NUMERICAL SOLUTION OF MATHEMATICAL MODELING AND ANALYSIS OF UNSTEADY BLOOD FLOW IN A PARALLEL PLATE CHANNEL WITH TRANSVERSE MAGNETIC FIELD AND HEAT SOURCE

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Abstract

In this study, we have developed a mathematical model to describe the behavior of unsteady blood flow through a parallel plate channel when subjected to a constant transverse magnetic field and influenced by a heat source. The model includes analytical expressions for the axial velocity, temperature distribution, and the normal velocity of the blood. These expressions are functions of two variables, y and t, and are derived to transform the system of partial differential equations into a set of ordinary differential equations, as per the conditions defined in our model. Our analysis focuses on investigating the impact of various parameters, namely the Hartmann number, heat source parameter, and Prandtl number, on the axial velocity, temperature distribution, and the normal velocity of the blood flow. To facilitate a better understanding of the problem, we have presented numerical solutions in graphical form. Overall, this mathematical model provides a simplified representation of the axial velocity, temperature distribution, and normal velocity of blood flow. It is intended to be a valuable resource not only for researchers in the field of Physiological fluid dynamics but also for medical practitioners seeking insights into blood flow behavior."

1. INTRODUCTION

Numerous researchers have delved into the study of blood flow dynamics, with a particular emphasis on its interaction with magnetic fields in recent decades. The application of magnetohydrodynamics (MHD) to physiological flow problems has garnered increasing interest due to its potential medical applications. Several studies have highlighted blood as an electrically conducting fluid [1-4]. In the presence of an external magnetic field, the electromagnetic force, known as the Lorentz force, acts on the blood, opposing its motion. Consequently, this magnetic field can be harnessed for therapeutic purposes, including the treatment of cardiovascular diseases and conditions marked by accelerated blood circulation, such as haemorrhages and hypertension. Biological systems, in general, experience the influence of external magnetic fields on blood flow within the human arterial system. Multiple mathematical models have been explored by various researchers to elucidate the nature of blood flow under the influence of these external magnetic fields. Tzirtzilakis [5] investigated a mathematical model of bio magnetic fluid dynamics (BFD) designed to describe Newtonian blood flow under the influence of a magnetic field. This model aligns with the principles of ferro dynamics and magnetohydrodynamics, taking into account both the magnetization and electrical conductivity of blood. Ramamurthy and Shanker [6] studied magnetohydrodynamic effects on blood flow through porous channels, considering blood as a Newtonian and conducting fluid. Das and Saha [7] explored arterial MHD pulsatile blood flow subjected to periodic body acceleration, while Madhu et al. [8] investigated blood flow in very narrow capillaries influenced by a transverse magnetic field, assuming the existence of a lubricating layer between red blood cells and the tube wall. Rathod and Tanveer [9] delved into the pulsatile flow of blood, treated as a couple stress fluid, through a porous medium under the influence of periodic body acceleration and a magnetic field. Singh and Rathee [10] provided an analytical solution for a two-dimensional blood flow model with variable viscosity through an indented artery, considering the effect of low-density lipoproteins and a magnetic field, emphasizing the heightened risk for circulatory issues among hypertensive patients. Dulal and Ananda [11] analyzed the effect of a uniform transverse magnetic field on the pulsatile motion of blood through an axi-symmetric tube, and Zamir and Roach [12] explored blood flow downstream of a two-dimensional bifurcation with symmetrical steady flow. Heat transfer within biological systems holds significance in diagnostic and therapeutic applications involving temperature fluctuations. The cardiovascular system, in particular, responds sensitively to environmental changes, adapting blood flow characteristics to meet the body's shifting demands. In addition to its role in transporting oxygen, metabolites, and other substances, blood flow modulates heat transfer within the body. Adhikary and Misra [13] presented an exact solution for oscillatory fluid flow and heat transfer in a porous oscillating channel under the influence of an external magnetic field. Lagendijk [14] investigated the impact of blood flow in large vessels on temperature distribution in hyperthermia, and Wang [15] modelled blood flow in small tubes using a two-fluid model, considering fully developed flow with constant heat flux convective heat transfer.

In this current study, we present a mathematical model that describes unsteady blood flow through an extremely narrow parallel plate channel influenced by a heat source and an external transverse magnetic field. Our work builds upon the extensive study by Madhu et al. [8] by incorporating heat transfer under the specific conditions outlined in our model. The primary objective of our research is to derive analytical expressions for the axial velocity, temperature distribution, and normal velocity, utilizing novel boundary conditions and converting the system of partial differential equations into a system of ordinary differential equations. Additionally, we aim to investigate the impact of the magnetic field (Hartmann number (Ha)), heat source parameter (α), and Prandtl number (Pr) on the axial velocity, temperature distribution, and normal velocity. Consequently, our mathematical model provides a straightforward representation of these key parameters in blood flow, offering valuable insights not only to researchers in the field of Physiological fluid dynamics but also to medical practitioners.

2. MATHEMATICAL FORMULATION

In the scenario illustrated in Figure 1, we are examining the flow between two parallel plates that are non-conducting. In this analysis, we assume that blood behaves as a Newtonian fluid, displaying properties such as incompressibility, homogeneity, and constant viscosity. Furthermore, we introduce the influence of a magnetic field, which is applied perpendicular to the direction of blood flow.

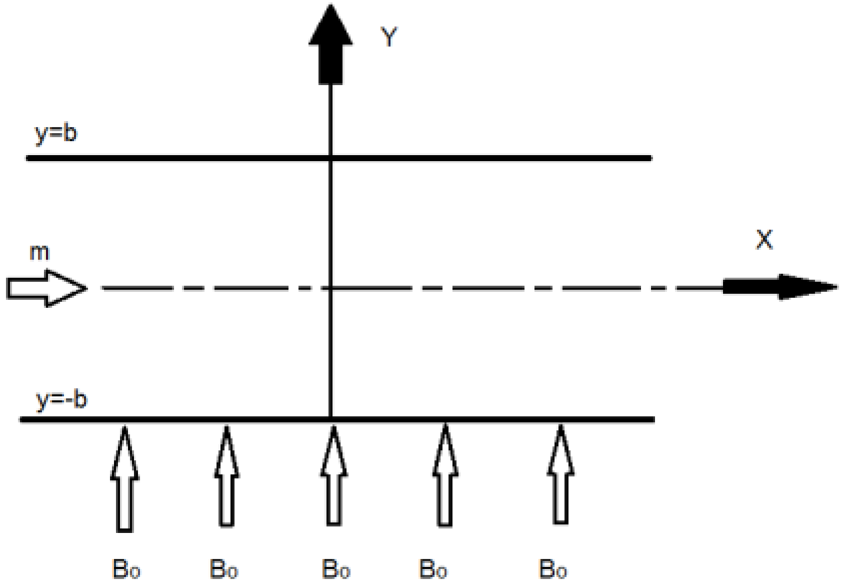

Figure 1: Geometry of the problem

In this context, we can describe the velocity components as 'u' and 'v,' corresponding to the flow in the axial (x-direction) and normal (y-direction) directions, respectively, at a given time 't' within the flow field. When considering the presence of a transverse magnetic field, the governing equations for the two-dimensional boundary layer can be expressed as follows:

$$\frac{\partial u}{\partial t}+\frac{1}{\rho}\frac{dp}{dx}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho}+g\beta(T-T_0) \tag{1}$$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{2}$$

$$\frac{\partial T}{\partial t}=\frac{K'}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho C_p}(T-T_0) \tag{3}$$

Introduce the following non-dimensional variables

$$x^*=\frac{x}{b},y^*=\frac{y}{b},u^*=\frac{u}{\left(\frac{m}{2\rho b}\right)},v^*=\frac{v}{\left(\frac{m}{2\rho b}\right)},$$

$$t^*=\frac{t}{\left(\frac{\rho b^2}{\mu}\right)},h^*(x,t)=\frac{\frac{dp}{dx}}{\left(\frac{\mu m}{2\rho^2 b^3}\right)},\theta^*=\frac{\theta(2\rho^2 b^3)}{\mu m} \tag{4}$$

Substitute these values into equations (1) to (3), we get

$$\frac{\partial u}{\partial t}+h=\frac{\partial^2 u}{\partial y^2}-Ha^2 u+g\beta\theta \tag{5}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\vartheta P_r}\frac{\partial^2\theta}{\partial y^2}+\frac{N}{\vartheta P_r}\theta \quad (7)$$

From equation (7), we can discern that the temperature distribution, denoted as θ, possesses a first derivative with respect to time 't'. Building upon this insight and utilizing the separation of variables technique for solving partial differential equations, we arrive at the following equation.

This equation leads us to the observation that the solution can be expressed as $\theta_1 = e^{-\lambda^2 t}$.

In a similar manner, the axial velocity 'u' follows the same concept, and as a result, the solution to the problem takes the form outlined in section 3. The boundary conditions are defined as follows:

$$\begin{aligned}\theta = e^{-\lambda^2 t}, u = e^{-\lambda^2 t} \; at\; y = -1\\ \theta = 0, \quad u = 0 \quad at\; y = 1\end{aligned} \quad (8)$$

3. SOLUTION OF THE PROBLEM

The equations (5) to (7) with corresponding boundary conditions (8) solved by Crank- Nicolson finite difference method. We divide the range of the problem into finite number n equal sub intervals of fixed width k.To prove convergence of finite difference scheme, the computation is carried out for slightly changed value of k by running same program. No significant change was observed in the values of u and θ. Also, after each cycle of iteration the convergence check is performed, the tolerance is set at 10^{-6} is satisfied at all points. Thus, it is concluded that the finite difference scheme is convergent and stable.

4. RESULTS AND DISCUSSION

The flow investigation in this study is focused on examining the impact of specific factors, namely the heat source and magnetic field, on the flow dynamics. The primary objective of this research is to elucidate the roles played by various parameters, including the heat source parameter, magnetic field strength (Hartmann number), Prandtl number, and decay parameter, in influencing temperature distribution, axial velocity, and normal velocity. To comprehensively assess these effects, numerical algorithms have been developed to numerically evaluate the analytical findings.

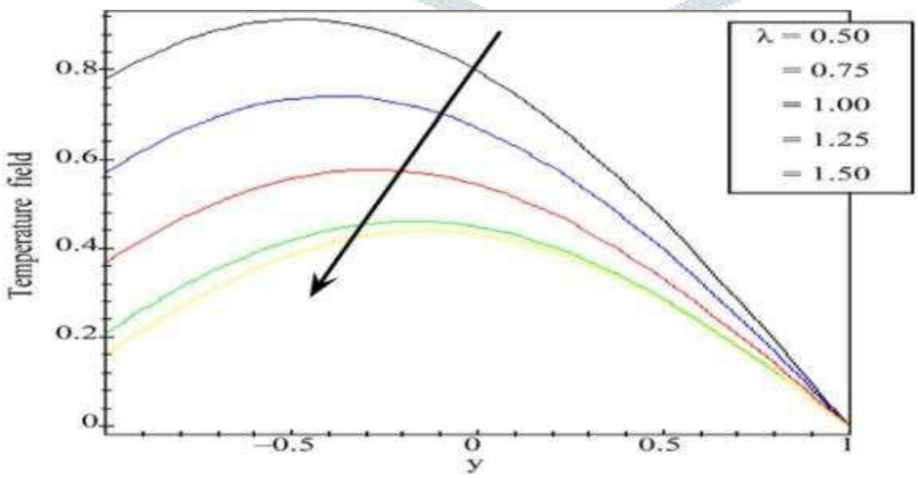

Figure 2. Temperature distribution for different values of Prandtl number

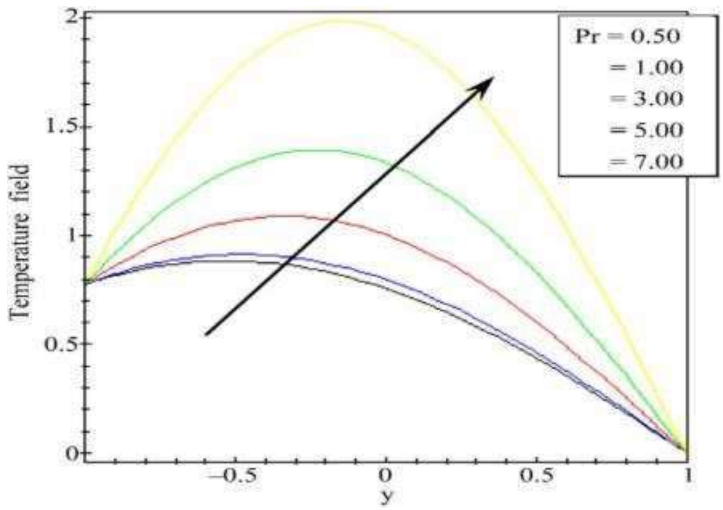

Figure 3.Temperature distribution for different values of heat source

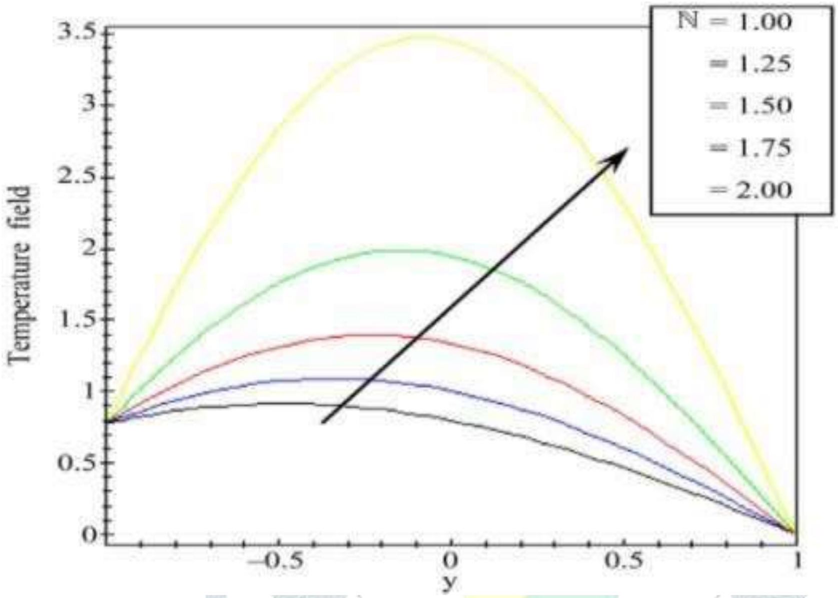

Figure 4. Temperature distribution for different values of decay parameter

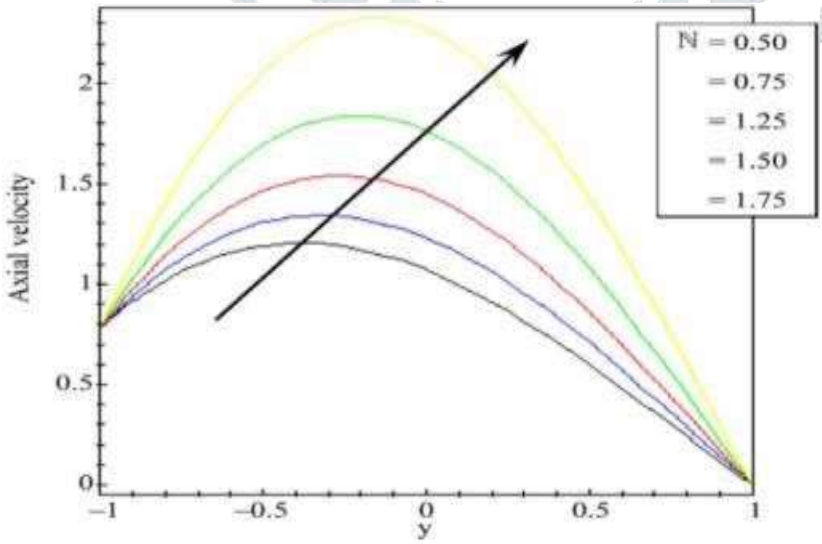

Figure 5. Axial velocity distribution for different values of heat source

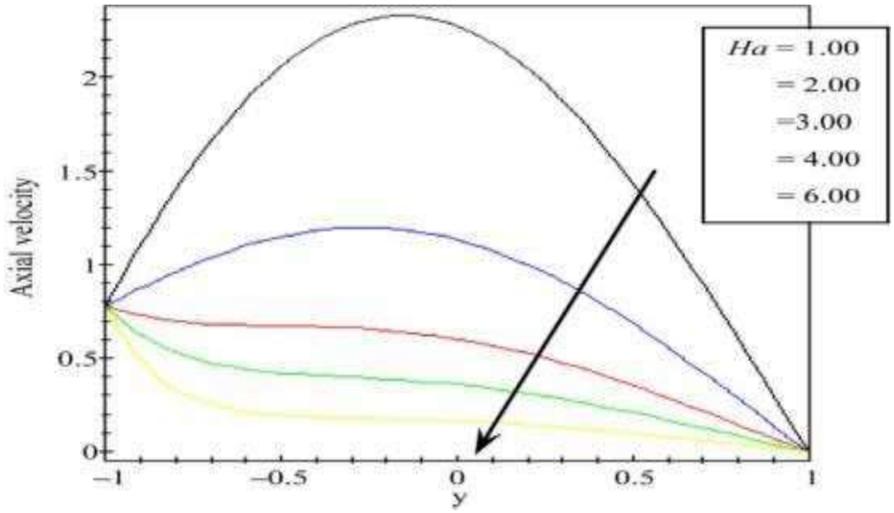

Figure 6. Axial velocity distribution for different values of Hartmann number

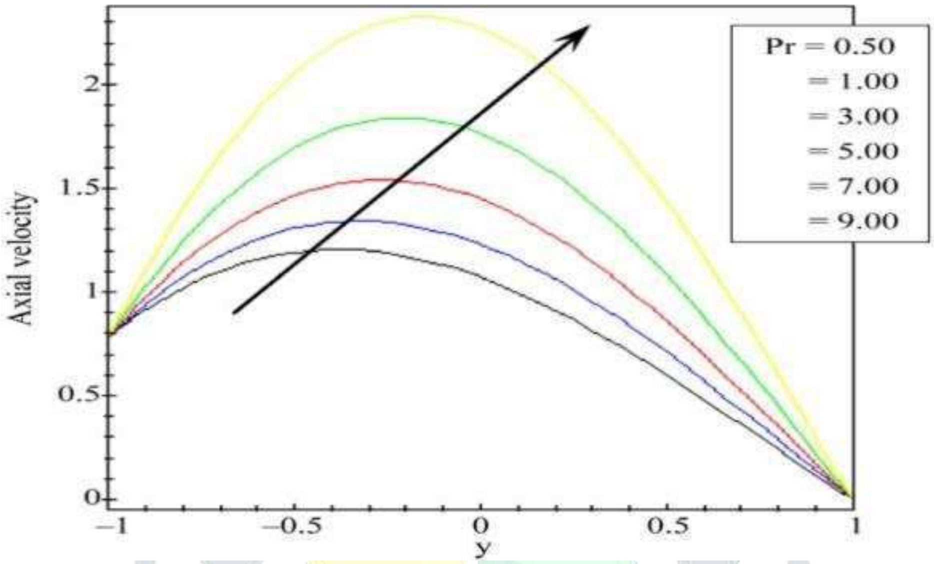

Figure 7. Axial velocity distribution for different values of Prandtl number

In Figure 2, we delve into the variation of temperature distribution as a function of the spatial coordinate (y) at a fixed time (t = 1.0) with specific values set for other parameters (λ = 0.5, ν = 0.5, Pr = 1.0). Multiple curves are plotted for different values of the heat source parameter (Λ = 1.00, 1.25, 1.50, 1.75, 2.00). It is evident that, for a given y value, the temperature field experiences an increase as the heat source parameter (Λ) is raised. Additionally, the temperature field exhibits a peak at y = 0 before gradually decreasing.

Moving on to Figure 3, we investigate the temperature field distribution for various Prandtl numbers (Pr = 0.50, 1.00, 3.00, 5.00, 7.00) while maintaining constant values for other parameters (t = 1.00, λ = 0.50, ν = 0.50, Λ = 1.00). The results show that the temperature field increases with higher Prandtl numbers (Pr). The influence of Prandtl number on the temperature field is akin to that of the heat source parameter.

Figure 4 explores the impact of the decay parameter on the temperature field distribution under specific conditions (t = 1.00, Λ = 1.00, ν = 0.50, Pr = 1.00). It is evident that the temperature field diminishes with an increase in the decay parameter. The most pronounced effect of the decay parameter is observed at y = -1, whereas there is minimal impact on the temperature distribution around y = 1.

In Figure 5, we present the axial velocity distribution for various values of the heat source parameter (∞ = 0.50, 0.75, 1.00, 1.25, 1.50) at a specific time (t = 1.0), with fixed parameters λ = 0.5, ν = 0.5, β = 0.50, g = 9.81, h = 0.50, Ha = 1.00, and Pr = 1.00. Notably, we observe that the axial velocity increases as the heat source parameter (∞) is elevated. Specifically, the axial velocity initially rises from y = -1, attains its maximum at y = 0, and then decreases until y = 1.

In Figure 6, we examine the impact of the magnetic field on the axial velocity for different values of the Hartmann number (Ha = 1.00, 2.00, 3.00, 4.00, 6.00) at t = 1.00, with constant parameters λ = 0.50, ν = 0.50, β = 0.50, g = 9.81, h = 0.50, ∞ = 1.50, and Pr = 1.00. Notably, the magnetic field induces a decrease in the axial velocity. Specifically, for Ha = 1.00, the axial velocity increases from y = -1, reaches its maximum at y = 0, and then decreases until y = 1. However, at Ha = 6.00, the axial velocity exhibits a continuous decrease along the spatial coordinate y.

Figure 7 displays the influence of the Prandtl number on the axial velocity distribution at t = 1.00, with constant parameters λ = 0.50, ν = 0.50, β = 0.50, g = 9.81, h = 0.50, ∞ = 1.00, and Ha = 1.00. It is evident that the axial velocity increases with an increase in the Prandtl number (Pr).

In this current investigation, we introduce a mathematical model to describe the unsteady flow of blood through an extremely narrow parallel plate channel, taking into account both heat generation and the influence of an external transverse magnetic field. This study builds upon the comprehensive work conducted by Madhu et al. [8] and extends their findings to encompass heat transfer under the specific conditions outlined in our model. Notably, the presence of the magnetic field and the heat source significantly impacts the flow characteristics.

Our mathematical model provides a straightforward representation of the axial velocity, temperature distribution, and normal velocity of blood flow. We obtain analytical expressions for these variables by considering that they depend solely on the spatial coordinate (y) and time (t), in addition to the corresponding boundary conditions. This transformation allows us to convert the governing partial differential equations into a set of ordinary differential equations.The temperature field within the channel exhibits an increase when the heat source parameter and Prandtl number are raised, while it decreases with an increase in the decay parameter.As for the axial velocity of the blood flow, it experiences an augmentation with higher values of the heat source parameter and Prandtl number. Conversely, an increase in the Hartmann number and decay parameter leads to a reduction in the axial velocity.

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