ALMOST ii- NORMAL SPACES

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Abstract. The aim of this paper is to introduced and study a new class of almost normal spaces, called almost ii-normal spaces by using ii-open sets due to Amir A.Mohammed and B.S Abdullah **[1]** and obtained several properties of such a space. Moreover, we obtain some new characterizations and preservation theorems of almost ii-normal spaces.

1.Introduction

In this paper, we introduced the concept of almost ii-normal by using ii-open set due to Amir A.Mohammed and B.S.Abdullah [**1**]and obtained several properties of such a space. In 1970, Singal and Arya [**6**] introduced the concept of almost normal spaces as a generalization of normal - spaces by using regularly closed sets and obtained several properties of such a space. Recently, Hamant Kumar and M.C.Sharma [**2**] introduced a new class of spaces, namely almost γ-normal and mildly γ-normal spaces are weaker form of γ- normal spaces .We show that these normal spaces, namely almost γ -normal and mildly γ-normal spaces are regularly open hereditary are give relationship of almost γ-normal and mildly γ -normal spaces and some known weaker form of almost normal and mildly normal spaces and obtain characterizations and preservation theorems of almost γ-normal and mildly γ -normal spaces . We introduced the concepts of gii- closed, rgii-closed, regularly ii closed sets, M-ii-closed, M-ii- open, almost ii-irresolute functions. Moreover, we obtain some new characterizations and preservation theorems of almost ii-normal spaces. Throughout this paper, (X, τ), (Y, σ) spaces always mean topological spaces X, Y respectively on which no separation axioms are assumed unless explicitly stated.

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2. Preliminaries

2.1. Definition. A subset A of a topological space X is called

1.**α-closed** [5] if $cl(int(cl(A))) \subseteq A$.

2. **gα-closed** [**4**]ifα-cl(A) ⊂U, whenever A ⊂U, and U is α-open in X .

3. **rgα-closed** [**7**]if α-cl(A) ⊂ U, whenever A ⊂U, and U is regularly α-open in X.

4. **ii-closed** [**1**] if int(cl(A)) ∩ cl(δ-int(A)) ⊆ A.

5. **gii-closed** if ii-cl(A) ⊂U, whenever A ⊂U and U is ii-open in X.

6. **regularly ii-open** if there is a regularly open set U such that U ⊂ A ⊂ii-cl(U) .

7.**rgii-closed** if ii-cl(A)⊂ U,whenever A ⊂U,and U is regular ii-open in X.

The complement of α-closed (resp. gα-closed, rgα-closed,ii-closed, gii-closed, rgii-closed) set is said to be **α-open** (resp.**gα-open**,**rgα-open**,**ii-open, gii-open**, **rgii-open**) set.The complement of regularly ii-open set is said to be **regularly ii-closed** set.

Definitions stated in preliminaries and above, we have the following diagram:

$$\begin{array}{ccccccc} \text{closed} & \Rightarrow & \alpha\text{-closed} & \Rightarrow & g\alpha\text{-closed} & \Rightarrow & rg\alpha\text{-closed} \\ & & \Downarrow & & \Downarrow & & \Downarrow \\ & & \text{ii-closed} & \Rightarrow & \text{gii-closed} & \Rightarrow & \text{rgii-closed}. \end{array}$$

However the converses of the above are not true may be seen by the following examples .

2.2. Example. Let X = {a,b,c,d } and τ= { ϕ, {a}, {b}, {a,b},{a,b, c},X}.Then the set A = {c} is α-closed set as well as ii-closed set but not closed set in X .

2.3. Example. Let X = { a, b, c , d, e } and τ = { ϕ, {a}, {d}, {e} , { a, d},{ a, e}, {d, e}, {a, d , e}, X }. Then the set A = {a, d, e} is rgα-closed set as well as rgii-closed set but not gα-closed set and not gii-closed set in X .

2.4.Example. Let X = { a, b, c , d } and τ = { ϕ,{a}, {b},{ a, b},{ a, b ,c} , X }. Then the set A = {c} is gii-closed set but not closed set in X .

3. Almost ii- Normal Spaces

3.1. Definition. A topological space X is said to be **almost - normal** [**6**] (resp.**almost ii-normal**) if for every pair of disjoint sets A and B, one of which is closed and other is regularly closed, there exist disjoint open(resp.ii-open)sets U and V of X such that A ⊂ U and B ⊂ V.

3.2. Example. Let X = { a, b, c, d } and τ = { ϕ, {a}, { b}, {a, b}, {c, d},{a, c, d}, {b, c, d}, X }. Then A = {b} is closed and B = {a} is regularly closed sets there exist disjoint open sets U = {b, c, d} and V = { a} of X such that A $\subset$ U and B $\subset$ V. Hence X is almost normal as well as almost ii-normal because every open set is ii-open set.

By the definitions and examples stated above, we have the following diagram :

normal $\Rightarrow$ almost - normal $\Rightarrow$almost ii-normal .

3.3.Lemma. A subset A of a topological space X is rgii-open iff F $\subset$ ii-int (A) whenever F is regularly closed and F $\subset$ A.

3.4.Theorem. For a topological space X, the following are equivalent :

(a) X is almost ii- normal.

(b) For every closed set A and every regularly closed set B, there exist disjoint gii-open sets U and V such that A $\subset$U and B $\subset$V.

(c) For every closed set A and every regularly closed set B, there exist disjoint rgii-open sets U and V such that A $\subset$U and B $\subset$V .

(d) For every closed set A and every regularly open set B containing A, there exists a gii-open set U of X such that A $\subset$U $\subset$ii-cl(U)$\subset$ B .

(e) For every closed set A and every regularly open set B containing A, there exists a rgii-open set U of X such that A $\subset$U $\subset$ii-cl(U) $\subset$ B .

(f) For every pair of disjoint sets A and B, one of which closed and other is regularly closed, there exist ii-open sets U and V such that A $\subset$ U and B $\subset$ V and U $\cap$V = ϕ .

Proof. (a) $\Rightarrow$ (b), (b) $\Rightarrow$ (c), (d) $\Rightarrow$ (e), (c) $\Rightarrow$ (d) , (e) $\Rightarrow$ (f) and (f) $\Rightarrow$ (a).

(a) $\Rightarrow$ (b). Let X be a almost ii-normal. Let A be a closed and B be a regularly closed sets in X. By assumption, there exist disjoint ii-open sets U and V such that A $\subset$ U and B $\subset$V. Since every ii-open set is gii-open set , U, V are gii- open sets such that A $\subset$U and B $\subset$V .

(b) $\Rightarrow$ (c). Let A be a closed and B be a regularly closed sets in X. By assumption, there exist disjoint gii-open sets U and V such that A $\subset$ U and B $\subset$V. Since every gii-open set is rgii-open set, U, V are rgii-open sets such that A $\subset$U and B $\subset$V .

(d) $\Rightarrow$(e). Let A be any closed set and B be any regularly open set containing A. By assumption, there exists a gii-open set U of X such that A $\subset$U $\subset$ii-cl(U)

$\subset$ B . Since every gii-open set is rgii-open set , there exists a rgii- open set U of X such that A $\subset$ U $\subset$ ii-cl(U) $\subset$ B .

(c) $\Rightarrow$(d). Let A be any closed set and B be a regularly open set containing A. By assumption, there exist disjoint rgii-open sets U and W such that A $\subset$U and X – B $\subset$W. By **Lemma 3.3**, we get, X – B $\subset$ ii-int(W) and ii-cl(U) $\cap$ ii-int(W) = ϕ . Hence, A $\subset$ U $\subset$ ii-cl(U) $\subset$ X –ii-int(W) $\subset$ B .

(e) $\Rightarrow$(f). For any closed set A and any regularly open set B containing A . Then A $\subset$ X – B and X – B is a regularly closed. By assumption, there exists a rgii-open set G of X such that A $\subset$ G $\subset$ ii-cl(G) $\subset$ X– B. Put U = ii-int(G),V = X –ii-cl(G).Then U and V are disjoint ii-open sets of X such that A $\subset$U and B $\subset$ V.

(f) $\Rightarrow$ (a) is obvious.

3.5.Definition. A function f : X $\rightarrow$Y is called **rc-continuous** [**3**] if for each regular closed set F in Y, f^{-1}(F) is regularly closed in X .

3.6.Definition. A function f : X $\rightarrow$Y is called **M -ii-open** (resp. **M -ii-closed**) if f (U)$\in$iiO(Y) (resp. f(U) $\in$iiC(Y)) for each U $\in$iiO(X) (resp.U $\in$iiC(X)).

3.7.Definition.A function f : X $\rightarrow$ Y is called **almost ii-irresolute** if for each x $\in$X and each ii-neighbourhood V of f (x), ii-cl(f^{-1}(V)) is a ii-neighbourhood of x.

4.Preservation Theorems

4.1.Theorem. If f : X $\rightarrow$ Y is continuous M-ii-open rc-continuous and almost ii-irresolute surjection from an almost ii-normal space X onto a space Y, then Y is almost ii-normal.

Proof. Let A be a closed set and B be a regularly open set containing A. Then by rc-continuity of f, f^{-1}(A) is a closed set contained in the regularly open set f^{-1}(B). Since X is almost ii-normal, there exists a ii-open set V in X such that f^{-1}(A) $\subset$V $\subset$ii-cl (V) $\subset$ f^{-1}(B) by **Theorem 3.4**. Then,

f(f^{-1}(A)) $\subset$ f(V) $\subset$ f(ii-cl(V)) $\subset$ f(f^{-1}(B)). Since f is M-ii-open and almost ii-irresolute surjection, it follows that f(V)$\in$iiO(Y) , we obtain

A $\subset$ f (V) $\subset$ ii-cl(f (V)) $\subset$ B. Then again by **Theorem 3.4** ,Y is almost ii- normal.

4.2.Theorem. If f : X $\rightarrow$ Y is rc-continuous M -ii-closed map from an almost ii-normal space X onto a space Y, then Y is almost ii- normal

.**Proof.** Easy to verify.

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