Review of Ginzburg Landau theory of Type II Superconductivity

Shambhoo Nath Prabhakar
Vill- Andhar, Po- Makhdumpur, Dist.- Gaya (Bihar)
Pin- 824234

Abstract :

Type II superconductors also have zero resistance, but their perfect diamagnetism occurs only below the lower critical field BC_1. Then one defines the ratio of λ and ξ as a Ginzburg-Landau parameter k k plays a very important role in type II superconductors. The density of super electrons ns which characterizes the superconducting state, increase from zero at the interface with a normal material to a constant value for inside, and the length scale for this to occur is the coherence length As external magnetic field B decays exponentially to zero inside a superconductor

Keywords : Phase transition, super conductivity, Helmholtz free energy, Gibbs density.

1. Introduction :

One of the most fertile approach to superconductivity has been developed from Ginzburg and Landau[1]. Landau[2] had develped a general theory for second order phase transition based on the idea that a phase transition could be characterized by some kind of order parameter. The critical insight in GL theory was that for a superconductor the order parameter must be identified with the macroscopic wave function Ψ . This means that an order parameter is complex and varies in space.

The Landau theory is concerned with the temperature region near Tc in which ϕ is small. The stable phase is the one in which the free energy F is a minimum. The first basic assumption of the theory, is that in the region where ϕ is small F can be expanded as a power series in :

F = F_n+ $\lambda\phi$ + $\alpha\phi^2$+ $\gamma\phi^3$ + $\frac{1}{2}\beta\phi^4$, (1)

There are systems in which such a simple expansion is not possible, but it does work for superconductivity. In equation (1), all the coefficient, λ, α etc. are to be taken as functions of T, since the equilibrium values of F and ϕ are functions of T. The second basic assumption is that the coefficients can be expanded in powers of (T-T). Again this simple assumption does of hold for all system, but again it does hold for superconductivity. We can use this assumption to simplity (1) considerably. First, the equilibrium phase corresponds to a minimum in F:

$\frac{\partial F}{\partial \phi}$ = 0 (2)

In the normal phase, we must have a minimum at ϕ = 0, which implies $\lambda = 0$ for T > T_c. Since we are assuming that x =0 T for all T. Furthermore, in most systems, including superconductors and He II, the term in ϕ^3 does not occur. We are thus left with

F = F_n+ α $(T)\phi^2\frac{1}{2}$+ $+\frac{1}{2}\beta(T)\phi^4$, (3)

where we now show the temperature dependences explicitly.

We now find the temperature dependence of a and J3 . Equation (2) for the value ϕ_0, of ϕat the minimum gives

$\alpha\phi_0 + \beta\phi_0^3 = 0$ (4)

with solutions ϕ_0 = 0 and ϕ_0^2 = -α/β Now we want ϕ_0 = 0 to be the only solution for T > Tc , whereas for T < Tc. we must have a solution with $\phi_0 \neq 0$. We can achieve this if we take the temperature dependence so that –α/β is negative for T> Tc, the positive for T < Tc. In addition, we must have β positive at all temperatures, since if β were negative F would decrease indefinitely for large values of ϕ. We therefore want a to be positive for T > Tc, and negative T< Tc . The simplest temperature dependences that give this are

$$\alpha(T) = A(T - T_c) \quad (5)$$

$$\beta(T) = \beta(T - T_c)\beta \quad (6)$$

With α and β given by equation (5) and (6) the solution for ϕ_0, from equation (5), is

$$\phi_0 = 0 \qquad T > T_C$$

$$\phi_0 = \pm A^{1/2} (T_C - T)^{1/2}/\beta^{1/2} \qquad T > T_C \quad (7)$$

The two signs for T<Tc correspond to the two branches shown in figure 2A. In addition, ϕ_0 = o remains a solution below Tc, but as we shall see shortly, it corresponds there to a maximum, not a minimum. Equation (7) gives a rapid, parabolic, increase in ϕ_0 = o as T decreases from Tc; It is instructive to consider the free energy F as well as ϕ. First, the value of F at the minimum. F_{min}, is given by substituting ϕ_0 in equation (3)

$$F_{min} = F_n. \qquad T > T_C \quad (8)$$

$$F_{min} = F_n - \frac{1}{2}\alpha^2\beta = F_n - \frac{1}{2}A^2 \quad (T_C - T)^2/BT < T_c \quad (9)$$

Note in particular, that F_{min}, decreases rather slowly from F_n as T decreases below T_c. The solution ϕ_n = o below Tc gives F = F_n as we said, it gives a maximum, not a minimum. The fact that F_{min} changes only slowly, while ϕ_n changes rapidly, with T, means that a thermal fluctuation which involves a large change in ϕ_n, need only require a small changes of free energy. This is the reason for the sensitivity of second-order phase changes to fluctuation effects.

2. Discussion of Ginzburg-Landau Equation

The extension of the Landau theory to superconductors involves treating the wave function Ψ as an order parameter. This introduces two complications: first, the order parameter becomes a function of position, in general; second, we must include explicitly the coupling of the supercurrent to the magnetic field and the magnetic-field energy. Both of these complications arise in the treatment of the mixed-state vortex lattice.

Let us first make Iv a function of position; we can do this by treating equation (3) as an expression for the free-energy density at the point r, which we integrate over the volume of the specimen to get the total free energy. Furthermore, with Ψ a function of position, we can expect a 'kinetic-energy' term in the energy proportional to $|\nabla\Psi|^2$ The free-energy density is therefore

$$f(r) = f_n + \alpha|\Psi(r)|^2 + \frac{1}{2}\beta|\Psi(r)|^4 + \frac{\hbar^2}{2m}\nabla\Psi(r)|^2 \qquad (10)$$

We use modulus signs because y is complex. Following the usual convention in GL theory, we write the coefficient of the gradient terms as $\hbar^2/m$, where m is the mass of the electron. As de Gennes (1966)[3] emphasises, there is no physical content to this choice: in the end it simply determines the normalisation of Ψ In fact. we did effectively make a different choice when we wrote London's equation using 2m, the mass of a Cooper pair. We allowed for that my noralising

Ψ so that $|\Psi|^2$ was $n_s/2$, the density of Cooper pairs. Here therefore, we should interpret $|\Psi|^2$ as n_s, the electron density, although in fact we shall not use this explicitly. We can take the coefficient of $|\nabla\Psi|^2$ independent of T, like β, because we are dealing with a small temperature region near T_c and the $|\nabla\Psi|^2$ term must always be positive.

Even at this stage, we can see the principal physical consequence of adding the gradient term: it prevents y from chafing too rapidly, since a high value of the gradient would give a large contribution to the free energy. On dimensional grounds, we can expect an appropriate ratio of coefficients to define a fundamental length $\xi(T) = (\hbar^2/2m|a|)^{1/2}$ is central to GL theory. It is clear that variations of which are rapid within a distance $\xi(T)$ will not occur. We shall find that $\xi(T)$ is the coherence length for variations of the order parameter in the sense it is the core radius of a vortex line.

The inclusion of magnetic-field in the free energy requires a little care. The total induction B in the superconductor is the sum of the induction produced by the applied field H_o, and that produced by the suprecurrent J_c.

Curl (B- $\mu_0 H_0) = \mu_0 J_c$ (11)

We extend the free energy of equation (10) to include magnetic-field effects by making the usual replacement.

$\nabla \rightarrow \nabla \pm \frac{2ie}{h} A$

with the + sign, if ∇ acts on Ψ and — sign if ∇ acts on Ψ . We also add the magnetic-field energy, to get

$$\int(r) = f_n + \alpha \left|\Psi(r)\right|^2 + \frac{1}{2} + \beta \left|\Psi(r)\right|^4$$
$$+ \frac{1}{2} \left|(\mathrm{ih}\nabla - 2\mathrm{eA})\Psi\right|^2 + \beta^2/2\mu_0 - \mu_0 - H_0^2/2 \qquad (12)$$

The integral of f(r) over the volume of the specimen is he Helmholtz free energy F = U – T ∑. To maintain consistency with the thermodynamic equation, we subtract $\frac{1}{2}\mu_0 H_0^2$, which is the magnetic energy of the coils generating the applied field H_o. The internal energy U is the energy of the superconductor in the presence of the magnetic field, so that

$$dU = Td\sum + H_0 . dm \qquad (13)$$

To find the stable state at temperatuare T and field H_o, we must minimise the Gibbs free energy.

$$G(T, H_o) = U - T\sum - H_0 M. \qquad (14)$$

Thus, finally, we have the result that we must minimise G:

$$G = \int g(r) d^3 r \qquad (15)$$

with

$$g(r) = f_n + \alpha \left|\Psi(r)\right|^2 + \frac{1}{2}\beta\left|\Psi(r)\right|^4 + \frac{1}{2}\left|(-i\hbar\nabla - 2eA)\Psi\right|^2 + \beta^2/2\mu_0 - H_{0.}B + \frac{1}{2}\mu_0 H_0^2 \qquad (16)$$

In some accounts of GL theory, it is stated that the free energy G = $\int f\,(r)d^3r$ should be minimised; the Helmholtz energy density (r) differs from the Gibbs density g(r) by H_o. M. We shall seee that one gets the same equation for Ψ(r) and for the super current whether one minimises F of G. However,in discussing the parallel critical fields of thin films, for example, it is important to use the correct function G.

The energy G is sn integral involving two functions Ψ(r) and A(r) (recall that B= curl A.) This contrast with the ordinary Landau theory, where G is a function of the variable ϕ. In that case, the equation for a minimum was simply $\partial G/\partial\phi = 0$. Now, since G depends on functions Ψ and A*, we must use the Euler-Lagrange equations of the calculus of variations. Since Ψ is complex, we can minimise with respect to either Ψ; the Ψ* equation is

$$\frac{\partial g}{\partial \Psi} - \sum \frac{\partial g}{\partial x_i}\frac{\partial g}{\partial(\nabla_j \Psi^*)} = 0 \qquad (17)$$

where $\nabla_j\Psi^*$ is the component of the gradient in the direction j. This equation is sometimes written formally as $\delta G/\delta\Psi^* = 0$, where $\delta G/\delta\Psi^*$ stands for the left-hand side of equation (17). After some manipulatation, and with the restriction that we use the gauge div A=0, equation (17) becomes

$$\frac{1}{2}\,(-h\nabla - 2eV)^2\alpha\Psi + \beta\left|\Psi\right|^2\Psi = 0. \qquad (18)$$

This is the first GL equation. If we had minimised with respect to y rather than Ψ^*we would simply have found the complex conjugate equation.

3. Conclusion :

From the above investigations on type II superconductivity we can draw the following conclusion.

I. Ginzburg- Landau phenomenological theory works quite well in explaining the various properties of type II superconductors.

References

1. V. C. Ginzburg and L.D. Landau zh. Eksp. Teor. Fiz. 20, 1064 (1950).

2. L.D. Landau, Zh. Eksp. Teor. Fiz 34, 1240 (1958).

3. P. G. de Gennes Superconductivity of Metals and alloys (New York: Benjamin 1966).

4. T. E. Faber, Prov. R. Soc. A. 214, 392 (1952).

5. T.E. Faber, Prov. R. Soc. A 231, 353 (1955).

6. T.E. Fabber, Proc. R. Soc. A241, 531 (1957).

7. J. Faber and D. S. Melachlen Phys. Rev. 177, 763 (1969).

8. D. Saint-James and P G de Gennes, Phys. Lett. 7, 306 (1963).

9. J. Feber and D.S. Malachlean, Phys. Rev 140, A1638 (1965).

10. L.J. Campbell, J. Low Temp. Phys. JLTP 8, 105 (1972).