

BIPOLAR SINGLE VALUED NEUTROSOPHIC RELATIONS

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Abstract: This paper deals with bipolar single valued neutrosophic relations and their properties with suitable examples along with the characterizations.

Keywords— bipolar single valued neutrosophic relation, bipolar single valued neutrosophic inverse, bipolar single valued neutrosophic identity, bipolar single valued neutrosophic kernel and bipolar single valued neutrosophic closure.

I. INTRODUCTION

The introduction of the fuzzy sets is by L.A.Zadeh [12] in 1965. W.R.Zhang [13] who introduced the notion of bipolar fuzzy sets. The concept of intuitionistic fuzzy sets was introduced by Atanassov [2] in 1986 as a generalization of fuzzy sets. Smarandache [7] proposed neutrosophic sets (NSs) by combining the non-standard analysis, a tri-component logic/set/probability theory and philosophy. "It is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra" In order to practice NSs in real-life applications conveniently, Wang et al. [10] introduced single valued neutrosophic sets (SVNSs). Single valued neutrosophic sets is a generalization of intuitionistic fuzzy sets in which three membership functions are independent and their values belong to the unit interval [0, 1]. The concept of Intuitionistic fuzzy relations was given by Burillo P and Bustince H [3]. An intuitionistic fuzzy relation was examined by Annie Varghese and Sunny Kuriakose [1]. Neutrosophic Relation was studied and presented by Salama et al[8]. Single valued neutrosophic relation was inspected by H.L. Yang et al[11]. Latreche A, et al[6] give a form of Single valued neutrosophic mappings defined by single valued neutrosophic relations with applications. Bipolar single-valued neutrosophic set was introduced by Mohana et al. [7] and also they give bipolar single-valued neutrosophic topological spaces.

Here in this paper, we introduce bipolar single valued neutrosophic relations and their properties with the counter

II. PRELIMINARIES

2.1 Definition [9]: Let X be a non-empty fixed set. A neutrosophic set B is an object having the form $B = \{ \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle \mid x \in X \}$ Where $\mu_B(x)$, $\sigma_B(x)$ and $\gamma_B(x)$ which represent the degree of membership function, the degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set B .

2.2 Definition [10]: Let a universe X of discourse. Then $A_{NS} = \{ \langle x, F_A(x), T_A(x), I_A(x) \rangle \mid x \in X \}$ defined as a single-valued neutrosophic set where truth-membership function $T_A: X \rightarrow [0, 1]$, an indeterminacy-membership function $I_A: X \rightarrow [0, 1]$ and a falsity-membership function $F_A: X \rightarrow [0, 1]$. No restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0 \leq \sup T_A(x) \leq \sup I_A(x) \leq \sup F_A(x) \leq 3$.

$\tilde{A} = \langle T, I, F \rangle$ is denoted as a single-valued neutrosophic number.

2.3 Definition [11]: Let A and B be two SVNSs in U . If for any $u \in U$, $T_A(u) \leq T_B(u)$, $I_A(u) \geq I_B(u)$ and $F_A(u) \geq F_B(u)$, then we called A is contained in B , i.e. $A \subseteq B$. If $A \subseteq B$ and $B \subseteq A$, then we called A is equal to B , denoted by $A = B$.

2.4 Definition [11]: Let A be a SVNSs in U . The complement of A is denoted by A^c , where $\forall u \in U$, $T_{A^c}(u) = F_A(u)$, $I_{A^c}(u) = 1 - I_A(u)$ and $F_{A^c}(u) = T_A(u)$.

2.5 Definition [11]: Let A and B be two SVNSs in U .

(1) The union of A and B is a SVNS C , denoted by $C = A \cup B$, where $\forall u \in U$, $T_C(u) = \max \{ T_A(u), T_B(u) \}$, $I_C(u) = \min \{ I_A(u), I_B(u) \}$ and $F_C(u) = \min \{ F_A(u), F_B(u) \}$.

(2) The intersection of A and B is a SVNS D , denoted by $D = A \cap B$, where $\forall u \in U$, $T_D(u) = \min \{ T_A(u), T_B(u) \}$, $I_D(u) = \max \{ I_A(u), I_B(u) \}$ and $F_D(u) = \max \{ F_A(u), F_B(u) \}$.

2.6 Definition [11]: A SVNS R in $U \times U$ is called a single valued neutrosophic relation (SVNR) in U , denoted by $R = \{ \langle (u, v), T_R(u, v), I_R(u, v), F_R(u, v) \rangle \mid (u, v) \in U \times U \}$, where $T_R: U \times U \rightarrow [0, 1]$,

$I_R : U \times U \rightarrow [0, 1]$ and $F_R : U \times U \rightarrow [0, 1]$ denote the truth-membership function, indeterminacy membership function and falsity-membership function of R, respectively.

2.7 Definition [11]: . Let R be a SVN in U, the complement and inverse of R are defined as follows, respectively

$$R^c = \{ \langle (u, v), T_{R^c}(u, v), I_{R^c}(u, v), F_{R^c}(u, v) \rangle \mid (u, v) \in U \times U \}, \text{ where } \forall (u, v) \in U \times U,$$

$$T_{R^c}(u, v) = F_R(u, v), I_{R^c}(u, v) = 1 - I_R(u, v) \text{ and } F_{R^c}(u, v) = T_R(u, v).$$

$$R^{-1} = \{ \langle (u, v), T_{R^{-1}}(u, v), I_{R^{-1}}(u, v), F_{R^{-1}}(u, v) \rangle \mid (u, v) \in U \times U \}, \text{ where } \forall (u, v) \in U \times U,$$

$$T_{R^{-1}}(u, v) = T_R(v, u), I_{R^{-1}}(u, v) = I_R(v, u) \text{ and } F_{R^{-1}}(u, v) = F_R(v, u).$$

2.8 Definition [11]: . Let R be a SVN in U.

- (1) If $\forall u \in U, T_R(u, u) = 1$ and $I_R(u, u) = F_R(u, u) = 0$, then R is called a reflexive SVN.
- (2) If $\forall u, v \in U, T_R(u, v) = T_R(v, u), I_R(u, v) = I_R(v, u)$ and $F_R(u, v) = F_R(v, u)$, then R is called a symmetric SVN.
- (3) If $\forall u \in U, T_R(u, u) = 0$ and $I_R(u, u) = F_R(u, u) = 1$, then R is called an anti-reflexive SVN.
- (4) If $\forall u, v, w \in U, \forall v \in U (T_R(u, v) \wedge T_R(v, w)) \leq T_R(u, w), \wedge_{v \in U} (I_R(u, v) \vee I_R(v, w)) \geq I_R(u, w)$ and $\wedge_{v \in U} (F_R(u, v) \vee F_R(v, w)) \geq F_R(u, w)$, then R is called a transitive SVN, where “ \vee ” and “ \wedge ” denote maximum and minimum, respectively.

2.9 Definition [11]: Let R, S be two SVNs in U. If $\forall u, v \in U, T_R(u, v) \leq T_S(u, v), I_R(u, v) \geq I_S(u, v)$ and $F_R(u, v) \geq F_S(u, v)$, then we call R is contained in S (or R is less than S), denoted by $R \subseteq S$ (or $R \leq S$).

2.10 Definition [11]: Let R be a SVN in U.

- (1) The maximal anti-reflexive SVN contained in R is called anti-reflexive kernel of R, denoted by ar(R).
- (2) The maximal symmetric SVN contained in R is called symmetric kernel of R, denoted by s(R).

2.11 Definition [11]: Let R be a SVN in U. The minimal reflexive SVN containing R is called reflexive closure of R, denoted by r(R).

2.12 Definition [11]: . Let R be a SVN in U. The minimal symmetric SVN containing R is called symmetric closure of R, denoted by s(R).

2.13 Definition [11]: We will call neutrosophic relation R from set X to set Y (or between X and Y) is a neutrosophic set in the direct product $X \times Y = \{ (x, y) : x \in X, y \in Y \}$, that is, every neutrosophic sub sets of $X \times Y$ that is, to every expression R given $R = \{ \langle x, y \rangle, (\langle \mu_R(x, y), \sigma_R(x, y), \gamma_R(x, y) \rangle : x \in X, y \in Y) \}$ where $X \times Y$ is characterized by membership function $\mu_R(x)$, the degree of indeterminacy $\sigma_R(x)$, and the degree of non-membership $\gamma_R(x)$ respectively of each element $x \in X, y \in Y$ to the set X and Y. where $\mu_R(x), \sigma_R(x), \gamma_R(x) : X \times Y \rightarrow]0^-, 1^+[$. Given sets $X = \{ x_1, x_2, \dots, x_m \}, Y = \{ y_1, y_2, \dots, y_n \}$, a neutrosophic relation in $X \times Y$ can be expressed by an $m \times n$ matrix. This kind of matrix, which expressed of neutrosophic relation, is called a neutrosophic matrix. Since the triple $(\mu_R, \sigma_R, \gamma_R)$ has values with in the interval $]0^-, 1^+[$, the elements of the neutrosophic matrix also have values within $]0^-, 1^+[$. In order to express neutrosophic relation R for $(\mu_R(x_i, y_j), \sigma_R(x_i, y_j), \gamma_R(x_i, y_j))$. The neutrosophic relation is defined as neutrosophic subsets of $X \times Y$, having the form $R = \{ \langle x, y \rangle, (\langle \mu_R(x_i, y_j), \sigma_R(x_i, y_j), \gamma_R(x_i, y_j) \rangle : x \in X, y \in Y) \}$. Where the triple $(\mu_R(x), \sigma_R(x), \gamma_R(x))$ has values with in the interval $]0^-, 1^+[$, the elements of the neutrosophic matrix also have values with in $]0^-, 1^+[$. Given a neutrosophic relation between X and Y we can define R^{-1} between Y and X by means of $\mu_{R^{-1}}^+(y, x) = \mu_R^+(x, y), \sigma_{R^{-1}}^+(y, x) = \sigma_R^+(x, y), \gamma_{R^{-1}}^+(y, x) = \gamma_R^+(x, y) \in X \times Y$ to which are will call inverse neutrosophic relation of R.

2.14 Definition [5]: In X, a bipolar neutrosophic set B is defined in the form

$B = \langle x, (T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x)) : x \in X \rangle$ where $T^+, I^+, F^+ : X \rightarrow [1, 0]$ and $T^-, I^-, F^- : X \rightarrow [-1, 0]$. The positive membership degree denotes the truth membership $T^+(x)$, indeterminate membership $I^+(x)$ and false membership $F^+(x)$ of an element $x \in X$ corresponding to the set A and the negative membership degree denotes the truth membership $T^-(x)$, indeterminate membership $I^-(x)$ and false membership $F^-(x)$ of an element $x \in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set.

2.15 Definition [7]: A Bipolar Single-Valued Neutrosophic set (BSVN) S in X is defined in the form of

$$BSVN(S) = \langle v, (T_{BSVN}^+, T_{BSVN}^-), (I_{BSVN}^+, I_{BSVN}^-), (F_{BSVN}^+, F_{BSVN}^-) : v \in X \rangle$$

where $(T_{BSVN}^+, I_{BSVN}^+, F_{BSVN}^+) : X \rightarrow [0, 1]$ and $(T_{BSVN}^-, I_{BSVN}^-, F_{BSVN}^-) : X \rightarrow [-1, 0]$. In this Definition, there T_{BSVN}^+ and T_{BSVN}^- are acceptable and unacceptable in past. Similarly I_{BSVN}^+ and I_{BSVN}^- are acceptable and unacceptable in future. F_{BSVN}^+ and F_{BSVN}^- are acceptable and unacceptable in present respectively.

2.16 Definition [7]: Let two bipolar single-valued neutrosophic sets $BSVN_1(S)$ and $BSVN_2(S)$ in X defined as $BSVN_1(S) = \langle v, (T_{BSVN_1}^+(1), T_{BSVN_1}^-(1)), (I_{BSVN_1}^+(1), I_{BSVN_1}^-(1)), (F_{BSVN_1}^+(1), F_{BSVN_1}^-(1)) : v \in X \rangle$ and $BSVN_2(S) = \langle v, (T_{BSVN_2}^+(2), T_{BSVN_2}^-(2)), (I_{BSVN_2}^+(2), I_{BSVN_2}^-(2)), (F_{BSVN_2}^+(2), F_{BSVN_2}^-(2)) : v \in X \rangle$. Then the operators are defined as follows:

(i) Complement

$$BSVN^c(S) = \langle v, (1 - T_{BSVN}^+, (-1 - T_{BSVN}^-), (1 - I_{BSVN}^+), (-1 - I_{BSVN}^-), (1 - F_{BSVN}^+), (-1 - F_{BSVN}^-) : v \in X \rangle$$

(ii) Union of two BSVN

$$BSVN_1(S) \cup BSVN_2(S) = \left\langle \begin{array}{l} \max(T_{BSVN_1}^+(1), T_{BSVN_2}^+(2)), \min(I_{BSVN_1}^+(1), I_{BSVN_2}^+(2)), \min(F_{BSVN_1}^+(1), F_{BSVN_2}^+(2)) \\ \max(T_{BSVN_1}^-(1), T_{BSVN_2}^-(2)), \min(I_{BSVN_1}^-(1), I_{BSVN_2}^-(2)), \min(F_{BSVN_1}^-(1), F_{BSVN_2}^-(2)) \end{array} \right\rangle$$

(iii) Intersection of two BSVN

$$BSVN_1(S) \cap BSVN_2(S) = \left\langle \begin{array}{l} \min(T_{BSVN_1}^+(1), T_{BSVN_2}^+(2)), \max(I_{BSVN_1}^+(1), I_{BSVN_2}^+(2)), \max(F_{BSVN_1}^+(1), F_{BSVN_2}^+(2)) \\ \min(T_{BSVN_1}^-(1), T_{BSVN_2}^-(2)), \max(I_{BSVN_1}^-(1), I_{BSVN_2}^-(2)), \max(F_{BSVN_1}^-(1), F_{BSVN_2}^-(2)) \end{array} \right\rangle$$

2.17 Definition [7]: Let two bipolar single-valued neutrosophic sets be $BSVN_1(S)$ and $BSVN_2(S)$ in X defined as $BSVN_1(S) = \langle v, (T_{BSVN_1}^+(1), T_{BSVN_1}^-(1)), (I_{BSVN_1}^+(1), I_{BSVN_1}^-(1)), (F_{BSVN_1}^+(1), F_{BSVN_1}^-(1)) : v \in X \rangle$ and $BSVN_2(S) = \langle v, (T_{BSVN_2}^+(2), T_{BSVN_2}^-(2)), (I_{BSVN_2}^+(2), I_{BSVN_2}^-(2)), (F_{BSVN_2}^+(2), F_{BSVN_2}^-(2)) : v \in X \rangle$.

Then $S_1 = S_2$ if and only if

$$T_{BSVN_1}^+(1) = T_{BSVN_2}^+(2), I_{BSVN_1}^-(1) = I_{BSVN_2}^-(2), F_{BSVN_1}^+(1) = F_{BSVN_2}^+(2),$$

$$T_{BSVN_1}^-(1) = T_{BSVN_2}^-(2), I_{BSVN_1}^+(1) = I_{BSVN_2}^+(2), F_{BSVN_1}^-(1) = F_{BSVN_2}^-(2) \text{ for all } v \in X.$$

2.18 Definition [7]: Let two bipolar single-valued neutrosophic sets be $BSVN_1$ and $BSVN_2$ in X defined as $BSVN_1(S) = \langle v, (T_{BSVN_1}^+(1), T_{BSVN_1}^-(1)), (I_{BSVN_1}^+(1), I_{BSVN_1}^-(1)), (F_{BSVN_1}^+(1), F_{BSVN_1}^-(1)) : v \in X \rangle$ and $BSVN_2(S) = \langle v, (T_{BSVN_2}^+(2), T_{BSVN_2}^-(2)), (I_{BSVN_2}^+(2), I_{BSVN_2}^-(2)), (F_{BSVN_2}^+(2), F_{BSVN_2}^-(2)) : v \in X \rangle$.

Then $S_1 \subseteq S_2$ if and only if

$$T_{BSVN_1}^+(1) \leq T_{BSVN_2}^+(2), I_{BSVN_1}^-(1) \geq I_{BSVN_2}^-(2), F_{BSVN_1}^+(1) \geq F_{BSVN_2}^+(2),$$

$$T_{BSVN_1}^-(1) \leq T_{BSVN_2}^-(2), I_{BSVN_1}^+(1) \geq I_{BSVN_2}^+(2), F_{BSVN_1}^-(1) \geq F_{BSVN_2}^-(2) \text{ for all } v \in X.$$

2.19 Definition [4]: Let 0_{BSVN} and 1_{BSVN} be BSVNs in X defined as

$$0_{BSVN} = \langle x, 0, 1, 1, -1, 0, 0 : x \in X \rangle \text{ is said to be Null or Empty bipolar single-valued neutrosophic set.}$$

$$1_{BSVN} = \langle x, 1, 0, 0, 0, -1, -1 : x \in X \rangle \text{ is said to be Absolute or Unit bipolar single-valued neutrosophic set.}$$

III. BIPOLAR SINGLE-VALUED NEUTROSOPHIC RELATION

Definition 3.1 A BSVN set R in $X \times X$ is called a bipolar single-valued neutrosophic relation (BSVNR) in X , denoted by

$$R = \langle (x, y), T_R^+(x, y), T_R^-(x, y), I_R^+(x, y), I_R^-(x, y), F_R^+(x, y), F_R^-(x, y) : x \in X, y \in X \rangle,$$

where $\forall (x, y) \in X \times X$ and also $T_R^+, I_R^+, F_R^+ : X \times X \rightarrow [0, 1], T_R^-, I_R^-, F_R^- : X \times X \rightarrow [-1, 0]$. We will denote with $BSVNR(X \times X)$ the set of all bipolar single-valued Neutrosophic relation in $X \times X$.

Definition 3.2 Let R be a BSVNR in X , the complement and inverse of R are defined as follows,

$$(a) R^c = \langle (x, y), (1 - T_R^+(x, y)) = T_{R^c}^+(x, y), (-1 - T_R^-(x, y)) = T_{R^c}^-(x, y), (1 - I_R^+(x, y)) = I_{R^c}^+(x, y),$$

$$(-1 - I_R^-(x, y)) = I_{R^c}^-(x, y), (1 - F_R^+(x, y)) = F_{R^c}^+(x, y), (-1 - F_R^-(x, y)) = F_{R^c}^-(x, y) \rangle$$

$$(b) R^{-1} = \langle (x, y), T_{R^{-1}}^+(x, y), T_{R^{-1}}^-(x, y), I_{R^{-1}}^+(x, y), I_{R^{-1}}^-(x, y), F_{R^{-1}}^+(x, y), F_{R^{-1}}^-(x, y) \rangle \text{ where}$$

$$\forall (x, y) \in X \times X, T_{R^{-1}}^+(x, y) = T_R^+(y, x), T_{R^{-1}}^-(x, y) = T_R^-(y, x), I_{R^{-1}}^+(x, y) = I_R^+(y, x),$$

$$I_{R^{-1}}^-(x, y) = I_R^-(y, x), F_{R^{-1}}^+(x, y) = F_R^+(y, x), F_{R^{-1}}^-(x, y) = F_R^-(y, x).$$

Example 3.3 Let $X = \{x, y\}$. A BSVNR R in X is given in TABLE I. By

TABLE I

R	X	Y
x	(0.2, -0.3),(0.1,-0.6),(0.4,-0.5)	(0.1, -0.6),(0.2,-0.3),(0.5,-0.4)
y	(0.1, -0.2),(0.2,-0.4),(0.6,-0.7)	(0.1, -0.5),(0.7,-0.8),(0.9,-0.1)

A BSVNR R^c in X is given in TABLE II,

TABLE II

R	X	Y
x	(0.8, -0.7),(0.9,-0.4),(0.6,-0.5)	(0.9, -0.4),(0.8,-0.7),(0.5,-0.6)
y	(0.9, -0.8),(0.8,-0.6),(0.4,-0.3)	(0.9, -0.5),(0.3,-0.2),(0.1,-0.9)

A BSVNR R^{-1} in X is given in TABLE III,

TABLE III

R	X	Y
x	(0.2, -0.3),(0.1,-0.6),(0.4,-0.5)	(0.1, -0.2),(0.2,-0.4),(0.6,-0.7)
y	(0.1, -0.6),(0.2,-0.3),(0.5,-0.4)	(0.1, -0.5),(0.7,-0.8),(0.9,-0.1)

Definitions 3.4 Let R and S are two BSVNR in X. Then

(a) The union $R \cup S$ of R and S is defined by

$$R \cup S = \{ \langle (x, y), \max \{T_R^+(x, y), T_S^+(x, y)\}, \min \{I_R^+(x, y), I_S^+(x, y)\}, \min \{F_R^+(x, y), F_S^+(x, y)\}, \max \{T_R^-(x, y), T_S^-(x, y)\}, \min \{I_R^-(x, y), I_S^-(x, y)\}; \min \{F_R^-(x, y), F_S^-(x, y)\} \rangle \mid (x, y) \in X \times X \}.$$

(b) The intersection $R \cap S$ of R and S is defined by

$$R \cap S = \{ \langle (x, y), \min \{T_R^+(x, y), T_S^+(x, y)\}, \max \{I_R^+(x, y), I_S^+(x, y)\}, \max \{F_R^+(x, y), F_S^+(x, y)\}, \min \{T_R^-(x, y), T_S^-(x, y)\}, \max \{I_R^-(x, y), I_S^-(x, y)\}; \max \{F_R^-(x, y), F_S^-(x, y)\} \rangle \mid (x, y) \in X \times X \}.$$

Definition 3.5 Let R is a BSVNR in X.

(a) If $\forall x, y \in X, T_R^+(x, y) = 0_R, T_R^-(x, y) = -1_R,$
 $I_R^+(x, y) = 1_R, I_R^-(x, y) = 0_R,$
 $F_R^+(x, y) = 1_R, F_R^-(x, y) = 0_R,$

Then R is called a null BSVNR and it is denoted by 0_R .

(b) If $\forall x, y \in X, T_R^+(x, y) = 1_R, T_R^-(x, y) = 0_R,$
 $I_R^+(x, y) = 0_R, I_R^-(x, y) = -1_R,$
 $F_R^+(x, y) = 0_R, F_R^-(x, y) = -1_R,$

Then R is called an absolute BSVNR and it is denoted by 1_R .

Definition 3.6 If $\forall x, y \in X, T_R^+(x, y) = \begin{cases} 1_R & x = y \\ 0_R & x \neq y \end{cases}, T_R^-(x, y) = \begin{cases} 0_R & x = y \\ -1_R & x \neq y \end{cases},$

$$I_R^+(x, y) = \begin{cases} 0_R & x = y \\ 1_R & x \neq y \end{cases}, \quad I_R^-(x, y) = \begin{cases} -1_R & x = y \\ 0_R & x \neq y \end{cases},$$

$$F_R^+(x, y) = \begin{cases} 0_R & x = y \\ 1_R & x \neq y \end{cases}, \quad F_R^-(x, y) = \begin{cases} -1_R & x = y \\ 0_R & x \neq y \end{cases},$$

Then R is called an identity BSVNR and it is denoted by Id_R .

By use of Definitions 3.2 and 3.5, the complement $(Id_R)^c$ of Id_R is a BSVNR satisfying:

$$\forall x, y \in X, \quad T_{(Id_R)^c}^+(x, y) = \begin{cases} 0_R x = y \\ 1_R x \neq y \end{cases}, \quad T_{(Id_R)^c}^-(x, y) = \begin{cases} -1_R x = y \\ 0_R x \neq y \end{cases},$$

$$I_{(Id_R)^c}^+(x, y) = \begin{cases} 1_R x = y \\ 0_R x \neq y \end{cases}, \quad I_{(Id_R)^c}^-(x, y) = \begin{cases} 0_R x = y \\ -1_R x \neq y \end{cases},$$

$$F_{(Id_R)^c}^+(x, y) = \begin{cases} 1_R x = y \\ 0_R x \neq y \end{cases}, \quad F_{(Id_R)^c}^-(x, y) = \begin{cases} 0_R x = y \\ -1_R x \neq y \end{cases}.$$

Definition 3.7 Let R be a BSVNR in X .

(a) If $\forall x \in X, T_R^+(x, x) = 1_R, T_R^-(x, x) = 0_R,$

$$I_R^+(x, x) = 0_R, I_R^-(x, x) = -1_R,$$

$$F_R^+(x, x) = 0_R, F_R^-(x, x) = -1_R, \text{ then } R \text{ is called a reflexive BSVNR.}$$

(b) If $\forall x, y \in X, T_R^+(x, y) = T_R^+(y, x), T_R^-(x, y) = T_R^-(y, x),$

$$I_R^+(x, y) = I_R^+(y, x), I_R^-(x, y) = I_R^-(y, x),$$

$$F_R^+(x, y) = F_R^+(y, x), F_R^-(x, y) = F_R^-(y, x), \text{ then } R \text{ is called a symmetric BSVNR.}$$

(c) If $\forall x \in X, T_R^+(x, x) = 0_R, T_R^-(x, x) = -1_R,$

$$I_R^+(x, x) = 1_R, I_R^-(x, x) = 0_R,$$

$$F_R^+(x, x) = 1_R, F_R^-(x, x) = 0_R, \text{ then } R \text{ is called an anti-reflexive BSVNR.}$$

(d) If $\forall x, y, z \in X, \bigvee_{v \in X} (T_R^+(x, y) \wedge (T_R^+(y, z)) \leq T_R^+(x, z),$

$$\bigwedge_{v \in X} (I_R^+(x, y) \vee (T_R^+(y, z)) \geq T_R^+(x, z)$$

$$\bigwedge_{v \in X} (F_R^+(x, y) \vee (T_R^+(y, z)) \geq T_R^+(x, z)$$

$$\bigvee_{v \in X} (T_R^-(x, y) \wedge (T_R^-(y, z)) \leq T_R^-(x, z)$$

$$\bigwedge_{v \in X} (T_R^-(x, y) \vee (T_R^-(y, z)) \geq T_R^-(x, z)$$

$$\bigwedge_{v \in X} (T_R^-(x, y) \vee (T_R^-(y, z)) \geq T_R^-(x, z), \text{ then } R \text{ is called a transitive BSVNR,}$$

where \vee and \wedge denote maximum and minimum, respectively.

Definition 3.8 Let R, S be two BSVNRs in X . If $\forall x, y \in X,$

$$T_R^+(x, y) \leq T_S^+(x, y), \quad T_R^-(x, y) \leq T_S^-(x, y)$$

$$I_R^+(x, y) \geq I_S^+(x, y) \quad I_R^-(x, y) \geq I_S^-(x, y)$$

$$F_R^+(x, y) \geq F_S^+(x, y) \quad F_R^-(x, y) \geq F_S^-(x, y), \text{ then it is called } R \text{ is contained in } S \text{ (} R \text{ is less than) and it is denoted by}$$

$$R \subseteq S \text{ (} R \leq S).$$

Theorem 3.9 Let R, S, P are three BSVNRs in X . Then follows,

(1) R is symmetric iff $R = R^{-1}$.

(2) $(R^c)^{-1} = (R^{-1})^c$.

(3) $(R^{-1})^{-1} = R, (R^c)^c = R$.

(4) $R \cup S \supseteq R, R \cup S \supseteq S$.

(5) $R \cap S \subseteq R, R \cap S \subseteq S$.

(6) If $R \subseteq S$, then $R^{-1} \subseteq S^{-1}$.

(7) If $P \supseteq S$ and $P \supseteq R$, then $P \supseteq R \cup S$.

(8) If $P \subseteq S$ and $P \subseteq R$, then $P \subseteq R \cap S$.

(9) If $R \subseteq S$, then $R \cup S = S$ and $R \cap S = R$.

(10) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}, (R \cap S)^{-1} = R^{-1} \cap S^{-1}$.

$$(11) (R \cup S)^c = R^c \cap S^c, (R \cap S)^c = R^c \cup S^c.$$

Proof. Obviously, (1) and (3)-(9) are hold. To show that (2), (10) and (11).

$$(2) \forall x, y \in X, T_{(R^c)^{-1}}^+(x, y) = T_{R^c}^+(y, x) = 1 - T_R^+(y, x) = 1 - T_{R^{-1}}^+(x, y) = T_{(R^{-1})^c}^+(x, y),$$

$$T_{(R^c)^{-1}}^-(x, y) = T_{R^c}^-(y, x) = -1 - T_R^-(y, x) = -1 - T_{R^{-1}}^-(x, y) = T_{(R^{-1})^c}^-(x, y),$$

$$\text{Similarly for } I_{(R^c)^{-1}}^+(x, y) = I_{(R^{-1})^c}^+(x, y), I_{(R^c)^{-1}}^-(x, y) = I_{(R^{-1})^c}^-(x, y), F_{(R^c)^{-1}}^+(x, y) = F_{(R^{-1})^c}^+(x, y),$$

$$F_{(R^c)^{-1}}^-(x, y) = F_{(R^{-1})^c}^-(x, y). \text{ Hence } (R^c)^{-1} = (R^{-1})^c.$$

$$(10) \forall x, y \in X,$$

$$T_{(R \cup S)^{-1}}^+(x, y) = T_{(R \cup S)}^+(y, x) = \max \{T_R^+(y, x), T_S^+(y, x)\} = \max \{T_{R^{-1}}^+(x, y), T_{S^{-1}}^+(x, y)\} = T_{R^{-1} \cup S^{-1}}^+(x, y)$$

$$I_{(R \cup S)^{-1}}^+(x, y) = I_{(R \cup S)}^+(y, x) = \min \{I_R^+(y, x), I_S^+(y, x)\} = \max \{I_{R^{-1}}^+(x, y), I_{S^{-1}}^+(x, y)\} = I_{R^{-1} \cup S^{-1}}^+(x, y)$$

$$F_{(R \cup S)^{-1}}^+(x, y) = F_{(R \cup S)}^+(y, x) = \min \{F_R^+(y, x), F_S^+(y, x)\} = \max \{F_{R^{-1}}^+(x, y), F_{S^{-1}}^+(x, y)\} = F_{R^{-1} \cup S^{-1}}^+(x, y)$$

$$T_{(R \cup S)^{-1}}^-(x, y) = T_{(R \cup S)}^-(y, x) = \max \{T_R^-(y, x), T_S^-(y, x)\} = \max \{T_{R^{-1}}^-(x, y), T_{S^{-1}}^-(x, y)\} = T_{R^{-1} \cup S^{-1}}^-(x, y)$$

$$I_{(R \cup S)^{-1}}^-(x, y) = I_{(R \cup S)}^-(y, x) = \min \{I_R^-(y, x), I_S^-(y, x)\} = \max \{I_{R^{-1}}^-(x, y), I_{S^{-1}}^-(x, y)\} = I_{R^{-1} \cup S^{-1}}^-(x, y)$$

$$F_{(R \cup S)^{-1}}^-(x, y) = F_{(R \cup S)}^-(y, x) = \min \{F_R^-(y, x), F_S^-(y, x)\} = \max \{F_{R^{-1}}^-(x, y), F_{S^{-1}}^-(x, y)\} = F_{R^{-1} \cup S^{-1}}^-(x, y)$$

Hence $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$. Similarly, we can show $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$.

$$(11) \forall x, y \in X,$$

$$T_{(R \cup S)^c}^+(x, y) = 1 - T_{(R \cup S)}^+(x, y) = \min \{1 - T_R^+(x, y), 1 - T_S^+(x, y)\} = \min \{T_{R^c}^+(x, y), T_{S^c}^+(x, y)\} = T_{R^c \cup S^c}^+(x, y) I_{(R \cup S)^c}^+(x, y) = 1 - I_{(R \cup S)}^+(x, y) = \max \{1 - I_R^+(x, y), 1 - I_S^+(x, y)\} = \max \{I_{R^c}^+(x, y), I_{S^c}^+(x, y)\} = I_{R^c \cup S^c}^+(x, y)$$

$$F_{(R \cup S)^c}^+(x, y) = 1 - F_{(R \cup S)}^+(x, y) = \max \{1 - F_R^+(x, y), 1 - F_S^+(x, y)\} = \max \{F_{R^c}^+(x, y), F_{S^c}^+(x, y)\} = F_{R^c \cup S^c}^+(x, y)$$

$$\text{Similarly for } T_{(R \cup S)^c}^-(x, y) = T_{R^c \cup S^c}^-(x, y), I_{(R \cup S)^c}^-(x, y) = I_{R^c \cup S^c}^-(x, y), F_{(R \cup S)^c}^-(x, y) = F_{R^c \cup S^c}^-(x, y).$$

Hence $(R \cup S)^c = R^c \cap S^c$. Similarly, we can show $(R \cap S)^c = R^c \cup S^c$.

Remark 3.10 According to Theorem 3.9 (1) and (2), the complement of a symmetric BSVNR is also a symmetric BSVNR.

IV. KERNELS OF BIPOLAR SINGLE-VALUED NEUTROSOPHIC RELATION

In this section, we will define and investigate anti-reflexive kernel and symmetric kernel of a BSVNR and their properties.

Definition 4.1 Let R be a BSVNR in X.

- The maximal anti-reflexive BSVNR contained in R is called anti-reflexive kernel of R and it is denoted by $ar(R)$.
- The maximal symmetric BSVNR contained in R is called symmetric kernel of R and denoted by $s(R)$.

Theorem 4.2 Let R is a BSVNR in X. Then

$$(1) ar(R) = R \cap (Id_R)^c.$$

$$(2) s(R) = R \cap R^{-1}.$$

Proof. (1) By Theorem 3.9 (5), $R \cap (Id_R)^c \subseteq R$. By the Definition of Id_R , $\forall x \in X$, then $T_R^+(x, x) = 1_R$,

$$T_R^-(x, x) = 0_R, I_R^+(x, x) = 0_R, I_R^-(x, x) = -1_R, F_R^+(x, x) = 0_R, F_R^-(x, x) = -1_R. \text{ Then } T_{(Id_R)^c}^+(x, x) = 0_R,$$

$$T_{(Id_R)^c}^-(x, x) = -1_R, I_{(Id_R)^c}^+(x, x) = 1_R, I_{(Id_R)^c}^-(x, x) = 0_R, F_{(Id_R)^c}^+(x, x) = 1_R, F_{(Id_R)^c}^-(x, x) = 0_R. \text{ Hence}$$

$$T_{R \cap (Id_R)^c}^+(x, x) = \min \{T_R^+(x, x), T_{(Id_R)^c}^+(x, x)\} = 0_R, I_{R \cap (Id_R)^c}^+(x, x) = \max \{I_R^+(x, x), I_{(Id_R)^c}^+(x, x)\} = 1_R,$$

$$F_{R \cap (Id_R)^c}^+(x, x) = \max \{F_R^+(x, x), F_{(Id_R)^c}^+(x, x)\} = 1_R, T_{R \cap (Id_R)^c}^-(x, x) = \min \{T_R^-(x, x), T_{(Id_R)^c}^-(x, x)\} = 0_R, I_{R \cap (Id_R)^c}^-(x, x) =$$

$$\max \{I_R^-(x, x), I_{(Id_R)^c}^-(x, x)\} = 1_R, F_{R \cap (Id_R)^c}^-(x, x) = \max \{F_R^-(x, x), F_{(Id_R)^c}^-(x, x)\} = 1_R. \text{ Then by the Definition 3.7 (c), } R \cap$$

$(Id_R)^c$ is an anti-reflexive BSVNR in X.

If K is an anti-reflexive BSVNR in X and $K \subseteq R$. Obviously, $K \subseteq (Id_R)^c$. Hence $K \subseteq R \cap (Id_R)^c$. So

$$ar(R) = R \cap (Id_R)^c.$$

(2) By Theorem 3.9 (9) and (3), $(R \cap R^{-1})^{-1} = R^{-1} \cap (R^{-1})^{-1} = R^{-1} \cap R = R \cap R^{-1}$, which implies that $R \cap R^{-1}$ is a symmetric BSVNR in X . By Theorem 3.9 (5), $R \cap R^{-1} \subseteq R$.

If K is a symmetric BSVNR in X and $K \subseteq R$. Then by Theorem 3.9 (6), $K^{-1} \subseteq R^{-1}$. Then by Theorem 3.9 (1) and (5), $K = K^{-1} \subseteq R \cap R^{-1}$. So $s(R) = R \cap R^{-1}$.

Example 4.3 Consider X and R in Example 3.3. By Theorem 4.2, we can obtain $ar(R)$ and $s(R)$ which are given in TABLE IV and V, respectively.

TABLE IV

R	X	Y
x	(0, -1),(1,0),(1,0)	(0.1, -0.6),(0.2,-0.3),(0.5,-0.4)
y	(0.1, -0.2),(0.2,-0.4),(0.6,-0.7)	(0, -1),(1,0),(1,0)

TABLE V

R	X	Y
x	(0.2, -0.3),(0.1,-0.6),(0.4,-0.5)	(0.1, -0.6),(0.2,-0.3),(0.5,-0.4)
y	(0.1, -0.6),(0.2,-0.3),(0.5,-0.4)	(0.1, -0.5),(0.7,-0.8),(0.9,-0.1)

Theorem 4.4 The anti-reflexive kernel operator (ar) of the BSVNR has the following properties:

- (1) $ar(0_R) = 0_R$ and $ar((Id_R)^c) = (Id_R)^c$.
- (2) $\forall R \in BSVNR(X)$, $ar(R) \subseteq R$.
- (3) $\forall R, S \in BSVNR(X)$, $ar(R \cup S) = ar(R) \cup ar(S)$, $ar(R \cap S) = ar(R) \cap ar(S)$.
- (4) $\forall R, S \in BSVNR(X)$, if $R \subseteq S$, then $ar(R) \subseteq ar(S)$.
- (5) $\forall R \in BSVNR(X)$, $ar(ar(R)) = ar(R)$.

Proof. (1) By the anti-reflexivity of 0_R and $(Id_R)^c$, obviously, $ar(0_R) = 0_R$ and $ar((Id_R)^c) = (Id_R)^c$.

(2) $\forall R \in BSVNR(X)$, by Theorems 4.2 (1) and 3.9 (5), $ar(R) = R \cap (Id_R)^c \subseteq R$.

(3) $\forall R, S \in BSVNR(X)$, by Theorem 4.2 (1), $ar(R \cup S) = (R \cup S) \cap (Id_R)^c = (R \cap (Id_R)^c) \cup (S \cap (Id_R)^c) = ar(R) \cup ar(S)$,

$ar(R \cap S) = (R \cap S) \cap (Id_R)^c = (R \cap (Id_R)^c) \cap (S \cap (Id_R)^c) = ar(R) \cap ar(S)$.

(4) $\forall R, S \in BSVNR(X)$, if $R \subseteq S$, by (3) and Theorem 3.9 (4) and (9), $ar(S) = ar(R \cup S) = ar(R) \cup ar(S) \supseteq ar(R)$.

(5) $\forall R \in BSVNR(X)$, by Theorem 4.2 (1), $ar(R) = R \cap (Id_R)^c$. Hence $ar(ar(R)) = ar(R \cap (Id_R)^c) = (R \cap (Id_R)^c) \cap (Id_R)^c = R \cap (Id_R)^c = ar(R)$.

Theorem 4.5 The symmetric kernel operator (s) has the following properties:

- (1) $s(0_R) = 0_R$, $s(X_R) = X_R$, $s(Id_R) = Id_R$.
- (2) $\forall R \in BSVNR(X)$, $s(R) \subseteq R$.
- (3) $\forall R, S \in BSVNR(X)$, $s(R \cap S) = s(R) \cap s(S)$.
- (4) $\forall R, S \in BSVNR(X)$, if $R \subseteq S$, then $s(R) \subseteq s(S)$.
- (5) $\forall R \in BSVNR(X)$, $s(s(R)) = s(R)$.

Proof. (1) By the symmetry of 0_R , X_R and Id_R , we have $s(0_R) = 0_R$, $s(X_R) = X_R$ and $s(Id_R) = Id_R$.

(2) $\forall R \in BSVNR(X)$, by Theorems 4.2 (2) and 3.9 (5), $s(R) = R \cap R^{-1} \subseteq R$.

(3) $\forall R, S \in BSVNR(X)$, by Theorems 4.2 (2) and 3.9 (10), we have $s(R \cap S) = (R \cap S) \cap (R \cap S)^{-1} = (R \cap S) \cap (R^{-1} \cap S^{-1}) = (R \cap R^{-1}) \cap (S \cap S^{-1}) = s(R) \cap s(S)$.

(4) $\forall R, S \in BSVNR(X)$, if $R \subseteq S$, by (3) and Theorem 3.9 (5) and (9), $s(R) = s(R \cap S) = s(R) \cap s(S) \subseteq s(S)$.

(5) $\forall R \in BSVNR(X)$, by Theorem 4.2 (2), $s(R) = R \cap R^{-1}$. Hence $s(s(R)) = s(R \cap R^{-1}) = (R \cap R^{-1}) \cap (R \cap R^{-1})^{-1} = (R \cap R^{-1}) \cap (R^{-1} \cap R) = R \cap R^{-1} = s(R)$.

V. CLOSURES OF BIPOLAR SINGLE-VALUED NEUTROSOPHIC RELATION

In this section, the concepts of reflexive closure and symmetric closure of a BSVNR are defined and investigate their properties.

Definition 5.1 Let R be a BSVNR in X . The minimal reflexive BSVNR containing R is called reflexive closure of R and it is denoted by $r(R)$.

Definition 5.2 Let R be a BSVNR in X . The minimal symmetric BSVNR containing R is called symmetric closure of R and it is denoted by $s^-(R)$.

Theorem 5.3 Let R be a BSVNR in X . Then

- (1) $r(R) = R \cup Id_R$.
- (2) $s^-(R) = R \cup R^{-1}$.

Proof. (1) By Theorem 3.9 (4), $R \cup Id_R \supseteq R$ and $R \cup Id_R \supseteq Id_R$. Then $\forall x \in X$, we have

$$T_{R \cup Id_R}^+(x, x) \geq T_{Id_R}^+(x, x) = 1_R, I_{R \cup Id_R}^+(x, x) \leq I_{Id_R}^+(x, x) = 0_R, F_{R \cup Id_R}^+(x, x) \leq F_{Id_R}^+(x, x) = 0_R,$$

$$T_{R \cup Id_R}^-(x, x) \geq T_{Id_R}^-(x, x) = 1_R, I_{R \cup Id_R}^-(x, x) \leq I_{Id_R}^-(x, x) = 0_R, F_{R \cup Id_R}^-(x, x) \leq F_{Id_R}^-(x, x) = 0_R.$$

Therefore $R \cup Id_R$ is a reflexive BSVNR.

If K is a reflexive BSVNR in X and $K \supseteq R$. By the reflexivity of K , $K \supseteq Id_R$. Then by Theorem 3.9 (7), $K \supseteq R \cup Id_R$. So $r(R) = R \cup Id_R$.

(2) By Theorem 3.9 (10), $(R \cup R^{-1})^{-1} = R^{-1} \cup (R^{-1})^{-1} = R^{-1} \cup R = R \cup R^{-1}$, which implies that $R \cup R^{-1}$ is a symmetric BSVNR in X . Then by the Theorem 3.9 (4), $R \cup R^{-1} \supseteq R$.

If K is a symmetric BSVNR in X and $K \supseteq R$. By the Theorem 3.9 (6), $K^{-1} \supseteq R^{-1}$. According to Theorem 3.9 (1) and (4), $K = K^{-1} \supseteq R \cup R^{-1}$. Therefore $s^-(R) = R \cup R^{-1}$.

Example 5.4 Consider X and R given in Example 3.1 again. By Theorem 5.3, we can compute $r(R)$ and $s(R)$ which are given in TABLE VI and VII, respectively.

TABLE VI

R	X	Y
x	(1,0),(0,-1),(0,-1)	(0.1, -0.6),(0.2,-0.3),(0.5,-0.4)
y	(0.1, -0.2),(0.2,-0.4),(0.6,-0.7)	(1,0),(0,-1),(0,-1)

TABLE VII

R	X	Y
x	(0.2, -0.3),(0.1,-0.6),(0.4,-0.5)	(0.1, -0.2),(0.2,-0.4),(0.5,-0.7)
y	(0.1, -0.2),(0.2,-0.4),(0.5,-0.7)	(0.1, -0.5),(0.7,-0.8),(0.9,-0.1)

Theorem 5.5 The reflexive closure operator (r) has the following properties:

- (1) $r(X_R) = X_R, r(Id_R) = Id_R$.
- (2) $\forall R \in BSVNR(X), R \subseteq r(R)$.
- (3) $\forall R, S \in BSVNR(X), r(R \cup S) = r(R) \cup r(S), r(R \cap S) = r(R) \cap r(S)$.
- (4) $\forall R, S \in BSVNR(X)$, if $R \subseteq S$, then $r(R) \subseteq r(S)$.
- (5) $\forall R \in BSVNR(X), r(r(R)) = r(R)$.

Proof. (1) By the reflexivity of X_R and $Id_R, r(X_R) = X_R, r(Id_R) = Id_R$.

(2) $\forall R \in BSVNR(X)$, by Theorems 5.3 (1) and 3.9 (4), $r(R) = R \cup Id_R \supseteq R$.

(3) $\forall R, S \in BSVNR(X)$, by Theorem 5.3 (1),

$$r(R \cup S) = (R \cup S) \cup Id_R = (R \cup Id_R) \cup (S \cup Id_R) = r(R) \cup r(S),$$

$$r(R \cap S) = (R \cap S) \cup Id_R = (R \cup Id_R) \cap (S \cup Id_R) = r(R) \cap r(S).$$

(4) $\forall R, S \in BSVNR(X), R \subseteq S$, by (3) and Theorem 3.9 (4) and (9), we have

$$r(S) = r(R \cup S) = r(R) \cup r(S) \supseteq r(R).$$

(5) $\forall R \in BSVNR(X)$, by Theorem 5.3 (1), $r(R) = R \cup Id_R$. It follows that

$$r(r(R)) = r(R \cup Id_R) = (R \cup Id_R) \cup Id_R = R \cup Id_R = r(R).$$

Theorem 5.6 The symmetric closure operator (s) has the following properties:

- (1) $s^-(0_R) = 0_R, s^-(X_R) = X_R, s^-(Id_R) = Id_R$.
- (2) $\forall R \in BSVNR(X), s^-(R) \supseteq R$.
- (3) $\forall R, S \in BSVNR(X), s^-(R \cup S) = s^-(R) \cup s^-(S)$.
- (4) $\forall R, S \in BSVNR(X)$, if $R \subseteq S$, then $s^-(R) \subseteq s^-(S)$.
- (5) $\forall R \in BSVNR(X), s^-(s^-(R)) = s^-(R)$.

Proof. (1) By the symmetry of $0_R, X_R$ and Id_R , we have $s^-(0_R) = 0_R, s^-(X_R) = X_R$ and $s^-(Id_R) = Id_R$.

(2) $\forall R \in BSVNR(X)$, by Theorem 5.3 (2), $s^-(R) = R \cup R^{-1} \supseteq R$.

(3) $\forall R, S \in BSVNR(X)$, by Theorems 5.3 (2) and 3.1 (10), we have

$$s^-(R \cup S) = (R \cup S) \cup (R \cup S)^{-1} = (R \cup S) \cup (R^{-1} \cup S^{-1}) = (R \cup R^{-1}) \cup (S \cup S^{-1}) = s^-(R) \cup s^-(S).$$

(4) $\forall R, S \in BSVNR(X)$, if $R \subseteq S$, by (3) and Theorem 3.9 (4) and (9),

$$s^-(S) = s^-(R \cup S) = s^-(R) \cup s^-(S) \supseteq s^-(R).$$

(5) $\forall R \in \text{BSVNR}(X)$, by Theorem 5.3 (2), $s^-(R) = R \cup R^{-1}$. Hence
 $s^-(s^-(R)) = s^-(R \cup R^{-1}) = (R \cup R^{-1}) \cup (R \cup R^{-1})^{-1} = (R \cup R^{-1}) \cup (R^{-1} \cup R) = R \cup R^{-1} = s^-(R)$.

Lemma 5.7 $\forall R \in \text{BSVNR}(X)$, we have

$$(1) (r(R^c))^c = ar(R).$$

$$(2) r(ar(R)) = r(R).$$

$$(3) ar(r(R)) = ar(R).$$

Proof. (1) By Theorem 5.3 (1), $r(R^c) = R^c \cup \text{Id}_R$. By Theorems 3.9 (11) and 4.3 (1),

$$(r(R^c))^c = (R^c \cup \text{Id}_R)^c = (R^c)^c \cap (\text{Id}_R)^c = R \cap (\text{Id}_R)^c = ar(R).$$

$$(2) \text{ By Theorems 4.3 (1) and 5.3 (1), } r(ar(R)) = r(R \cap (\text{Id}_R)^c) = (R \cap (\text{Id}_R)^c) \cup \text{Id}_R \\ = (R \cup \text{Id}_R) \cap ((\text{Id}_R)^c \cup \text{Id}_R) = (R \cup \text{Id}_R) \cap X_R = r(R).$$

$$(3) \text{ By Theorems 4.3 (1) and 5.3 (1), } ar(r(R)) = ar(R \cup \text{Id}_R) = (R \cup \text{Id}_R) \cap (\text{Id}_R)^c \\ = (R \cap (\text{Id}_R)^c) \cup (\text{Id}_R \cap (\text{Id}_R)^c) = (R \cap (\text{Id}_R)^c) \cup 0_R = ar(R).$$

Lemma 5.8 $\forall R \in \text{BSVNR}(X)$, we have

$$(1) (s^-(R^c))^c = s(R).$$

$$(2) s^-(s(R)) = s(R).$$

$$(3) s(s^-(R)) = s^-(R).$$

Proof. (1) By Theorem 5.3 (2), $s^-(R^c) = R^c \cup (R^c)^{-1}$. By Theorems 3.9 and 4.3 (2),

$$(s^-(R^c))^c = (R^c \cup (R^c)^{-1})^c = (R^c)^c \cap ((R^c)^{-1})^c = R \cap R^{-1} = s^-(R).$$

(2) and (3), The proofs are straightforward and follow from the Definitions of symmetric kernel and symmetric closure.

VI. CONCLUSION

In this paper we introduced the bipolar single valued neutrosophic relations and their properties were studied along with the suitable examples.

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