ON RADIO SQUARE DIFFERENCE Dd-DISTANCE NUMBER OF CYCLE RELATED GRAPHS

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Abstract

A Radio square difference Dd-distance labeling of a connected graph G is an injective function f from the vertex set V(G) to N such that for two distinct vertices u and v of G, Dd(u, v) + |f(u)|^2 + |f(v)|^2 ≥ 1 + diamd(G), where Dd(u, v) denote the Dd-distance between u and v and also diamd(G) denotes the Dd-diameter of G. The radio square difference number of f, rsdn(G) is the maximum label assigned to any vertex of G. The radio square difference number of G, rsdn(G) is the maximum value of f of G. In this paper we find the radio square difference number of some basic graph.

Keywords. Dd-distance, SD labeling, Radio labeling, Radio square difference Dd-distance number.

Introduction

First introduced the idea of graph theory by Euler. By a graph G = (V(G), E(G)) we mean a finite undirected graph without loops or multiple edges. Let V(G) and E(G) denotes the vertex set and edge set of G. The order and size of G are denoted by p and q respectively. In 2001, Chatrani et al. [1] defined the concept of radio labelling of G. Radio labelling of graphs is motivated by restrictions inherent in assigning channel frequencies for radio transmitters.

The Dd-distance was introduced by A. Anto Kinsely and P. Siva Ananthi [2]. For a connected graph G, the Dd-length of a connected u - v is defined as Dd(u, v) = D(u, v) + deg(u) + deg(v). The Dd-radius denoted by r(G) is the minimum Dd-eccentricity among all vertices of u and v of G. That is r(G) = min{e(G): v ∈ V(G)}. Similarly the Dd-diameter Dd(G) is the maximum Dd eccentricity among all vertices of G. We observe that for any two vertices u, v of G. We have d(u, v) ≤ Dd(u, v). The equality holds if and only if u and v are identical. If G is any connected graph then the Dd-distance is metric on the set of vertices of G. We can check easily r(G) ≤ Dd(G) ≤ 2r(G). The concept of square difference labelling was introduced by J. Shiama in 2012.

The Radio Dd-distance was introduced by K. John Bosco and T. Nicholas in 2017. We introduce the concept of radio square difference Dd-distance in this paper.
Definition 1.1.

The concept of radio square difference $Dd$-distance coloring is a function $f:V(G) \to N$ such that $D^{Dd}(u, v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(G) + 1$ where $diam^{Dd}(G)$ is the maximum color assigned to any vertex of $G$. It is denoted by $rsdn^{Dd}(G)$.

Theorem 1.2

The radio square difference $Dd$-distance number of a cycle graph $C_n, rdsn^{Dd}(C_n) = n$.

Proof:

Let $\{v_1,v_2,\ldots,v_n\}$ be the vertex set, $E(C_n) = \{v_iv_{i+1}, v_1v_n / i = 1,\ldots,n-1\}$.

Then $D^{Dd}(v_i, v_{i+1}) = n + 3, v_i, v_{i+1} \in V(C_n) \land D^{Dd}(v_i, v_j) = n, 1 \leq i, j \leq n, i \neq j$,

so $diam^{Dd}(C_n) = n + 3$.

Then radio square difference $Dd$-distance condition becomes

$D^{Dd}(v_i, v_j) + |f(v_i)^2 - f(v_j)^2| \geq diam^{Dd}(C_n) + 1$ for any $v_i, v_j \in V(C_n)$.

Therefore, $f(v_i) = i, 1 \leq i \leq n$.

Hence, $rsdn^{Dd}(C_n) = n$.

Theorem 1.3

The radio square difference $Dd$-distance number of a gear graph $K_n, rdsn^{Dd}(G_n) = n$

Proof:

Let $\{v_0,v_1,\ldots,v_n\}$ and $\{u_1,u_2,\ldots,u_n\}$ be the vertex set where $v_0$ is the apex vertex and $E(G) = \{v_0v_1, v_iu_i / i = 1,2,\ldots,n\}$ be the edge set. $D^{Dd}(v_1, u_1) = 2n-1$,

$D^{Dd}(v_0, v_1) = 3n + 2$. Then $diam^{Dd}(G_n) = 3n+2$.

Then radio square difference $Dd$-distance condition becomes

$D^{Dd}(u, v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(G) + 1$, for any pair of vertices $(u, v)$ where $u \neq v$.

Now, $D^{Dd}(v_0,v_1) + |f(v_0)^2 - f(v_1)^2| \geq diam^{Dd}(G_n) + 1, f(u_i) = n + i + 1, 1 \leq i \leq n$.

Hence, $rsdn^{Dd}(G_n) = 2n + 1$.

Theorem 1.4

The $Dd$-radio square difference number of a crown graph, $rsdn^{Dd}(C_n \odot K_1) = 2n$.

Proof:

Let $V(C_n \odot K_1) = \{v_i, u_j / i, j = 1,2,\ldots,n\}$ be the vertex set and $E(C_n \odot K_1) = \{v_iv_j/i, j = 1,2,3,\ldots,n\}$ be the edge set. Then $D^{Dd}(v_1, v_2) = n + 5, D^{Dd}(v_n, u_1) = n + 4, D^{Dd}(u_1, u_2) = n + 3$. So $diam^{Dd}(C_n \odot K_1) = n + 5$.

Then radio square difference $Dd$-distance condition becomes
The radio square difference number of a Helm graph, $H_n$, $rsdn^D(H_n) = 2n + 1$, $n \geq 3$.

Proof:

Let $\{v_0, v_1, \ldots, v_n\}$ and $\{u_1, u_2, \ldots, u_n\}$ are the vertex set, where $v_0$ is the apex vertex and $E(G) = \{v_0v_1, v_iu_i/ i = 1,2, \ldots, n\}$ be the edge set.

$D^D(v_0, v_1) = 2n + 4, D^D(u_1, u_2) = n + 4, D^D(v_n, u_1) = n + 6$.

Then $diam^D(H_n) = 2n + 4$. The radio square difference $D^D$-distance condition,

$D^D(u, v) + |f(u)^2 - f(v)^2| \geq diam^D(H_n) + 1$ for any pair of vertices $(u, v)$ where $u \neq v$.

Now, $D^D(v_0, v_1) + |f(v_0)^2 - f(v_1)^2| \geq diam^D(H_n) + 1$.

Then, $f(v_i) = i + 1$ and $f(u_i) = n + i + 1$, $1 \leq i \leq n$.

Hence, $rsdn^D(H_n) = 2n + 1$.

Theorem 1.6

The radio square difference $D^D$-distance number of a wheel graph $W_n$,

$rsdn^D(W_n) = \begin{cases} 
  n + 1, & \text{if } n \leq 7 \\
  \frac{1}{2}(3n - 5), & \text{if } n \text{ is odd } n > 7 \\
  \frac{1}{2}(3n - 4), & \text{if } n \text{ is even } n \geq 8 
\end{cases}$

Proof:

Let $V(W_n) = \{v_0, v_1, v_2, \ldots, v_n\}$ be the vertex set where $v_0$ is the central vertex and $E(K_{1,n}) = \{v_0v_i/i = 1,2,3, \ldots, n\}$ be the edge set. $D^D(v_0, v_i) = n + 2, D^D(v_i, v_{i+1}) = 4, 1 \leq i \leq n$. so $diam^D(W_n) = n + 2$. By radio square difference $D^D$-distance condition,

$D^D(u, v) + |f(u)^2 - f(v)^2| \geq diam^D(W_n) + 1$, for any pair of vertices $(u, v)$ where $u \neq v$.

Now, $D^D(u, v) + |f(u)^2 - f(v)^2| \geq diam^D(W_n) + 1$.

Case (a) $n$ is odd

For $(v_0, v_1)$, $D^D(v_0, v_1) + |f(v_0)^2 - f(v_1)^2| \geq diam^D(W_n) + 1$.

Then, $f(v_i) = \frac{n-1}{2} + i - 2, 1 \leq i \leq n$.

Case (b) $n$ is even
Then, $f(v_i) = \frac{n}{2} + i - 2, \ 1 \leq i \leq n$

Hence $rsdn^{pd}(K_{1,n}) = \begin{cases} 
\frac{n}{2} + 1, \text{if } n < 7 \\
\frac{n}{2}(n - 1), \text{if } n \text{ is odd } n \geq 7 \\
\frac{3}{2}(3n - 4), \text{if } n \text{ is even } n \geq 8
\end{cases}$

Reference:


