ON RADIO SQUARE DIFFERENCE Dd-DISTANCE NUMBER OF CYCLE RELATED GRAPHS

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Abstract

A Radio square difference Dd-distance labeling of a connected graph G is an injective function f from the vertex set $V(G)$ to N such that for two distinct vertices u and v of $G, D^{Dd}(u,v)+|[f(u)]^2-[f(v)]^2| \geq 1+diam^{Dd}(G)$, where $D^{Dd}(u,v)$ denote the Dd-distance between $u$and $v$and also $diam^{Dd}(G)$denotes the Dd-diameter of G.The radio square difference number of $f, rsdn^{Dd}(f)$ is the maximum label assigned to any vertex of G. The radio square difference number of G, $rsdn^{Dd}(G)$ is the maximum value of f of G. In this paper we find the radio square difference number of some basic graph.

Keywords. Dd-distance, SD labeling, Radio labeling, Radio square difference Dd-distance number.

Introduction

First introduced the idea of graph theory by Euler. By a graph $G=(V(G),E(G))$ we mean a finite undirected graph without loops or multiple edges. Let $V(G)$ and $E(G)$ denotes the vertex set and edge set of G. The order and size of G are denoted by p and q respectively. In 2001,Chatrant et al.[1] defined the concept of radio labelling of G. Radio labelling of graphs is motivated by restrictions inherent in assigning channel frequencies for radio transmitters.

The Dd-distance was introduced by A. Anto Kinsely and P. Siva Ananthi [2]. For a connected graph G, the Dd-length of a connected $u-v$ is defined as $D^{Dd}(u,v)=D(u,v)+\deg(u)+\deg(v)$. The Dd-radius denoted by $r^{Dd}(G)$ is the minimum Dd-eccentricity among all vertices of u and v of G. That is $r^{Dd}(G)=\min\{e^{Dd}(G): v \in V(G)\}$. Similarly the Dd-diameter $D^{Dd}(G)$ is the maximum D^{Dd} eccentricity among all vertices of G. We observe that for any two vertices u, v of G. We have $d(u,v) \leq D^{Dd}(u,v)$. The equality holds if and only if u and v are identical. If G is any connected graph then the Dd-distance is metric on the set of vertices of G. We can check easily $r^{Dd}(G) \leq D^{Dd}(G) \leq 2r^{Dd}(G)$. The concept of square difference labelling was introduced by J. Shiama in 2012.

The Radio Dd-distance was introduced by K. John Bosco and T. Nicholas in 2017. We introduce the concept of radio square difference Dd-distance in this paper.

Definition 1.1.

The concept of radio square difference Dd-distance coloring is a function $f: V(G) \to N$ such that $D^{Dd}(u,v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(G) + 1$ where $diam^{Dd}(G)$ is the maximum color assigned to any vertex of G. It is denoted by $rsdn^{Dd}(G)$.

Theorem 1.2

The Radio square difference Dd-distance number of a cycle graph C_n, $rsdn^{Dd}(C_n) = n$.

Proof:

Let $\{v_1, v_2, \ldots, v_n\}$ be the vertex set, .$E(C_n) = \{v_i v_{i+1}, v_1 v_n / i = 1, \ldots, n-1\}$.

Then $D^{Dd}(v_i, v_{i+1}) = n + 3,\ v_i, v_{i+1} \in V(C_n)$, $D^{Dd}(v_i, v_j) = n, 1 \leq i, j \leq n, i \neq j,$

so $diam^{Dd}(C_n) = n + 3$.

Then radio square difference Dd-distance condition becomes

$D^{Dd}(v_i, v_j) + \left|f(v_i)^2 - f(v_j)^2\right| \geq diam^{Dd}(C_n)$ +1 for any $v_i, v_j \in V(C_n)$,

Therefore, $f(v_i) = i, 1 \leq i \leq n,$

Hence, $rsdn^{Dd}(C_n) = n$.

Theorem 1.3

The radio square difference Dd-distance number of a gear graph K_n, $rsdn^{Dd}(G_n) = n$

Proof:

Let $\{v_0, v_1, \ldots, v_n\}$ and $\{u_1, u_2, \ldots, u_n\}$ are the vertex set where v_0 is the apex vertex and

$E(G) = \{v_0 v_1, v_i u_i / i = 1,2, \ldots, n\}$ be the edge set. $D^{Dd}(v_1, u_1)$ = 2n-1,

$D^{Dd}(v_0, v_1) = 3n + 2$. Then $diam^{Dd}(G_n)$ = 3n+2.

Then radio square difference Dd-distance condition becomes

$D^{Dd}(u,v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(G)$ +1 ,for any pair of vertices (u, v) where $u \neq v$.

Now, $D^{Dd}(v_0, v_1) + |f(v_0)^2 - f(v_1)^2| \geq diam^{Dd}(G_n) + 1. f(u_i) = n + i + 1,\ 1 \leq i \leq n.$

Hence , $rsdn^{Dd}(G_n) = 2n + 1$.

Theorem 1.4

The Dd-radio square difference number of a crown graph, $rsdn^{Dd}(C_n \odot K_1)$ = 2n.

Proof:

Let $V(C_n \odot K_1) = \{v_i, u_j / i, j = 1,2, \ldots, n\}$ be the vertex set and

$E(C_n \odot K_1) = \{v_i\ v_j / \text{i} = 1,2,3, \ldots, \text{n}\}$ be the edge set. Then $D^{Dd}(v_1, v_2) = n + 5$, $D^{Dd}(v_n, u_1) = n + 4$, $D^{Dd}(u_1, u_2) = n + 3$. So $diam^{Dd}(C_n \odot K_1) = n + 5$

Then radio square difference Dd-distance condition becomes

$D^{Dd}(u,v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(G) + 1$

Now, $D^{Dd}(u,v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(C_n \odot K_1) + 1$

for any $u, v \in V\,(C_n \odot K_1),\ u \neq v$.

Then, $f(v_i) = i,\ i \leq i \leq n$ and $f(u_j) = n + j,\ 1 \leq j \leq n$,

Hence, $rsdn^{Dd}(C_n \odot K_1)$ = 2n.

Theorem 1.5:

The radio square difference number of a Helm graph, H_n , $rsdn^{Dd}(H_n)$ =2n+1 , n ≥ 3.

Proof:

Let $\{v_0, v_1, \ldots, v_n\}$ and $\{u_1, u_2, \ldots, u_n\}$ are the vertex set, where v_0 is the apex vertex and $E(G) = \{v_0 v_1, v_i u_i / i = 1,2,\ldots,n\}$ be the edge set.

$D^{Dd}(v_0, v_1) = 2n + 4, D^{Dd}(u_1, u_2) = n + 4. D^{Dd}(v_n, u_1) = n + 6.$

Then $diam^{Dd}(H_n)$ = 2n+4. The radio square difference Dd-distance condition,

$D^{Dd}(u,v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(H_n) + 1$ for any pair of vertices (u, v) where $u \neq v$.

Now, $D^{Dd}(v_0, v_1)$+$|f(v_0)^2 - f(v_1)^2| \geq diam^{Dd}(H_n)$ +1 ,

Then , $f(v_i)$ = i + 1 and $f(u_i) = n + i$ +1, 1 ≤ i ≤ n

Hence, $rsdn^{Dd}(H_n)$ = 2n + 1.

Theorem 1.6

The radio square difference Dd-distance number of a wheel graph W_n,

$$rsdn^{Dd}(W_n) = \begin{cases} n+1, & if\ \ n \leq 7 \\ \frac{1}{2}(3n-5)\,, & if\ n\ is\ odd\ \ n > 7 \\ \frac{1}{2}(3n-4), & if\ n\ is\ even\ n \geq 8 \end{cases}$$

Proof:

Let $V\,(W_n) = \{v_0, v_1, v_2, \ldots, v_n\}$ be the vertex set where v_0 is the central vertex and $E(K_{1,n}) = \{v_0 v_i / i = 1,2,3,\ldots,n\}$ be the edge set. $D^{Dd}(v_0, v_i) = n + 2, D^{Dd}(v_i, v_{i+1}) = 4,$ 1≤ $i \leq n$.so $diam^{Dd}(W_n) = n + 2.$ By radio square difference Dd-distance condition,

$D^{Dd}(u,v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(G) + 1$, for any pair of vertices (u, v) where $u \neq v$

Now, $D^{Dd}(u,v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(W_n) + 1$

Case (a) n is odd

For(v_0, v_1), $D^{Dd}(v_0, v_1) + |f(v_0)^2 - f(v_1)^2| \geq diam^{Dd}(W_n) + 1$.

Then, $f(v_i) = \frac{n-1}{2} + i - 2\ ,\ 1 \leq i \leq n$

Case (b) n is even

Then, $f(v_i) = \frac{n}{2} + i - 2, 1 \leq i \leq n$

Hence $rsdn^{Dd}(K_{1,n}) = \begin{cases} n+1, if \ n < 7 \\ \frac{3}{2}(n-1), \ if \ n \ is \ odd \ n \geq 7 \\ \frac{1}{2}(3n-4), if \ n \ is \ even \ n \geq 8 \end{cases}$

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