Expressions inspired from Rational Number Series

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Abstract
The author had submitted a paper on ‘Rational Number Series’[1]. The basic operation was from the expression \( \frac{(n-1)}{n} \). In this paper many expressions inspired from the Rational Number Series are presented.

Keywords
Expressions, rational number series, binomial expressions;

Introduction
Many expressions of significance are possible from the basic type \( \frac{(n-1)}{n} \). In this paper eleven expressions are presented. Many of them are binomial expressions.

Expression 1
\[
\sum_{n=3}^{\infty} \left( \frac{(n-1)}{n} - \frac{(n-3)}{(n-2)} \right) = \frac{3}{2}
\]

Expression 2
\[
\sum_{n=3}^{m} \left( \frac{(n-1)}{n} - \frac{(n-3)}{(n-2)} \right) = \frac{3m^2 - 7m + 2}{2(m-1)m}
\]

Expression 3
\[
\sum_{n=0}^{\infty} \left( \frac{(n+3)}{(n+4)} - \frac{(n+1)}{(n+2)} \right) = \frac{5}{6}
\]

Expression 4
\[
\sum_{n=0}^{m} \left( \frac{(n+3)}{(n+4)} - \frac{(n+1)}{(n+2)} \right) = \frac{5m^2 + 23m + 18}{6m^2 + 42m + 72}
\]
Expression 5
\[
\sum_{n=0}^{\infty} \left( \frac{(n+2)^k}{(n+3)^k} - \frac{(n+1)^k}{(n+2)^k} \right) = 1 - \frac{1}{\sum_{f=0}^{k}(f)}
\]

Expression 6
\[
\sum_{n=0}^{m} \left( \frac{(n+2)^k}{(n+3)^k} - \frac{(n+1)^k}{(n+2)^k} \right) = \frac{\sum_{t=0}^{k}(t)}{\sum_{s=0}^{k}(s)} m^l 2^{k-l} - \frac{1}{\sum_{s=0}^{k}(s)}
\]

Expression 7
\[
\sum_{n=0}^{m} \left( \frac{(n+1)^k}{(n+2)^k} - \frac{1}{n^k} \right) = \frac{\sum_{t=0}^{k}(t)}{\sum_{s=0}^{k}(s)} m^l \frac{3^{k-s}}{(-2)^{k-s}}
\]

Expression 8
\[
\sum_{n=0}^{m} \left( \frac{1}{(n+2)^k} - \frac{(n-1)^k}{(n+1)^k} \right) = \frac{\sum_{t=0}^{k}(t)}{\sum_{s=0}^{k}(s)} m^l - \frac{k}{m}
\]

Expression 9
\[
\sum_{n=0}^{m} \left( \frac{(n-1)^k}{n^k} - \frac{(n-2)^k}{(n-1)^k} \right) = \frac{\sum_{t=0}^{k}(t)}{\sum_{s=0}^{k}(s)} m^l (-1)^{k-l} - \frac{k}{m^k}
\]

Expression 10
\[
\sum_{n=0}^{m} \left( \frac{(n-2)^k}{(n-1)^k} - \frac{(n-3)^k}{(n-2)^k} \right) = \frac{\sum_{t=0}^{k}(t)}{\sum_{s=0}^{k}(s)} m^l (-2)^{k-l} - \frac{3^k}{k}
\]

Expression 11
\[
\prod_{n=1}^{\infty} \left( \frac{(n+3)}{(n+2)} - \frac{n}{(n+1)} \right) = \frac{1}{3}
\]

Conclusion
In total eleven expressions have been submitted in this paper. The concept of Rational Number Series is wide and has resulted in many expressions of significance. And many of them are binomial expressions.

References