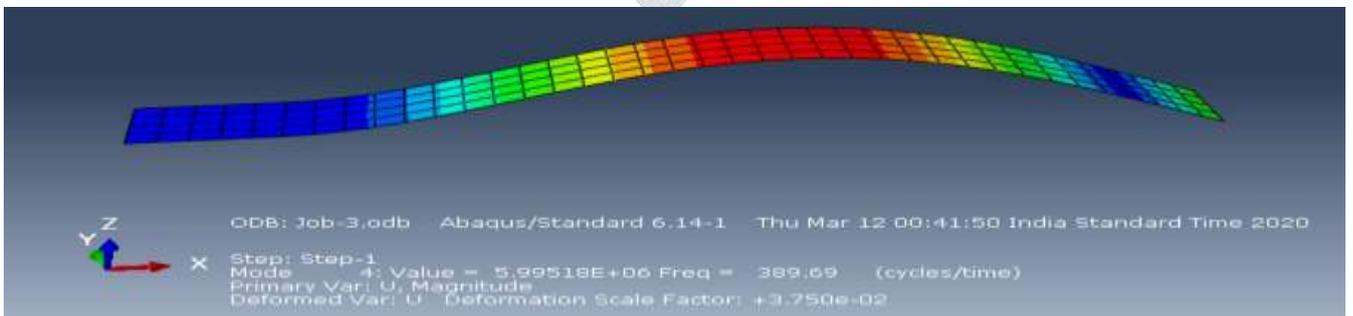
a) First mode at 47.00 Hz



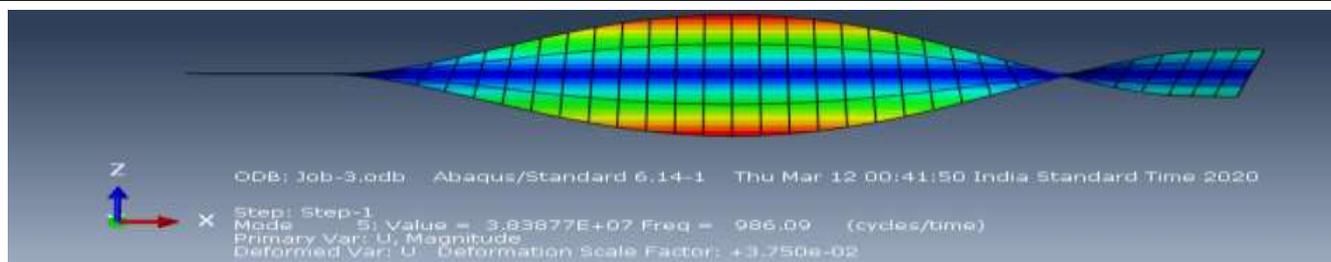
b) second mode at 249.51 Hz



c) third mode at 267.6 Hz



d) Fourth mode at 389.69 Hz

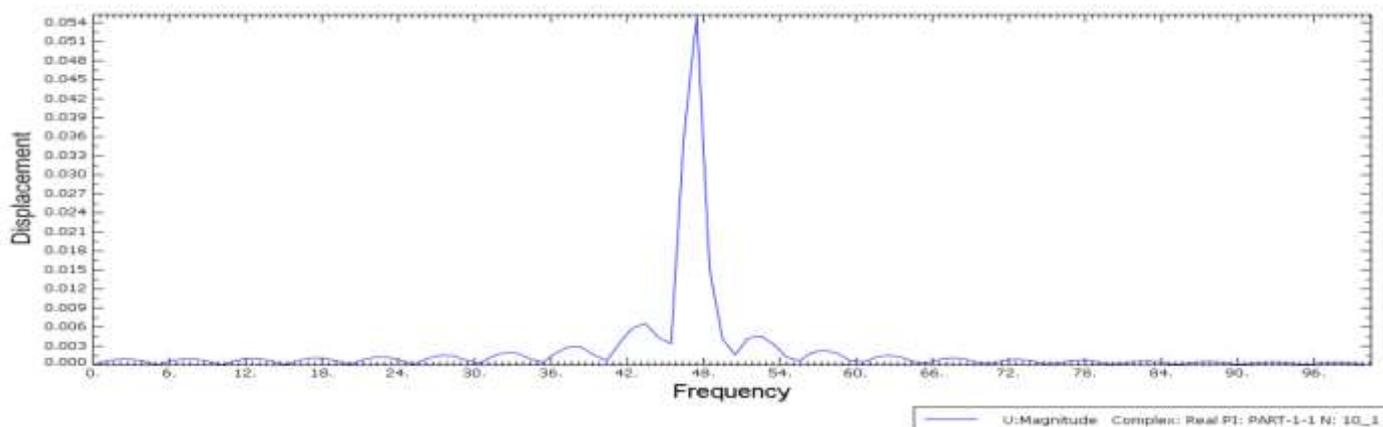


e) Fifth mode at 986.09 Hz

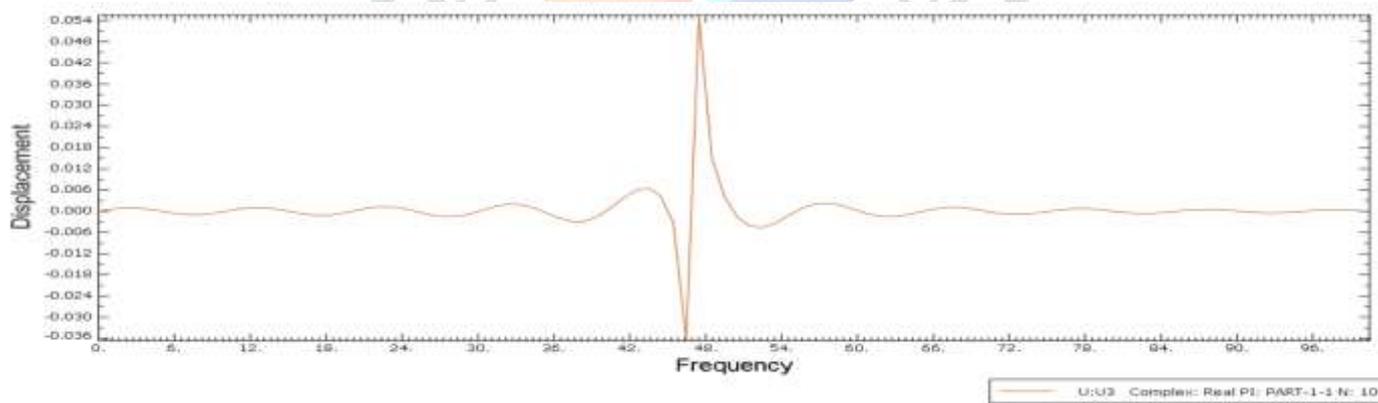
Figure 5: Mode Shape of cantilever beam

**ii. Displacement Vs. Frequency Response**

The beam tip displacement varies with varying frequency of excitation as shown in following graph. Reference node selected for displacement for the beam as shown above figure. The beam is excited between the frequency zone as 1 to 100 Hz as beam is transversally vibrate at required force direction. The response of displacement of tip node as shown in graph. The magnitude of displacement verses frequency for I mm base excitation is given below.



a) Frequency Vs Displacement (magnitude) of excitation at 1 mm amplitude



b) Frequency Vs Displacement at 1 mm amplitude

Figure 6: Frequency Vs Displacement plots

The modes shapes and the respective resonance frequencies for different stack sequences are tabulated below.

Table 1: First five natural frequencies for different beam stack sequences

Sr. No.	Stack Sequence	First mode	Second mode	Third mode	Fourth mode	Fifth mode
1	0/90/90/0	47.00	249.51	267.60	389.69	986.09
2	0/45/45/0	47.247	278.92	310.84	392.22	1153.5
3	0/30/30/0	47.92	291.22	311.38	411.25	1142.9

The maximum displacement of the tip at first natural frequencies for different stack sequences given in table 2. The amplitude of vibration selected 1 mm and 2 mm because generally system of vibration occurs in this range.

Table 2: Maximum Displacement verses base excitation

Sr. No.	Stack Sequence	Excitation amplitude	Tip Displacement
2	0/90/90/0	1 mm	56 mm
		2 mm	107 mm
2	0/45/45/0	1 mm	58 mm
		2 mm	111 mm
3	0/30/30/0	1mm	61 mm
		2 mm	117 mm

## VII. CONCLUSION

We have manufactured composite laminate beam using hand layup method. The composite beam with different stack sequences and fiber direction is produced. Carbon fiber and epoxy resin used to make this beam. The micromechanics approach is used to decide the properties of composite laminates. The composite beam is studied for various stack sequences and the FEA software, ABAQUS used to find out the natural frequencies and mode shape of the cantilever beam with tip mass. The composite beam is simulated for base excitation and plotted frequency verses displacement graph. The cross ply is shows more stiffness than general plies combination. The deflection also changes with change in stack sequences.

## REFERENCES

- [1] V. C. Meesala and M. R. Hajj, "Response variations of a cantilever beam–tip mass system with nonlinear and linearized boundary conditions". *J. Vib. and Con.*0(2018) 1-12.
- [2] J. Freundlich, "Transient vibrations of a fractional Kelvin-Voigt viscoelastic cantilever beam with a tip mass and subjected to a base excitation". *J. Sound Vib.* 438 (2019) 99-11.
- [3] Li Jun, Hua Hongxing, S. Rongying, "Dynamic finite element method for generally laminated composite beams". *Int. J. Mech. Sci.* 50 (2008) 466–480.
- [4] L. Fernando, D. Vandepitte, V. Tita, Ricardo de Medeiros, "Dynamic response of laminated composites using design of experiments: An experimental and numerical study", *Mech. Syst. Sig. Process.*115 (2019) 82-101.
- [5] P. Kesarwani, S. Jahan, K. Kesarwani, "Composites: Classification and its manufacturing process", *Int. J Appl. Res.* 2015; 1(9): 352-358.
- [6] A.S. Bassiouni, R.M. Gad-Elrab, T.H. Elmahdy, "Dynamic analysis for laminated composite beams". *Compos. Struct.* 44 (1999) 8 1-87.
- [7] D.K. Maiti, P. K. Sinha "Bending and free vibration analysis of shear deformable laminated composite beams by finite element method". *Composite Structures* 29 (1994) 421-431
- [8] F. Moleiro, M. Soares, J.N. Reddy, "A layerwise mixed least-squares finite element model for static analysis of multilayered composite plates". *Comput and Struct* 89 (2011) 1730–1742A. 2001.