TEAM SCHEDULING IN SPORTS BY ROUND ROBIN TOURNAMENTS USING GRAPH THEORY

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ABSTRACT

The main problem in sports scheduling is to determine the date and the venue in which each game of a tournament will be played. Applications are found in the scheduling of tournaments of sports such as football, baseball, basketball, cricket, and hockey.

Preparing a playing schedule is becoming more complex as it becomes harder to balance the objectives of maximizing revenue (broadcast and venue); maximizing travel efficiencies; maintaining "competitive fairness" and accommodating the needs of a diverse group of constituents that includes fans, coaches, players, media, sponsors and facility owners and operators. This paper provides an introductory review of fundamental problems in sports scheduling and their formulations in professional leagues of different sports disciplines such as football, baseball, basketball, cricket, and hockey.

Keywords
Scheduling, Tournaments, Games, Graphs, Combinations, Matches.

INTRODUCTION

The people think about Mathematics play a vital role in the Sciences and Engineering. Yet Mathematics plays a large role in sports. With the help of Mathematics, Coaches constantly try to find best result. There are also mathematical issues involved in scoring systems for some of the complex and subjective aspects of sports events.

However, the sheer magnitude of the number of games that must be played in league sports creates a large domain for Mathematics to assist in the efficient operation of sports such as baseball, football, basketball, soccer, and cricket. Some of the fascinating Mathematics of sports is scheduling related to optimization or quick starting methods.

The way of getting insight into a complex environment is to classify what one sees and study the objects in each of the categories separately as a way of simplifying things. In fact there are many types of tournaments:

ROUND ROBIN TOURNAMENT : Each team plays exactly n games against every other team

ELIMINATION TOURNAMENT : The tournament progresses *n* rounds where in each round some teams are eliminated and the surviving teams are paired in future rounds, where again losers are eliminated

TEAM SCHEDULING

Perhaps the very first question that arises in scheduling is to design the matches that must occur for a round robin tournament. In a single round robin tournament (SRRT) each team must play exactly one game against every other team.

Graph theory, a branch of combinatorial which draws heavily on geometrical ideas, uses diagrams consisting of dots (Vertex) and lines (Edges) helps in mathematical problems. The complete graph on *n* vertices has exactly one edge between every pair of vertices. These graphs are denoted K_n

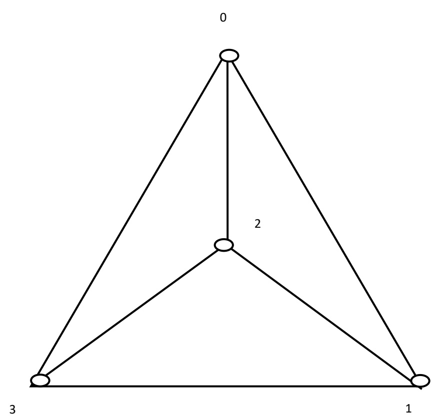

Figure 1

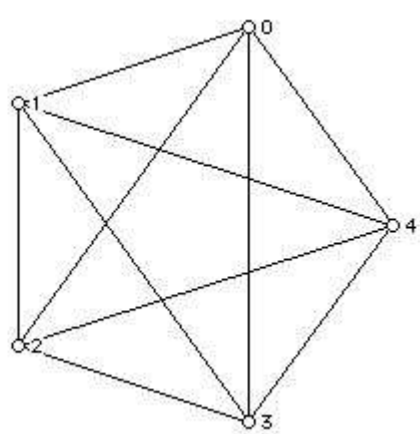

Figure 2

In both the cases, the vertices of the graph are labeled with the names of the teams involved in the tournament. The vertices (dots) of a complete graph represent the teams in a tournament and an edge joining two teams represents a match played by those two teams.

The number of edges of the complete graph with n vertices is $n(n-1)/2$, which is the number of matches that must played with every other team exactly once. In the graph K_n each vertex has n-1 edges at each vertex. The number of edges at a vertex of a graph is known as its degree.

CASE I

In case 1, 4 teams must play with each other.

This means a total of 4(3)/2 = 6 matches must be played.

These matches could be played in 6 time slots, say one a week for 6 weeks. However, it might be desirable if venues (rooms; playing fields) for the matches are available to have several matches per time slot and the games be completed over a shorter period of time. Thus, since there are 4 players, and 4/2 is 2, we could consider two matches per time slot, and complete the tournament in three weeks rather than 6 weeks. Two matches per time slot might mean that there would be two games at exactly the same time or that the games be played in the morning and afternoon on the same "court" of a single day.

Figure 3 shows the details of how the scheduling could work.

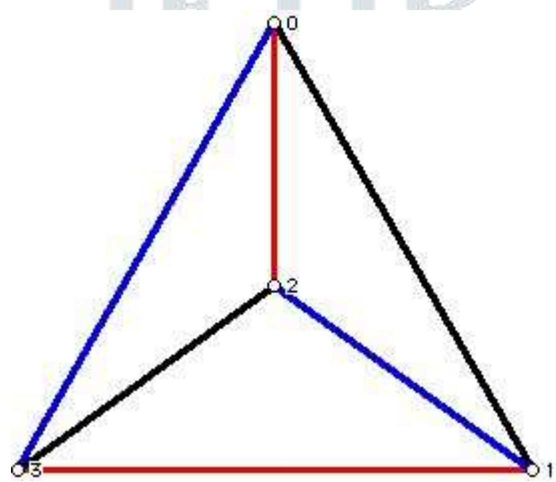

Figure 3

Edges in the graph that have the same color would occur during one time slot.

Thus, for time slot 1 (blue line) there would be matches between team 0 and team 3 and team 1 and team 2;

For time slot 2 (black line) there would be matches between team 0 and team 1 and team 2 and team 3;

For time slot 3 (red line) there would be matches between team 0 and team 2 and team 1 and team 3.

CASE II

In case 2, Since 5 is an odd number we cannot merely have all the teams play in pairs during a time slot. For this, the concept of a "bye" in sports scheduling refers to a team's not having to play a match during a particular time slot. If one has 5 teams, there are 10 matches that must be carried out for a round robin tournament where each team plays every other. Since 5/2 is not an integer, we cannot play 3 games per time slot but we can play 2 games per time slot (4 teams play) and assign a bye to one team. Thus, in five time slot we can schedule the whole tournament. Thus the way to schedule, for five time slots can be constructed and the team which has a bye in each time slot is slotted in Figure 4.

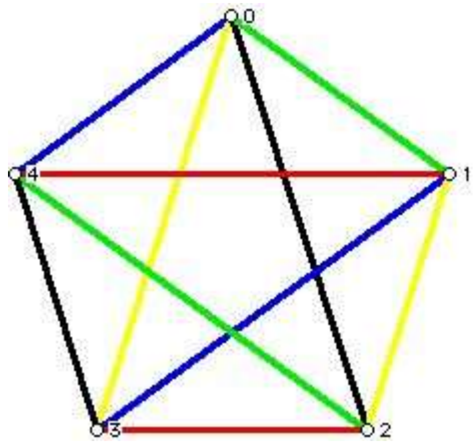

Figure 4

The edges in different colors indicate which teams play in a time slot.

In time slot one (Two yellow lines) there would be matches between team 0 and team 3 and team 1 and team 2; team 4 would get a bye.

In time slot two (Two blue lines) there would be matches between team 0 and team 4 and team 1 and team 3; team 2 would get a bye.

In time slot three (Two green lines) there would be matches between team 0 and team 1 and team 2 and team 4; team 3 would get a bye.

In time slot four (Two brown lines) there would be matches between team 1 and team 4 and team 2 and team 3; team 0 would get a bye.

In time slot five (Two black lines) there would be matches between team 0 and team 2 and team 3 and team 4; team 1 would get a bye.

ROUND ROBIN TOURNAMENT

We discuss the round robin tournaments with 6 teams. Fifteen matches are to be played. Since there are 6 teams there will be 15/3 = 5 time slots with 3 matches in each time slot.

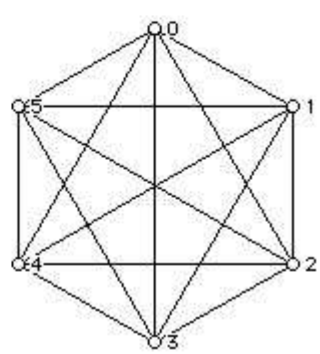

Figure 5

Consider the boundary edge 01 in Figure 5 of the regular hexagon we construct a matches by using the edges that do not meet this edge (that are parallel to it, as it were).We have the matches between 01, 25, and 34 in time slot one. Proceeding around the boundary we get another two groups of matches: 12, 03, 45, and 23, 14, 05 in time slot two. There are 6 edges remaining our aim is to group these into two sets of size 3. However, unfortunately the six edges that remains form two disjoint triangles: edges 02, 24, 04 and 13, 15, 35. Since we cannot pick two disjoint edges from either of these triangles we reach a dead end. There is no way,so we can take our initial group of teams for the first three time slots and extend the result to two more time slots.

We can try some alternate systematic way to schedule 6 teams in a round robin tournament. Here is a method that works and generalizes. Here we number the teams from 1 to *n* rather than from 0 to *n*-1. We will consider only the case with an even number of teams, since when there is an odd number of teams, as already explained we can add a fictional team and whenever a real team is asked to play the fictional team, the real team has a bye.

Consider the case with 6 teams. Construct an initial table with the first half of the teams listed consecutively in the first row and the last half of the teams listed in reverse order in the next row.

The teams that line up in the table will play in the first round.

Round 1:

1	2	3
6	5	4

In figure 6, the edge 1 to 6 is shown vertically in a diagram where the vertex 1 is placed at the "center" of a regular pentagon and the numbers 2 and 3 are listed clockwise starting from 1, while the numbers 5 and 4 are shown counterclockwise starting at 6. In the diagram the edges which make up the sides of the regular convex pentagon are omitted. Only the edges which make up the pairings in one round for the teams are shown. However, in the partition of the complete graph on 6 vertices into 3 disjoint edges. Since there are 5 rounds each with 3 edges we can account for all of the 15 edges in the complete graph on 6 vertices.

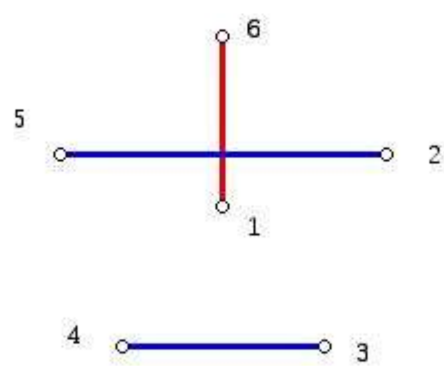

Figure 6

We will fix the pairings for the next round ,fix the first team in the cell in the first row and first column and rotate the other teams in the clockwise direction: 2, 3, 4, 5, and 6. On rotating one position clockwise and record the entries into a new table: 6, 2, 3, 4, 5.

Round 2:

1	6	2
5	4	3

After clockwise rotation of the vertices in the regular pentagon the diagram will be as follows

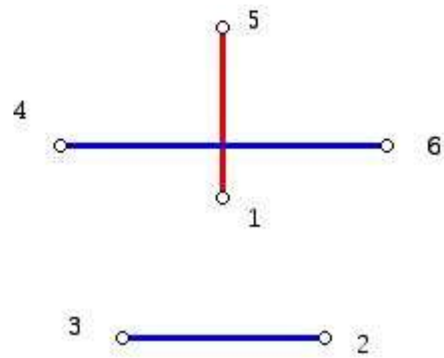

Figure 7

We repeat this rotation operation until we exhaust the new pairings and return to the start.

Round 3:

1	5	6
4	3	2

Then the corresponding labeled graph is

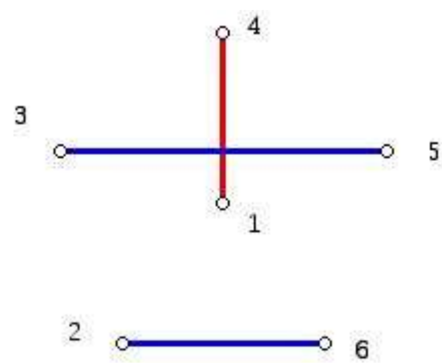

Figure 8

Round 4:

1	4	5
3	2	6

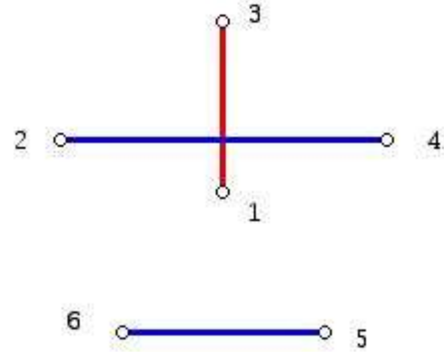

Figure 9

Round 5:

1	3	4
2	6	5

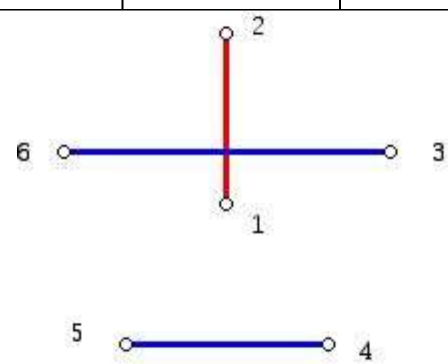

Figure 10

CONCLUSION:

Thus, the fundamental basis in sports scheduling and their formulations in professional leagues of different sports disciplines such as football, baseball, basketball, cricket, and hockey etc., has been discussed by round robin tournament using graph theory with odd and even number of teams.

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