NUMERICAL SOLUTION OF LONGITUDINAL PERTURBATION FLUID AND PARTICLE VELOCITY ON A DUSTY FLUID JET COMPRESSIBLE FLOW PROBLEM CONSIDERING FINITE VOLUME FRACTION

V.GANESH[1], B . K. RATH[2] , SONAL BHARTI[3] , D.K.DASH[4]

1.Research Scholar , SSSUTMS , Sehore , M.P & faculty in GIET University, Gunupur

2.Department of Mathematics, GIET University , GUNUPUR , Rayagada.

3..Dean School of Engineering, SSSUTMS , Sehore , M.P

4.Department of Mathematics, CCET(C.G), Bhilai, Raipur.

ABSTRACT

The numerical solution of compressible dusty fluid considering finite volume fraction has been studied. Assuming the velocity and temperature in the jet to differ only slightly from that of the surrounding stream, a perturbation method has been employed to linearize the basic equations. The linearized boundary layer equations have been solved by using double transform technique. Numerical computations have been made to discuss the profiles of velocity of the fluid and particle. It is observed that consideration of finite volume fraction reveals that the magnitude of perturbation velocity of both fluid and particle is reduced significantly.

KEYWORDS: Particulate suspension, Boundary layer characteristics, volume fraction, diffusion. Compressible fluid.

NOMENCLATURE

(u, v) Fluid velocity, (z, r) Denote the distance along and perpendicular to the jet axis, K for thermal conductivity, p_r for Prandtl number, (T,T_p) Temperatures of fluid and particle phase, T_0 for Undisturbed temperature of jet flow, $(\bar{u},\bar{v},\bar{u}_p,\bar{v}_p)$ Non-dimensional velocity components of fluid and particle phase respectively,($u_1,u_{p_1},v_1,v_{p_1},T_1,T_{p_1},\rho_{p_1}$) First order perturbation quantities, (u_0,u_{p_0}), The uniform velocity at the exit of jet flow, (T_0,T_{p_0}) Temperature at the exit of jet flow, $(\bar{T},\bar{T}_p)$ Dimensionless temperature components of fluid and particle phase, U for Undisturbed jet velocity, α Concentration parameter, (υ,υ_p), Kinematics coefficient of viscosity of fluid and particle phase respectively, (ρ, ρ_p) Density of fluid and particle phase respectively, $\bar{\rho}_p$ Dimensionless density of the particle phase, μ Coefficient of viscosity of fluid, μp Coefficient of viscosity of particle phase, Cs Specific heat of the solid particles, C_p Specific heat at constant pressure for the gas, φ volume fraction of dust particles, λ The momentum equilibration length, ρ_p density of the particles in the free-stream, ρ_{p_0} Undisturbed particle density in the jet, τ_T thermal equilibration time, τ_m The momentum equilibration time.

INTRODUCTION

Compressible jet mixing of a dusty fluid originating from a circular jet has been studied by Dutta and Das [1990], Heat transfer of an axially symmetrical jet mixing compressible dusty fluid, studied by D.K.Dash & Chand Ram [2014]. The Effect of Brownian Motion on the bulk stress in a suspension of spherical particles studied by Batchelor,G.K [1977] . Modeling Dispersion of suspended particulate matter (SPM) in Axi symmetric jet mixing studied by Panda T.C., Mishra S.K., Dash D.K.[2005] and many more has studied jet problem in case of negligible volume fraction of SPM. But this assumption leads to an error which ranges from insignificant to very large. Also this type of assumption is not justified when the fluid density is high or particle mass fraction is large. In the present paper the effect of finite volume fraction in an axially symmetrical jet mixing of a compressible fluid with suspended particulate matter has been studied. Assuming the velocity and temperature in the jet to differ only slightly from that of the surrounding stream, a perturbation method has been employed to linearize the governing differential equations. The resulting linearized equations have been solved by using double Transform technique. Numerical computation of the integrals giving velocity of both the fluid and particle phase have been made to discuss the results. Consideration of finite volume fraction reveals that the magnitude of perturbation velocity of both fluid and particle is reduced significantly

MATHEMATICAL FORMULATION

The equation governing the study compressible two-phase boundary layer flow in axi-symmetric case can be written in cylindrical polar coordinates as

$$(1-\phi)\left(u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}\right)=\frac{T}{r}\frac{\partial}{\partial r}\left(\sqrt{T}\,r\frac{\partial u}{\partial r}\right)+\frac{\rho_p T}{\tau_m}\left(u_p-u\right) \quad (1)$$

$$u_p\frac{\partial u_p}{\partial z}+v_p\frac{\partial u_p}{\partial r}=\frac{u-u_p}{\tau_m} \quad (2)$$

To study the boundary layer flow, we introduce the dimensionless variables are

$$z^*=\frac{z}{\lambda},\ r^*=\frac{r}{(\tau_m\upsilon_0)^{\frac{1}{2}}},u^*=\frac{u}{U},v^*=v\left(\frac{\tau_m}{\upsilon_0}\right)^{\frac{1}{2}},\ u_p^*=\frac{u_p}{U},v_p^*=v_p\left(\frac{\tau_m}{\upsilon_0}\right)^{\frac{1}{2}},\alpha=\frac{\rho_{p_0}}{\rho_0},\ \rho^*=\frac{\rho}{\rho_0}$$

$$\rho_p^*=\frac{\rho_p}{\rho_{p_0}},\ T^*=\frac{T}{T_0},\ T_p^*=\frac{T_p}{T_0},\ \lambda=\tau_m U,\ \tau_m=\frac{2}{3}\frac{C_p}{C_s}\frac{1}{p_r}\tau_T,p_r=\frac{\mu C_p}{K},\ \upsilon^*=\frac{\upsilon}{\upsilon_0}.$$

Assuming the flow from a circular, opening under full expansion i.e. no pressure variation occurs throughout the flow, and flow variables of the jet differ only slightly from +0that of the surrounding stream, it is possible to write the velocities, temperatures and particle density in the following form as $u=u_0+u_1,\ v=v_1,u_p=u_{p_0}+u_{p_1},\ v_p=v_{p_1},T=T_0+T_1,T_p=T_{p_0}+T_{p_1},\ \rho_p=\rho_{p_0}+\rho_{p_1}$ where $u_1<<u_0,u_{p_1}<<u_{p_0},T_1<<T_0,\ T_{p_1}<<T_{p_0},\ \rho_{p_1}<<\rho_{p_0}$.

The quantities with suffix '0' are the values at the opening and those with suffix '1' are perturbed quantities. Again, the suffix p denotes those variables for the particle. Under the above assumption the governing equation (1) to (2) can be written after dropping the * and suffix one in the non-dimensional linearized form as follows

$$(1-\phi)u_0 \frac{\partial u}{\partial z} = \frac{(T_0)^{\frac{3}{2}}}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right) + \alpha \rho_{p_0} T_0 (u_p - u) \quad (3)$$

$$u_{p_0} \frac{\partial u_p}{\partial z} = (u - u_p) + (u_0 - u_{p_0}) \quad (4)$$

The boundary conditions are

$$u(0,r) = u_{1_0},\ u_p(0,r) = u_{p_{10}},\ T\ (0,r) = T_{10},\ T_p(0,r) = T_{p_{10}},\ \rho_p(0,r) = \rho_{p_{10}} \text{ for } r \le 1$$

$$u(0,r) = u_p(0,r) = T\ (0,r) = T_p(0,r) = \rho_p(0,r) = 0 \text{ for } r > 1$$

$$v(0,r) = v_p(0,r) = \frac{\partial u}{\partial r}(z,0) = \frac{\partial T}{\partial r}(z,0) = 0$$

$$u\ (z,\infty) = u_p(z,\infty) = T\ (z,\infty) = T_p(z,\infty) = \rho_p(z,\infty) = 0 \quad (5)$$

METHOD OF SOLUTION

The governing equations (3) to (4) are linear partial differential equations having seven unknowns u, u_p, v, v_p, T, T_p and ρ_p. These equations have been solved by a double transform technique. First employing Hankel transform with respect to the variable r, which is denoted by * and then Laplace transform with respect to the variable z, which is denoted by – .

Taking Hankel and Laplace transforms respectively of both sides of the above equations and using the boundary conditions we obtain.

$$\bar{u}^* = \frac{J_1(p)\left[B u_{p_{10}} + (C+S)u_{10}\right]}{P F(S)} \quad (6)$$

$$\bar{u}_p^* = \frac{J_1(p)\left[C u_{10} + (A+S)u_{p_{10}}\right]}{P F(S)} \quad (7)$$

The inversion of (6) to (7) gives

$$u = \int_0^\infty \left[u_{10} \cosh QZ + \frac{1}{Q}\left\{\frac{(C-A)}{2}u_{10} + B u_{p_{10}}\right\}\sinh QZ\right] e^{-\frac{(A+C)Z}{2}} J_0(pr) J_1(p)dp \quad (8)$$

$$u_p = \int_0^\infty \left[u_{p_{10}} \cosh QZ + \frac{1}{Q}\left\{C u_{10} + \frac{(A-C)}{2} u_{p_{10}}\right\}\sinh QZ\right] e^{-\frac{(A+C)Z}{2}} J_0(pr) J_1(p)dp \quad (9)$$

Where $F(S) = S^2 + S(A+C) + C(A-B),\ Q = \sqrt{BC + \left(\frac{A-C}{2}\right)^2}$

$$A = \frac{P^2 T_0^{\frac{3}{2}} + \alpha \rho_{p_0} T_0}{(1-\phi)u_0},\ B = \frac{\alpha \rho_{p_0} T_0}{(1-\phi)u_0},\ C = \frac{1}{u_{p_0}}$$

DISCUSSION OF RESULT AND CONCLUSION

In the present paper numerical computation have been made by taking $P_r = 0.72$, $u_{10} = up_{10} = T_{10} = Tp_{10} = \rho_{p10} = 0.1$, $\varphi = 0.01$. The velocity at the exit are taken nearly equal to unity. The improper integrals have been evaluated on using the gauss-three point quadrature rule.

Table 1.1 and table 1.2 shows the profiles of axial fluid perturbation velocity u and longitudinal perturbation particle velocity u_p for α =0.1, 0.2,0.3 and for different values of Z. It is observed that the axial fluid velocity u is greater than the axial particle velocity u_p near the nozzle exit whereas u_p is greater in the mixing region downstream. Also the magnitude of both fluid and particles are changed as we proceed in the down stream direction.

Clearly, it indicates that, consideration of finite volume fraction reveals that the magnitude of perturbation velocity of both fluid and particle is reduced significantly.

Table1.1: Values of longitudinal perturbation fluid velocity u1

ALPHA=0.1			ALPHA=0.2			ALPHA=0.3		
Z=0.25	Z=0.50	Z=1.0	Z=0.25	Z=0.50	Z=1.0	Z=0.25	Z=0.50	Z=1.0
1.00E-01	1.02E-01	1.06E-01	1.00E-01	1.02E-01	1.07E-01	1.00E-01	1.02E-01	1.07E-01
1.01E-01	1.02E-01	1.06E-01	1.01E-01	1.02E-01	1.06E-01	1.01E-01	1.02E-01	1.07E-01
1.01E-01	1.01E-01	1.02E-01	1.01E-01	1.01E-01	1.01E-01	1.01E-01	1.01E-01	1.00E-01
9.72E-02	9.09E-02	8.40E-02	9.63E-02	8.95E-02	8.27E-02	9.51E-02	8.78E-02	8.12E-02
4.75E-02	4.71E-02	4.74E-02	4.74E-02	4.70E-02	4.74E-02	4.72E-02	4.68E-02	4.75E-02
2.28E-03	7.64E-03	1.47E-02	3.00E-03	8.76E-03	1.59E-02	3.93E-03	1.01E-02	1.72E-02
-3.03E-04	5.29E-05	2.10E-03	-2.72E-04	2.24E-04	2.73E-03	-2.35E-04	4.93E-04	3.56E-03
-2.31E-04	-2.25E-04	-5.47E-05	-2.28E-04	-2.21E-04	5.53E-05	-2.27E-04	-2.10E-04	2.37E-04
-1.60E-04	-1.84E-04	-1.80E-04	-1.69E-04	-1.84E-04	-1.73E-04	-1.76E-04	-1.84E-04	-1.56E-04
-1.73E-04	-1.85E-04	-1.89E-04	-1.78E-04	-1.85E-04	-1.89E-04	-1.81E-04	-1.85E-04	-1.88E-04

Table1.2: Values of longitudinal perturbation particle velocity UP1

ALPHA=0.1			ALPHA=0.2			ALPHA=0.3		
Z=0.25	Z=0.50	Z=1.0	Z=0.25	Z=0.50	Z=1.0	Z=0.25	Z=0.50	Z=1.0
6.62E-01	5.37E-01	3.66E-01	6.62E-01	5.37E-01	3.66E-01	6.62E-01	5.37E-01	3.67E-01
8.19E-01	6.59E-01	4.39E-01	8.19E-01	6.59E-01	4.40E-01	8.19E-01	6.59E-01	4.40E-01
8.32E-01	6.69E-01	4.45E-01	8.32E-01	6.69E-01	4.45E-01	8.32E-01	6.69E-01	4.45E-01
7.89E-01	6.34E-01	4.17E-01	7.89E-01	6.33E-01	4.17E-01	7.88E-01	6.33E-01	4.16E-01
3.84E-01	3.09E-01	2.05E-01	3.84E-01	3.09E-01	2.05E-01	3.84E-01	3.09E-01	2.05E-01
-2.41E-03	-7.20E-04	4.26E-03	-2.35E-03	-4.49E-04	4.90E-03	-2.25E-03	-1.15E-04	5.65E-03
-1.88E-02	-1.46E-02	-8.42E-03	-1.88E-02	-1.46E-02	-8.23E-03	-1.88E-02	-1.46E-02	-7.97E-03
-7.08E-03	-5.55E-03	-3.42E-03	-7.08E-03	-5.55E-03	-3.40E-03	-7.07E-03	-5.54E-03	-3.36E-03
3.49E-03	2.67E-03	1.54E-03	3.49E-03	2.67E-03	1.54E-03	3.49E-03	2.67E-03	1.54E-03
2.86E-03	2.18E-03	1.24E-03	2.85E-03	2.17E-03	1.24E-03	2.85E-03	2.17E-03	1.24E-03

REFERENCE :

[1] Batchelor,G.K.(1977), published the paper entitled " The Effect of Brownian Motion on the bulk stress in a suspension of spherical particles" in J.Fluid mech., 83,97-117.

[2] Bender,E.A. (1978) , published the paper entitled "An introduction to mathematical modeling", Wiley, New York.

[3] Buffo,Elwoods and Dyer James S. (1977), published the paper entitled "Model formulation and solutions methods" Wiley Hamilton, New York.

[4] Rath B.K.,Mahapatra,P.K,&Dash D.K,(2017), published the paper entitled " Effect of Volume fraction along with concentration parameter in the dusty incompressible fluid" in 162Advances and Application in Fluid Mechanics , Volume 20, No 1,2017 ,pp 117-125

[5] D.K.Dash & Chand Ram (2014), published the paper entitled "Heat transfer of an axially symmetrical jet mixing compressible dusty fluid" in International J. of Engg. Research & Indu. Appls.

[6] Panda T.C., Mishra S.K., Das A.K.(1999), published the paper entitled "Modelling the turbulent diffusion of SPM in Ekman boundary layer based on 1st order closure" in Acta Ciencia indica, 25m(4), 365-373.

[7] Panda T.C., Mishra S.K., Das A.K.(2001), published the paper entitled " Modelling of laminar diffusion of SPM in Ekman boundary layer" in AMSE, Vol.70 (5), 47-55.

[8] Panda T.C., Mishra S.K., Das A.K.(2001), published the paper entitled " Unsteady parallel flow of particulate suspension" in AMSE, Vol 69(1), 1-13.

[9] Panda T.C., Mishra S.K., Panda K.C.,(2002), published the paper entitled "Diffusion of SPM using two phase flow model" in Int.J.for Numericals methods in Fluids, New York.

[10] Panda T.C., Mishra S.K., Dash D.K.(2005), published the paper entitled "Modeling Dispersion of suspended particulate matter (SPM) in Axi symmetric jet mixing" in Far East J. Appl. Math. 20(3) (2005), 289 – 304.

[11] Rath, B.K., Behera, G.K., and Dash, D.K. (2015). published the paper entitled "Solution of Longitudinal velocity of the fluid and the particle of he dustu fluid with the effect of volume fraction in the incompressible fluid of SPM" in Adv. Appl. Fluid. Mech. 18, 155-162

[12] Dash D.K. & Rath B.K.,(2009) published the paper entitled" Effect of volume fraction in axi-symmetric jet mixing of incompressible flow In cylindrical polar co-ordinates" in Far east journal of applied mathematics, Vol 34, No. 1,2009, Pages 83-94.

[13] Dash D.K. & Rath B.K.,(2008) published the paper entitled "Modelling of boundary layer equation in axisymmetric, incompressible Flow In cylindrical polar foordinates and its simplification" in Impact, J.sci.Tech Volume2(1) 2008,FIJI ISLAND, page 1-8.

[14] Dash,D.K ; Rath,B.K., published a paper entitled "Effect of volume fraction on temperature in axi symmetric jet mixing of incompressible flow in cylindrical polar coordinates " in Applied Science Periodical in volume xiii, No -1 , February 2011