# Equilibrium Point in The Photogravitational Non-Planar Restricted Three Body Problem

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**Abstract.** In photogrvitational restricted three body problem there exist equilibrium points which, in addition to the five coplanar such points of the classical problem, lie out of the orbital plane. Radzievskii found two equilibrium points on OXZ plane symmetrical with respect to the orbital plane. We have found equilibrium points which lie out of the orbital plane, when both primaries are radiating.

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## 1. INTRODUCTION

In the photogravitational restricted three body problem there exist equilibrium points which, in addition to the five coplanar such points of the classical problem, lie out of the orbital plane.

The existence of these points was first pointed out by Radzievskii [1] who primarily studied the cases of sun-planet-particle. He found two equilibrium points, $L_6$ and $L_7$ on the OXZ plane symmetrical with respect to the orbital plane. Chernikov [2] added some notes in the case where the Poynting-Robertson effect is considered.

The stability of these points was first studied in the solar problem by Perezhogin[3], [4] in the whole range of existence when the smaller body is considered non-luminous.

In this paper we find the existence of out of plane equilibrium points. We suppose that both primaries are radiating. These points lie on the OXZ plane and appear in pairs with members symmetrical with respect to the OX-axis.

# 2. Out Of Plane Equilibrium Points

In a rotating, barycentric, dimensionless coordinate system with the two radiating bodies on the OX-axis, the equations of motion for the photogratival restricted three-body problem take the form

$$\ddot{X} - 2\dot{Y} = X - \frac{Q_1(X+\mu)}{r_1^3} - \frac{Q_2(X+\mu-1)}{r_2^3}$$

$$\ddot{Y} + 2\dot{X} = Y\left(1 - \frac{Q_1}{r_1^3} - \frac{Q_2}{r_2^3}\right)$$

$$\ddot{Z} = Z\left(-\frac{Q_1}{r_1^3} - \frac{Q_2}{r_2^3}\right)$$

where

$$Q_1 = q_1(1-\mu),\ Q_2 = q_2\mu$$

$$r_1^2 = (X+\mu)^2 + Y^2 + Z^2,\ r_2^2 = (X+\mu-1)^2 + Y^2 + Z^2$$

and $q_1, q_2\ (q_1 \le 1,\ q_2 \le 1)$ the radiation pressure constants corresponding to the bodies with masses $1-\mu$ and $\mu$ $\left(\mu \le \frac{1}{2}\right)$.

We suppose $\overline{R}_o = (X_0, Y_0, Z_0)$ $Z_0 \ne 0$ the position of out of plane equilibrium points. For the existence and position of equilibrium points, the following equations must be satisfied.

$$X_0 - \frac{Q_1(X_0+\mu)}{r_{10}^3} - \frac{Q_2(X_0+\mu-1)}{r_{20}^3} = 0 \qquad (2)$$

$$\frac{Q_1}{r_{10}^3} + \frac{Q_2}{r_{20}^3} = 0,\ Y_0 = 0$$

where

$$r_{10} = r_1(\overline{R_0}),\ r_{20} = r_2(\overline{R_0})$$

$$r_{10}^2 = (X_0+\mu)^2 + Z_0^2$$

$$r_{20}^2 = (X_0+\mu-1)^2 + Z_0^2$$

From equations (2), we have that

$$X_0 = \frac{Q_1}{r_{10}^3} = -\frac{Q_2}{r_{20}^3} \tag{3}$$

or $$X_0^{5/3} - 2(1-2\mu) X_0^{2/3} - \left(Q_1^{2/3} - Q_2^{2/3}\right) = 0 \tag{4}$$

These points on the OXZ plane appear in pairs with members symmetrical with respect to the OX-axis under the necessary condition that $Q_1 \; Q_2 < 0$

We define the auxiliary parameter

$$Q = \left(\frac{Q_1}{Q_2}\right)^{1/3} = \left(\frac{q_1(1-\mu)}{q_2\mu}\right)^{1/3} < 0$$

We suppose Q = -1

Then for $q_2 < -(1-2\mu)/16\mu$ and $\mu \neq \frac{1}{2}$ there exist a pair of equilibrium points named $L_6$, $L_7$

with coordinates $(X_0, Y_0, Z_0) = \left(\frac{1}{2} - \mu, 0, \pm\left[\left(\frac{2q_2\mu}{1-2\mu}\right)^{2/3} - \frac{1}{4}\right]^{1/2}\right)$

Thus, we conclude that out of plane equilibrium points are affected by radiation factor of smaller primary.

## 3.Stability Of Equilibrium Points.

The linearized system of equations about the out of plane equilibrium points take the form

$$F(D)\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \overline{O} \tag{6}$$

where

$$F(D) = \begin{bmatrix} D^2 - A_1 & -2D & -A_2 \\ 2D & D^2 - 1 & 0 \\ -A_2 & 0 & D^2 + A_1 - 1 \end{bmatrix} \tag{7}$$

$$A_1 = 1 + 3\left[\frac{Q_1}{r_{10}^5}(X_0+\mu)^2 + \frac{Q_2}{r_{20}^5}(X_0+\mu-1)\right]$$

$$A_2 = 3Z_0\left[\frac{Q_1}{r_{10}^5}(X_0+\mu)+\frac{Q_2}{r_{20}^5}(X_0+\mu-1)\right]$$

and D stands for differentiation with respect to the time. The characteristics equation of (6) is

$$\lambda^6 + 2\lambda^4 + b\lambda^2 + c = 0 \tag{8}$$

where

$$b = 1 + 9\frac{X_0 Z_0^{\,2}}{r_{10}^2 r_{20}^2}\left[5X_0 - (1-2\mu)\right]$$

$$c = \frac{3X_0 Z_0^2}{r_{10}^2 r_{20}^2}\left[5X_0 - (1-2\mu)\right] \tag{9}$$

As the roots $\lambda_i$ of 8 exist in pairs of opposite numbers, it is clear that the differential system (6) has bounded solutions only in the case that $\lambda_j \in I$, $(j = 1,........6)$ Putting $\lambda^2 = \rho$, we get the equation.

$$\rho^3 + 2\rho^2 + b\rho + c = 0 \text{......} \tag{10}$$

Stability holds for $\rho_i < 0, i = 1,2,3$ which is equivalent to the relations

$$\Delta < 0,\ b > 0,\ c > 0 \tag{11}$$

where

$$\Delta = \left(\frac{3b-8}{9}\right)^3 + \left(\frac{18b-27c-16}{54}\right)^2$$

or equivalently

$$0 < b \le \frac{4}{3} \text{ and } c_m(b) < c < c_M(b) \tag{12}$$

where

$$c_m(b) = \begin{cases} 0, & for\, b \le 1 \\ \frac{2}{27}\left[9b - 8 - (4-3b)^{3/2}\right., & for\ b > 1\left.\right] \end{cases} \tag{13}$$

and

$$c_M(b) = \frac{2}{27}\left[9b - 8 + (4-3b)^{3/2}\right] \tag{14}$$

For $L_6$ and $L_7$ equilibrium points we have $(\mu = 0.5)$

$$b = 1 + 9\frac{X_0^2\, Z_0^2}{r_{10}^2\, r_{20}^2} > 1 \tag{15}$$

$$c = 15\ \frac{X_0^2\ Z_0^2}{r_{10}^2\, r_{20}^2} = b - 1 > 0$$

while from (12) we have

$$b < b^* = \frac{-45 + \sqrt{5145}}{24} \tag{16}$$

Combining relations (15), (16) and the equations which determine Xo and Zo, we get the condition for stability for $\mu = 0.5$

$$\left[q_1^{\ 2/3} - q_2^{\ 2/3}\right]^{8/5} - 2\left(q_1^{\ 2/3} + q_2^{\ 2/3}\right) + \left[q_1^{\ 2/3} - q_2^{\ 2/3}\right]^{2/5} > 0 \tag{17}$$

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