A CASE FLOW ORIENTED EOQ MODEL BY DETERIORATING METHOD

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Abstract:

In this paper, we investigate the generalized inventory model is developed to determine an optimal ordering policy for deteriorating items with delayed payments permitted by the supplier under inflation and time discounting. Mathematical models have been derived for obtaining the optimal cycle time and optimal payment time for item so that the annual total relevant cost is minimized.

Keywords: Optimal ordering policy, Optimal cycle time and Optimal payment time.

1.Introduction:

In Inventory Management, Economic Order Quantity (EOQ) is the order quantity that minimizes the total holding costs and Ordering costs. It is the one of the Oldest classical production scheduling models. This model was developed by Ford. W.Harris in 1913,but R.H Wilson, a consultant who applied it extensively and K.Andler are given credit for their in-depth analysis. EOQ refers to the optimum amount of an item that should be ordered at any given point in time, such that the total annual cost of carrying and ordering that item is minimized. EOQ is also sometimes known as Optimum lot size.

2. Model Formulation:

The total time horizon H has been divided into n equal parts of length T so that $T = \frac{H}{n}$.

Hence, the reorder times over the planning horizon H are

$$T_j = j\,(\quad = 0,1,2,\ldots,n-1)$$

Let (t) be the inventory level during the first replenishment cycle. This inventory level during the first replenishment cycle. This inventory level is depleted by the effects of demand and deterioration.

The variation of (t) with respect to t is

$\frac{dI(t)}{dt} = -D - (t), \quad 0 \le t \le T$(2.1)

With the boundary condition $(T) = 0$

The solution of (2.1) can be represented by

$(t) = \frac{D}{\theta}[e^{\theta\left(\frac{H}{n}-t\right)-1})]$, $0 \le t \le T=\frac{H}{n}$(2.2)

Consequently initial inventory after replenishment becomes

$(0) = Q = \frac{D}{\theta}[e^{\theta\left(\frac{H}{n}-t\right)-1})]$, (2.3)

Since there are n replenishments in the entire horizon .

The present value of the total replenishment costs is given by

$C_R = A\sum_{j=0}^{n-1} e^{-jRT}$,

$=A[e^{-0} + e^{-RT} + \cdots + e^{-(n-1)RT}]$

$=A[1 + e^{-RT} + \cdots + e^{-nRT} + e^{RT}]$

$CR\ (1\text{-}e^{-R\left(\frac{H}{n}\right)}) = A[1 - e^{-RH}]$

$CR = \frac{A[1-e^{-RH}]}{1-e^{-R\left(\frac{H}{n}\right)}}$(2.4)

The present value of total purchasing costs is given by

$C_p = c\ \sum_{j=0}^{n-1} I(0)e^{-jRT}$,

$= c\ \sum_{j=0}^{n-1} \frac{D}{\theta}[e^{\theta\left(\frac{H}{n}\right)} - 1]e^{-jRT}$,

$C_p = \frac{cD}{\theta}\left[e^{\theta\left(\frac{H}{n}\right)} - 1\right]\{\frac{1-e^{\{-RH\}}}{1-e\text{^}\{-R(\frac{H}{n})\}}\}$ (2.5)

The present value of the holding costs during the first replenishment cycle is

$$h_1 = \int_0^T I(t)e^{-RT}dt$$

$$h_1 = \frac{hD}{\theta}\left\{\frac{e^{\theta\left\{\frac{R}{n}\right\}} - e^{-R\left\{\frac{H}{n}\right\}}}{\theta + R} + \frac{e^{\left\{-H\frac{R}{n}-1\right\}}}{R}\right\}$$

Hence
The present value of the total holding costs over the time horizon H is given by

$$C_h = \sum_{j=0}^{n-1} h_1 e^{-jRT}$$

$$= \frac{hD}{\theta}\left\{\frac{e^{\theta\left\{\frac{R}{n}\right\}} - e^{-R\left\{\frac{H}{n}\right\}}}{\theta + R} + \frac{e^{\left\{-H\frac{R}{n}-1\right\}}}{R}\right\}\{\frac{1 - e^{\{-RH\}}}{1 - e^{\wedge}\{-R(\frac{H}{n})\}}\}$$

The inventory model has the effect of delay in payments in two distinct types of cases.

Case: 1

Interest Charged and Interest Earned

In this case, the replenishment cycle time T is longer than or equal to M.

Therefore, the delay in payments is permitted and the total relevant cost includes both the interest charged and the interest earned.

The present value of the interest payable during the first replenishment cycle is

$$I_{p^1}^1 = cI_c \int_M^T I(t)e^{-RT}dt$$

$$= \frac{cI_cD}{\theta}\left[\frac{e^{\theta\left(\frac{H}{n}-n\right)-RM} - e^{-R\frac{H}{n}}}{\theta + R}\right] + \frac{cI_cD}{\theta R}\left(e^{-R\frac{H}{n}} - e^{-RM}\right)$$

Hence the present value of the total interest payable over the time horizon H is

$$I_{P^1}^H = \sum_{j=0}^{n-1} I_{P^1}^1 e^{-jRT}$$

$$= I_{P^1}^1 \sum_{j=0}^{n-1} e^{-jRT}$$

$$= \frac{cI_cD}{\theta}\left[\frac{e^{\theta\left(\frac{H}{n}-n\right)-RM} - e^{-R\frac{H}{n}}}{\theta + R}\right] + \frac{cI_cD}{\theta R}\left(e^{-R\frac{H}{n}} - e^{-RM}\right)\left\{\frac{1 - e^{-RH}}{1 - e^{-R^{H/n}}}\right\}$$

The present value of the interest earned during the first replenishment cycles

$$I_{e^1}^1 = cI_e \int_0^T Dte^{-Rt}dt$$

$$= \frac{cI_eD}{R}\left[\frac{1}{R}\left(1 - e^{-R\frac{H}{n}}\right) - Te^{-R\frac{H}{n}}\right]$$

Hence the present value of the total interest earned over the time horizon H is

$$I_{e^1}^H = \sum_{j=0}^{n-1} I_{e^1}^1 e^{-jRT}$$

$$= \frac{cI_eD}{R}\left[\frac{1}{R}\left(1 - e^{-R\frac{H}{n}}\right) - Te^{-R\frac{H}{n}}\right]\left(\frac{1 - e^{-RH}}{1 - e^{-R\frac{H}{n}}}\right)$$

The total present value of the costs over the time horizon H is

$$\boldsymbol{TVC_1(s,n) = C_R + C_P + C_h + I_{P^1}^H - I_{e^1}^H}$$

Case:2 **Interest Earned**

The interest charged during the time period (0,) is equal to zero when

M>Tbecause the supplier can be paid in full at the permissible delay,.

The interest earned in the first cycle is the interest earned during the time period (0, T) plus the interest earned from the cash invested during the time period (T,) after the inventory

Is exhausted at time$T$,and it is given by

$$I_{e_2}^1 = cI_e\left[\int_0^T Dte^{-RT}dt + (M - T)e^{-RT}\int_0^T Ddt\right]$$

$$= \frac{cI_eD}{R^2}\left(1 - e^{-R\frac{H}{n}}\right) - \frac{cI_eDH}{Rn}e^{-R\frac{H}{n}} + cI_eDe^{-R\frac{H}{n}}\left[M\frac{H}{n} - \left(\frac{H}{n}\right)^2\right]$$

The present value of the total interest earned over the time horizon$H$ is

$$I_{e^2}^H = \sum_{j=0}^{n-1} I_{e^2}^1 e^{-jRT}$$

$$= \frac{cI_cD}{R^2}\left(1 - e^{-R\frac{H}{n}}\right) - \frac{cI_eDH}{Rn}e^{-R\frac{H}{n}}$$

$$+cI_eDe^{-R\frac{H}{n}}\left[M\frac{H}{n} - \left(\frac{H}{n}\right)^2\right]\left\{\frac{1 - e^{-RH}}{1 - e^{-R\frac{H}{n}}}\right\}$$

Sincethereplenishmentcost,purchasingcost,andinventoryholdingcostoverthetimeh orizon$H$arethesameas case I,

The total present value of the costs,$TVC_2(s,n)$, isgivenby

$$TVC_2(n) = C_R + C_P + C_h - I_{e^2}^H$$

Sensitivity analysis

Tocharacterizetheeffectsofparameters$M$,,and$R$ontheoptimalsolutions,theset of values of M,θ and R are assumed as M = [30,45,60] days, θ=[0.05,0.1,0.15],and$R$=[0.05,0.1,0.15].The results of sensitivity analysis on parmeters M,θ and R are summarized in tables respectively.

Table 2.2 Sensitivity analysis on *M*

Credit period, *M*	**Numberof Replenishment *n***	**Cycle Time *T*(year)**	**Order Quantity, *Q*(units)**	**Totalpresent Valueofcosts, *TVC*(*n*)($)**
30	31	0.161	156.727	41486.79
45	29	0.172	167.676	41375.247
60	**38***	**0.132***	**127.571***	**41263.846***

*optimal solution

Thus the high value of credit M results in lower of cycle time (T) and order quantity(Q).

Therefore the total present of costs TVC decreases.

Table 2.3 Sensitivity analysis on θ

Deterioration rate, θ	Number of Replenishment n	Cycle Time T (year)	Order Quantity, Q (units)	Total present Value of costs, $TVC(n)$ ($)
0.05	23	0.217	209.834	40913.993
0.10	25	0.2	193.933	41123.573
0.15	**38***	**0.132***	**127.571***	**41263.846***

*optimal solution

Thus deterioration rate(θ) increases then cycle time and order quantity decrease, Therefore the total present value of costs increases.

Table 2.4 Sensitivity analysis on *R*

Net inflation Rate, R	Number of Replenishment n	Cycle Time T (year)	Order Quantity, Q (units)	Total present Value of costs, $TVC(n)$ ($)
0.05	25	0.2	194.909	46233.407
0.10	**38***	**0.132***	**127.571***	**41263.846***
0.15	39	0.128	124.268	37011.133

Thus net discount rate of inflation R increases then cycle time, order quantity, and the total present value of costs all decrease. So the effect of R on TVC is quite significant. It implies that the effect of inflation and time value of money on total present value of costs is significant.

Conclusion

Thus the model considers the effects of deterioration, inflation and permissible

delay in payments, based on the DCF approach we permit a proper recognition of the financial implication of the opportunity cost in inventory analysis. We have also present optimal solution procedure to find the optimal number of replenishment, cycle time and order quantity to minimize the total present value of costs. Numerical results indicates the increase in the permissible delay in both the cycle time and order quantity to increase, and results in positive change in total cost. when the trade credit period increase the retailer earns more by investing the cash from the sales of inventory resulting in lower costs.