High Compression Ratio for Color Image using Compressive Sensing Technique

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Abstract:- Compressive sensing (CS) is a modern detecting methodology, which compresses the flag being procured at the time of detection. Signals can have sparse or compressible representation either within the unique domain or in a few change domains. Depending on the sparsity of the signals, CS permits us to test the signal at a rate much underneath the Nyquist sampling rate. Also, the changed reproduction calculations of CS can reliably recreate the first signal back from less compressive estimations. For signal recreation, we utilize the l1 minimization strategy. This fact has fortified inquire about intrigued within the utilize of CS in a few areas, such as attractive reverberation imaging, high-speed video acquisition, and ultra-wideband communication.

Keyword:- Image Compression, CS, DTC, DWT, PSNR, CR.

I. INTRODUCTION

The sampling rate utilizing Nyquist-criterion is chosen by the most elevated recurrence part present in the signal, though; the testing rate in CS is administered by the sign sparsity. The CS estimations are non-versatile, i.e., not gaining from past estimations. The brought about less compressive estimations can be effortlessly put away or sent. This gives an impression of compacting the sign at the hour of procurement in particular and subsequently the name 'Compressive Sensing'. After the renowned Shanon examining hypothesis, the presentation of compressive detecting (CS) resembles a significant discovery in the sign preparing a network. CS was presented by Donoho [1], Candès, Romberg, and Tao [2] in 2004. They have built up their numerical establishment. CS is fundamentally utilized for the obtaining of signals which is either inadequate or compressible. Sparsity is the intrinsic property of that signal for which, the entire of the data contained in the signal can be spoken to just with the assistance of few huge segments when contrasted with the all-out length of the signal. Essentially, on the off chance that the arranged parts of a signal decay quickly obeying power law, at that point these signals are called compressible signals, allude. A signal can have sparse/compressible portrayal either in the first domain or in some changed domains like Fourier transform, cosine transform, wavelet transform, and so on.

II. IMAGE COMPRESSION USING DCT AND DWT

Discrete cosine transform:-

Discrete cosine transform is able to represent most of the visually significant information, in the visually significant information, in a typical image, with a limited number of DCT coefficients. It is the building block of JPEG compression standard. It is similar to a Fourier transform considering only the real part of the basis function.

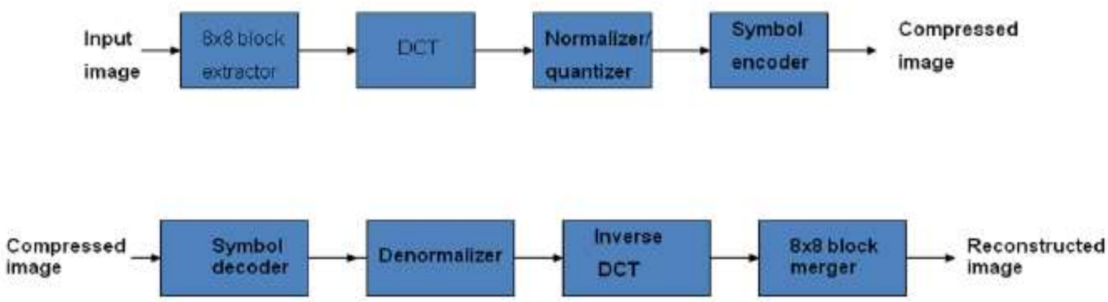
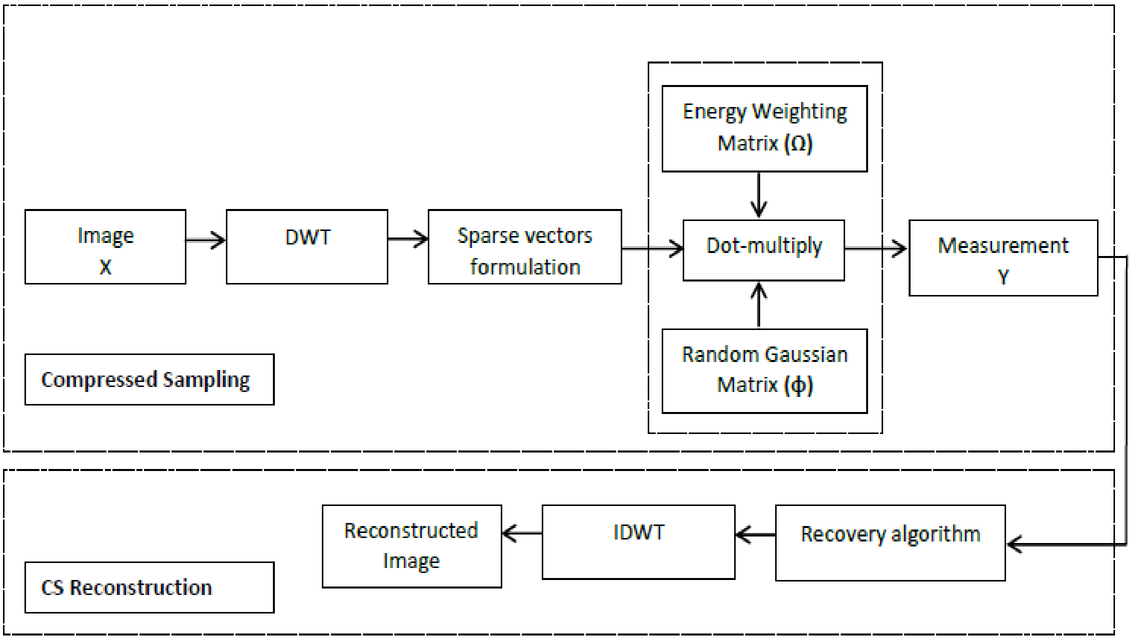


Figure 1: Image compression using DCT

The basic process of image compression using DCT, it is a general overview of JPEG image process. The image is broken into 8×8 block of pixels, then the DCT is applied to each block like left to right and top to bottom. Then each block will be compressed through the quantization. The array of compressed block that constitute is stored in a drastically reduced amount of space. Finally, the image is reconstructed through decompression or inverse discrete cosine transform. Then we get compressed image using DCT.

Discrete wavelet Transform: In the wavelet transform we present a signal in time-frequency form. Basically the wavelet transform is based on small waves, called wavelets, with verifying frequency, limited duration, and zero average values.

Figure 2: Image compression using DWT



Suppose that an image with dimensions M× N, the single stage two-dimensional DWT is calculated using one dimensional DWT. Here we use one dimensional law-pass and high-ass filter for image rows transformed. The filtered output is then down-sampled by a factor of two. The down-sampled filtered image is again transformed column-wise using the same filters followed by a decimation stage resulting into four decompositions: low-pass low-pass, high-pass, high-pass, high-pass low-pass and high-pass high-pass.
The process is repeated for the required levels by transforming the LL sub-band into four decompositions. The wavelet transform can be seen as a cascade of low pass and high pass filter resulting in an approximation signal and several detail signal.
The DWT has a slightly higher computational complexity than the DCT, but exhibits visually more pleasing artifices.
The low –pass wavelet coefficients carry much of the image energy. Since a low-frequency components energy contribution is much larger than the high-frequency components, image greatly depends upon these low-frequency components.

III. PROPOSED METHODOLOGY

Compressive sensing:- Compressive detecting may be a modern innovation where the information is procured in a compressed arrangement. These compressed estimations are at that point encouraged to a few non-linear optimization calculations (moreover called reversal calculation) to create the total flag. This portion is actualized in software. Beneath the reasonable conditions that the first flag and the estimation framework must fulfil, the recreation turns out to have exceptionally small or indeed zero blunder. This depends on the rule that, through streamlining, the sparsity of a signal can be misused to recuperate it from far fewer samples than needed by the Nyquist–Shannon sampling hypothesis. There are two conditions under which recuperation is conceivable. The first is sparsity, which requires the sign to be sparse in some areas, for example, Fourier, DCT, DWT. The subsequent one is ambiguity, which is applied through the isometric property, which is adequate for sparse signs.

The CS technique based on two fundamental principles:

1) The sparse portrayal of the signal of interest on some premise, which is known as the representation basis; and

2) The incoherence between the detecting matrix and the representation basis. The terms detecting (sensing), sparsity, and incoherence will be characterized in the following segment

CS help to recover the sparse signal using only few samples in contract to Nyquist sampling theory, where sampling of the signal is performed at a rate lager than highest frequency present in the signal. Compressive sensing theory overview can be found in [3], [4], [5], [6].
Let us consider x is a 2-dimentional signal or image which can be viewed as an N × 1 column vector R^N with elements x[n], n= 1,2,3.............,N.

So, x to be actual signal of dimensions $N \times 1$; $x \in R^N$ To obtain M non adaptive linear measurements from x, we can multiply x by a fat matrixΦ.
The sampling process is presented as:

$$y = \Phi\, x \qquad (1)$$

Here, Φ is called sampling matrix or measurement matrix with dimensions $M \times N$.And y is compressed measurement vector of dimension $M \times 1$ (See Fig1) where$M \ll N$.

According to conventional Compressive sensing theory [7], [8], for the recovery from few measurements, x should be sparse in some certain transform domain (For example DCT, DWT, curvelet etc).
Suppose that s to be the sparse representation of a non-sparse signal x in transform domain (ψ), then the signal x can be expressed as:

$$x = \sum_{i=1}^{N} s_i \Psi i \quad \text{or} \quad x = \psi s \tag{2}$$

Where s is the sparse $N \times 1$, column vector of weighting coefficients $s_i = \langle x, \Psi_i \rangle = \Psi_i^T x$ and $.T$ denotes transposition, So x and s are equivalent representation of the signal, with x in the time or space domain and s in the Ψ domail.

$$s = \{s_1, s_2 \dots, s_N\} \tag{3}$$

Such that $s_i = \langle x, \Psi_i \rangle$ and Ψ is $N \times N$ basis matrix it is also called universal bass matrix, so we can written as:

$$\Psi_{N \times N} = \{\Psi_1, \Psi_2 \dots \dots, \Psi_N\} \tag{4}$$

Where Ψ_i is the i^{th} column vector of basis matrix.
The signal x is K sparse if it is a linear combination of only k basis vector, that is K of the s_i coefficient are nonzero and $(N - K)$ are zero. The case of interest is when $K \ll N$. The signal x is compressible if the representation has just a few large coefficients and many small coefficients.

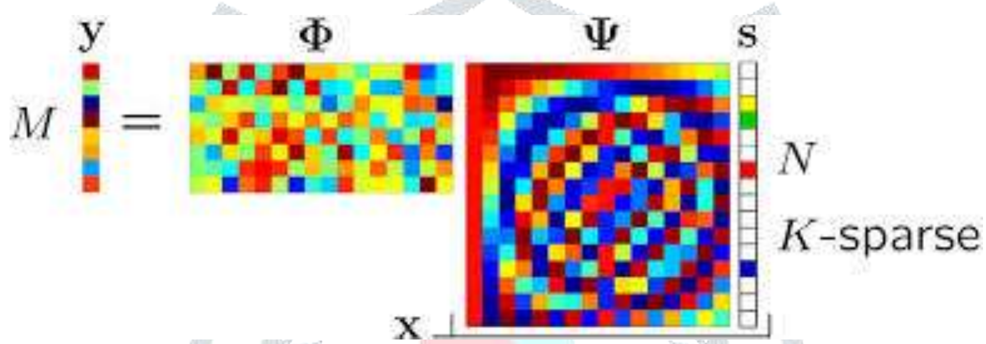


Figure 3: Compressive sensing measurement process with a random Gaussian measurement matrix Φ and basis matrix ψ

The fact that compressible signal are well approximated by K - sparse representations form that foundation of transform coding [9]. In data acquired; the complete set of transform coefficients $\{s_i\}$ is computed by $s = \Psi^T x$; the K largest coefficients are located and the $(N - K)$ smallest coefficients are discarded; and the K values and locations of the largest coefficient are encoded.
Consider a general linear measurement process that computes $M < N$ inner products between x and a collection of vectors $\{\phi_i\}_{i=1}^{M}$ as in $y_j = \langle x, \phi_j \rangle$. Measurements y_j arrange in an $M \times 1$ vector y and the measurement vectors ϕ_j^T as rows in an $M \times N$ matrix Φ.
Then the sampling process becomes:

$$y = \Phi x \tag{5}$$

$$y = \Phi \Psi s = \Theta s \tag{6}$$

Where Θ is the $M \times N$ sensing matix, and *the measurment process is not adaptive, meaning that* Φ is fixed and it is not depends on the signal x. A stable measurement matrix Φ such that the salient information in any K sparse or compressible signal is not damage by the dimensionality reduction from $x \in \mathbb{R}^N$ to $y \in \mathbb{R}^M$ and reconstraction algorithm to recover x from only $M \approx K$ measurements y.

$$1 - \epsilon \leq \frac{\|\Theta v\|_2}{\|v\|_2} \leq 1 + \epsilon \tag{7}$$

The matrix Θ must preserve the lengths of these particular K -sparse vectors. In general the location of K nonzero entries in s are not known. A sufficient condition for a stable solution for both K -sparse and compressible signals is that Θ satisfies for an arbitrary $3K$ -sparse vector v.
This condition, referred to as the restricted isometric property (RIP)[1]. A related condition, referred to incoherence. In addition to the Sparsity principle, CS relies on another principle which is the "Incoherence" between the sensing matrix Φ and the sparsity basis Ψ. The incoherence principal is also related to an equivalent property, which is called restricted isometric property.
Both RIP and Incoherence can be achieved with high probability simply by selecting Φ as a random matrix. We can consider Φ is independent and identically distributed random variable from a Gaussian probability density function with mean zero and variance 1/N [10], [11], [12].
The Gaussian measurement matrix Φ has two interesting and useful properties:

1. The matrix Φ is incoherent with the basis $\Psi = 1$ of delta spikes with high probability.

2. The matrix Φ is universal in the sense that $\Theta = \Phi\Psi$ will be independent and identically distributed Gaussian and thus have the RIP with high probability regardless of the choice of orthogonal basis Ψ.

IV. RECONSTRUCTION ALGORITHM

There are several techniques to recover the signal or image like minimumL0 norm, minimum L1 norm, and minimum L2 norm. The main aim to find the signal sparse coefficient vector in the $(N - M)$ dimensional translated null space H =N (Θ) + s.

Minimum L2 norm reconstruction:

We define the lp norm of the vector s as –

$$\left(\|S\|_p\right)^P = \sum_{i=1}^{N}|s_i|^p \tag{8}$$

The classical approach to inverse problems of this type is to find the vector in the translated null space with the smallest L2 norm by solving

$$\hat{s} = argmin\ \|\acute{s}\|_2 \ \ such\ that\ \ \Theta\, \acute{s} = \text{y}. \tag{9}$$

This optimization has the convenient closed-form solution $\hat{s} = \Theta^T = (\Theta\ \Theta^T)^{-1}y$

Unfortunately, L2 minimizations will almost never a K -sparse solution, returning instead a nonsparse $\hat{s}$ with many nonzero elements.

Minimum L0 norm reconstruction:

Since the L2 norm measures signal energy and not signal sparsity, consider the L0 norm that counts the number of non-zero entries in S. Hence a K -sparse vector has L0 norm equal to K.

Then the modified optimization uncompressed size can recover a K -sparse signal exactly with high probability using only $M = K + 1$ iid Gaussian measurements. Unfortunately, solving this question is numerically unstable and requiring an exhaustive enumeration of all $\binom{N}{k}$ possible location of the nonzero entries in s.

Minimum L1 norm reconstruction:

The optimization based on the L1 norm

$$\hat{s} = \text{argmin}\ \|\acute{s}\|_1 \ \text{such that}\ \ \Theta\, \acute{s} = \text{y}. \tag{10}$$

Now we can exactly recover K -sparse signal and closely approximate compressible signal with high probability using only $M \geq cK\ \log(N/K)$ iid Gaussian measurements [1,2]. This is a convex optimization problem that conveniently reduces to a linear program known as basis pursuit. Other related reconstruction algorithm proposed in [6,7]

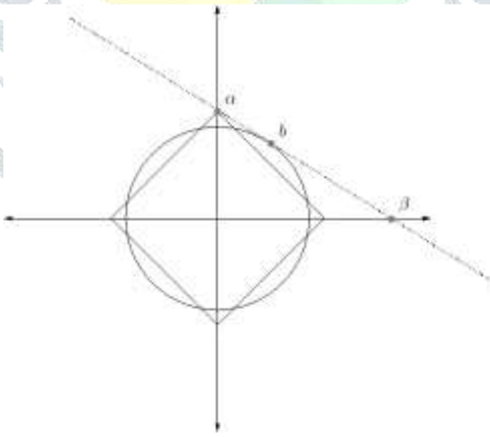


Figure 4: Sparsity and the l1 norm.

Hence, we can say that we get batter reconstruction by L1 norm minimum algorithm. The geometry of the compressive sensing problem in $\mathbb{R}^N$ helps visualize why L2 reconstruction fail to find the sparse solution that can be identified by L1 reconstruction. The set of K -spase vector s in $\mathbb{R}^N$is a highly nonlinear space consisting of all K -dimensional hyperplanes that are aligned with the coordinate axes.

For completeness , and since the paper about image compression, we present in the following a brief discussion of the two popular transforms used in image compression, mainly the DCT and the DWT.

V. EXPERIMENTAL RESULT

DCT, DWT and CS image compression technique is simulated in MATLAB. We applied compression technique and reconstruction algorithm of an image then we get the compressed image, and we find the PSNR and CR of the compressed image.

Compression Ratio:

CR is the ratio of the size of compressed image to the size of original image. Compression ratio should be as high as possible to achieve better compression.

$$\text{Compression ratio} = \frac{\text{Uncompressed size}}{\text{Compressed size}}$$

Mean Square error:

MSE is cumulative difference between the original image and compressed image. MSE should be as minimum as passable for better quality of image.

Peak Signal to noise ratio (PSNR):

PSNR is an important parameter for image compression. It is measurement of the peak error present between the compressed image and original image. For better quality of image PSNR should be as high as possible.

$$PSNR = 10.\log_{10}\left(\frac{MAX_I^2}{MSE}\right)$$

$$PSNR = 20.\log_{10}\left(\frac{MAX_I}{\sqrt{MSE}}\right)$$

$$PSNR = 20.\log_{10}(MAX_I) - 10.\log_{10}(MSE)$$

Here, we compare the PSNR and CR of the reconstructed image and check from which technique we get high PSNR and CR of the compressed image.

a) Original image

b) DCT Compressed Image **c) DWT Compressed Image** **d) CS Compressed Image**

Now we compare all the reconstructed image with the original image and find which image has more information and high quality.

Table 1: Comparison Result

Compression Techniques	PSNR(db)	CR
DCT	41.784	1.26
DWT	40.812	2.31
Compressive sensing	48.872	4.36

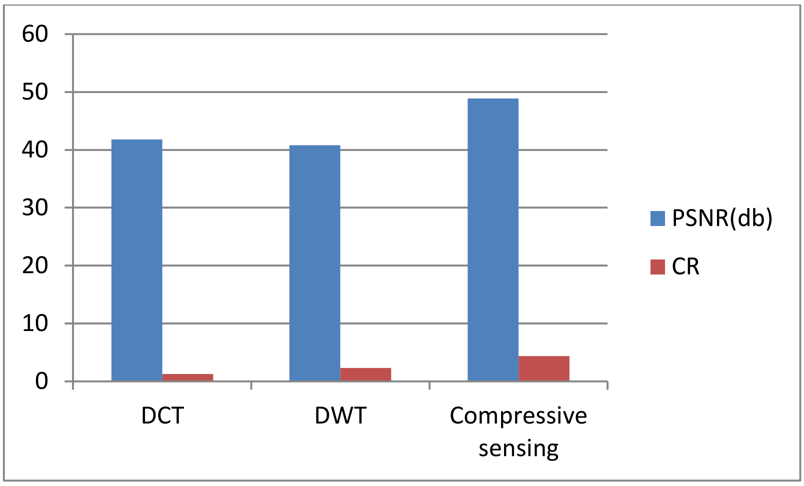


Figure 5: Graphical Representation

VI. CONCLUSIONS

In this paper our main object is to compress the image by using compressive sensing and achieve high quality of image with high PSNR and CR. In the compressive sensing technique we sparse the signal and find the K-nonzero sparse matrix, then applies the compressive sensing technique. Once compression process is completed then we reconstruct the image by using L1 norm minimum reconstruction algorithm. Using L1 norm minimum algorithm we get batter reconstruction of the image. And also we discuss about the other very famous image compression technique like DCT and DWT. We find the compression ratio and PSNR through DCT and DWT and compare those results with the compressive sensing results.

Now, we clearly say that CS is the best technique for image compression as compare to DCT and DWT, because in DCT less compression ratio and bad quality of image. But in DWT we get little bit good quality of image but size of the image is not too reduced. In case of CS we get high compression ratio with high PSNR and the quality of the image is more batter and the image size is very less. CS is a very fast compression technique using CS we get very good quality of image in very few time. Compression algorithms basically depends on the three factors i.e. quality of image, amount of compression and speed of compression.

VII. FUTURE SCOPE

CS has gained a wider acceptance in a shorter time span, as a sampling technique for sampling the signals at their information rate. CS take the advantage of sparsiy or compressibility of the underlying signal to simultaneously sample and compress the signal.

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