EFFECT OF CONVECTIVE MHD IN NON-NEWTONIAN CASSON FLUID

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Abstract:

In this paper, we analyze the effect of heat transfer on free convection flow of casson fluid over a vertical plate with Hall effect has been studied. A similarity analysis approach was used to transform the system of PDE, analytical solutions are acquired with the aid of solving the ODE to research the rate and temperature fields. Variations of thrilling parameters on the velocity, heat transfer, pores and skin friction are determined by using plotting graphs. Further, it was changed into concluded that the casson fluid parameter and Hall parameter has an retarding influence on velocity profile and additionally inside the skin friction.

Keywords: Casson fluid, Hall effect, MHD, Free convection, Heat and mass transfer.

INTRODUCTION:

The study of convective heat transfer move wonders in nature are frequently gone to by mass transfer. Convective mass transfer makes the help of different strategies in the chemical engineering. This shows like adequate reason to contain mass transfer in heat convection too. A similarity occurs between convective mass transfer and convective heat transfer. This similarity is instructively real huge since it gives an opportunity to arrange the comprehension of heat transfer and to learn mass transfer with the least retention muthukumaraswamy et al. [1] considered mass transfer effects on exponentially accelerated isothermal vertical plate. Haque et al. [2] examined MHD micro polar fluid flow with constant heat and mass fluxes whereas, Das et al. [3] discussed heat and mass transfer of a second grade MHD fluid over a convectively heated stretching sheet. Nadeem and Akbar [4] considered MHD on the peristaltic flow in an annulus with heat and mass transfer. Simultaneously, unsteady free convention MHD flow with heat and mass transfer from numerous geometrices in permeable media has few desiging and geophysical applications for example underground energy transport drying of permeable solids, thermal insulation and geothermal supplies. Hayat et al [5] contemplated MHD second grade fluid flow in permeable channel. Rassouline-Mousavi et al. [6-7] talked about constrained convection in a vertical plate filled with a Darcy-Brinkman-Forchheimar permeable medium Hussnan et al. [8] characterized the arrangement of unsteady MHD free convention flow in a permeable medium with constant mass diffusion. Nadeem et al. [9] and Khalid et al. [10] talked about unsteady free convective MHD casson fluid flow with heat and mass transfer through permeable medium. Das et al. [11] contemplated magneto-nanofluid flow past an impulsively started permeable plate.

The aim of this investigation is to review the MHD flow, heat and mass transfer characteristics of an electrically conducting incompressible Casson fluid during a channel with stretching walls within the presence of a reaction subject to a consistent transverse magnetic field during a situation where the surface velocity of the channel varies linearly with distance from the origin. The target of the study is to research the flow of blood in arteries whose walls are stretchable by modelling blood as a Casson fluid. The constitutive equation for the Casson fluid model suggested by Nakamura and Sawada [12]; Eldabe and Silwa [13]; Das [14]; Boyd [15] is employed. The non-linear coupled equations are very complex to seek out exact solution hence we employed a numerical method to solve the problem.

FORMULATION OF THE PROBLEM:

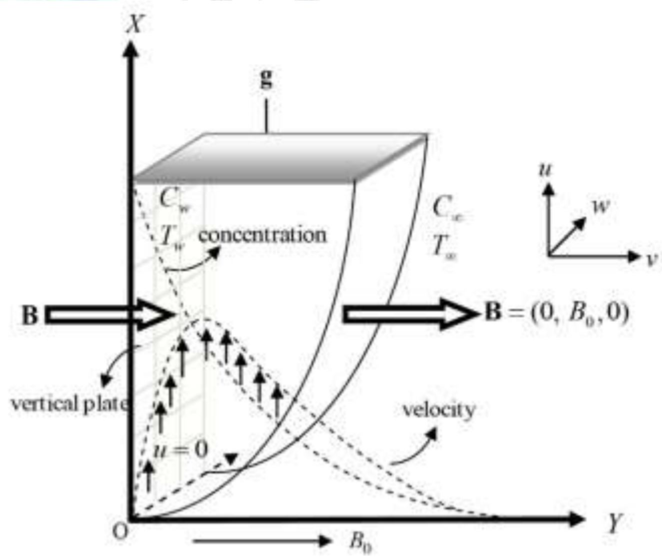
Consider the flow of an incompressible viscous fluid past a flat sheet which coincide with the plane y=0.The fluid flow is restricted to y>0.Two equal and opposite forces are carried along the y-axis simply so that the wall is stretched retaining the origin fixed. The rheological equation of the state for an isotrophic and incompressible flow of a casson fluid is as follows

$$\tau_{ij} = \begin{cases} 2(\mu_B + p_y/\sqrt{2\pi})e_{ij}, & \pi > \pi_c \\ 2(\mu_B + p_y/\sqrt{2\pi_c})e_{ij}, & \pi < \pi_c \end{cases}$$

Here $\pi = e_{ij}e_{ij}$ and e_{ij} are the (i, j)th factor of the deformation rate, π is the product of the component of deformation the Non-Newtonian model, μ_B is plastic dynamic viscosity of the Non-Newtonian fluid and P_y is that the yield stress of the fluid. The fluid flow is assumed to be within the X direction which is taken along the plate within the upward direction and Y axis is normal to it as verified in figure 1. Instantaneously at time t>0 the plate temperature and concentration are raised to $T_w > T_\infty$ and $C_w > C_\infty$ respectively, which are thereafter sustained. Here Tw, Cw are temperature and centralization outside the physical phenomenon.

A uniform magnetic field B is imposed parallel to the Y-axis. Due to the consideration of the thermal radiation, the rosseland approximation for thermal radiation, $q_r = -4\sigma^*/3k^*(\delta T^4/\delta y)$ is introduced, which is thereafter takes the form, $q_r = -16\sigma^*/3k^*\, T^3_\infty\, \delta T/\delta y$ in association with the Taylor and series for T^4 about T_∞ Implies $T^4 = 4T^3_\infty - 3T^4_\infty$. In the case of Hall effect, a Z-component for the velocity is anticipated to arise. Thus the fluid velocity vector

$q = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$.



The non-dimensional variables that have been used within the governing equations can be written as follows:

$$X = \frac{xU_0}{\upsilon},\ \ Y = \frac{yU_0}{\upsilon},\ \ U = \frac{u}{U_0},\ \ V = \frac{v}{U_0},\ \ W = \frac{w}{U_0},\ \ \text{Ʈ} = \frac{tU_0{}^2}{\upsilon}$$

$$Q = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \varphi = \frac{(C - C_\infty)}{(C_w - C_\infty)}$$

Using those above dimensionless variables, the received dimensionless equations are given as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \left(1+\frac{1}{\beta}\right)\frac{\partial^2 U}{\partial Y^2} + G_r + G_r^* - \frac{M}{(1+m^2)}(U+mW) \tag{2}$$

$$\frac{\partial W}{\partial \tau} + U\frac{\partial W}{\partial X} + V\frac{\partial W}{\partial Y} = \left(1+\frac{1}{\beta}\right)\frac{\partial^2 W}{\partial Y^2} - \frac{M}{(1+m^2)}(W - mU) \tag{3}$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \left(\frac{Q_r+1}{P_r}\right)\frac{\partial^2 \theta}{\partial Y^2} + E_c\left(1+\frac{1}{\beta}\right)\left[\left(\frac{\partial U}{\partial Y}\right)^2 + \left(\frac{\partial W}{\partial Y}\right)^2\right] + \frac{J_h}{(1+m^2)}(U^2+W^2) - \bar{Q}\theta \tag{4}$$

$$\frac{\partial \phi}{\partial \tau} + U\frac{\partial \phi}{\partial X} + V\frac{\partial \phi}{\partial Y} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial Y^2} + S_r\frac{\partial^2 \phi}{\partial Y^2} \tag{5}$$

Where, $\beta = \frac{\mu_b\sqrt{2\pi}}{p_y}$ is the Casson fluid parameter.

The initial and boundary conditions can be written as follows:

$\tau > 0,\ U=0,\ W=0,\ \theta=1,\ \phi=1$ at $Y=0$ and $U\to 0,\ W\to 0,\ \theta\to 0,\ \phi\to 0$ at $Y\to\infty$

The non-dimensional parameters are given as follows:

Grashof number, $G_r = \frac{g\beta_T(T-T_\infty)v}{U_0^3}$; Modified Grashof number, $G_r^* = \frac{g\beta_c(C-C_\infty)v}{U_0^3}$;
Magnetic parameter, $M = \frac{\sigma B_0^2 v}{\rho U_0^2}$; Prandtl number, $P_r = \frac{\rho C_p v}{K}$; Eckert number, $E_c = \frac{U_0^2}{C_p(T_w-T_\infty)}$;
Radiative parameter, $Q_r = \frac{16\,\sigma^* T_\infty^3}{3K^* K}$; Schmidt number, $S_c = \frac{v}{D_m}$; Heat source parameter, $\widetilde{Q} = \frac{Qv}{\rho C_p U_0^2}$;
Soret number $S_r = \frac{D_T}{v}\frac{(T_w - T_\infty)}{(C_w - C_\infty)}$ and Joule Heating parameter $J_h = ME_c$.

SOLUTION OF THE PROBLEM:

To solve the governing non-linear coupled dimensionless PDE (1) to (5) with the related initial and boundary conditions, the precise finite difference method has been used. To get the difference equations within the region of the flow is split into a grid or mesh of lines parallel to X and Y axis where X axis is taken along the plate and Y axis is normal to the plate as shown within the figure.

It is assumed that the maximum length of the physical phenomenon is X_{max}=60 (i.e) X varies from 0 to 60 and therefore number of grid spacing in X direction is m=60, hence the constant mesh size along X axis becomes ΔX=1.0 ($0\le X\le 60$) and Y_{max} =20 (i.e) Y varies from 0 to 20 and the number of grid spacing in Y direction is n=60, hence the constant mesh size along Y axis becomes ΔY=0.33 (0≤Y≤20) with a smaller time step $\Delta\boldsymbol{\tau}$=0.001.

Let U',W',θ' and ϕ ' denote the values of U,W,θ and ϕ at the end of a time step respectively.Using the unique finite difference technique to the system of PDE (1) to (5) the obtained suitable finite difference equations are given as follows:

$$\frac{U_{i,j}-U_{i-1,j}}{\Delta X}+\frac{V_{i,j}-V_{i-1,j}}{\Delta Y}=0 \tag{6}$$

$$\frac{U'_{i,j}-U_{i,j}}{\Delta\tau}+U_{i,j}\frac{U_{i,j}-U_{i-1,j}}{\Delta X}+V_{i,j}\frac{U_{i,j}-U_{i,j-1}}{\Delta Y}=\left[1+\frac{1}{\beta}\right]\left[\frac{U_{i,j+1}-2U_{i,j}+U_{i,j-1}}{(\Delta Y)^2}\right]+G_r+G_r^*-\frac{M}{(1+m^2)}(U_{i,j}+mW_{i,j}) \tag{7}$$

$$\frac{W'_{i,j}-W_{i,j}}{\Delta\tau}+U_{i,j}\frac{W_{i,j}-W_{i-1,j}}{\Delta X}+V_{i,j}\frac{W_{i,j}-W_{i,j-1}}{\Delta Y}=\left[1+\frac{1}{\beta}\right]\left[\frac{W_{i,j+1}-2W_{i,j}+W_{i,j-1}}{(\Delta Y)^2}\right]-\frac{M}{(1+m^2)}(W_{i,j}+mW_{i,j}) \tag{8}$$

$$\frac{\theta'_{i,j}-\theta_{i,j}}{\Delta\tau}+U_{i,j}\frac{\theta_{i,j}-W_{i-1,j}}{\Delta X}+V_{i,j}\frac{\theta_{i,j}-\theta_{i-1,j}}{\Delta Y}=\left[\frac{Q_r+1}{P_r}\right]\left[\frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i,j-1}}{(\Delta Y)^2}\right]+\frac{J_h}{(1+m^2)}\left[(U_{i,j})^2+(W_{i,j})^2\right]+ E_c\left(1+\frac{1}{\beta}\right)\left[\left(\frac{U_{i,j-1}-U_{i,j-1}}{\Delta Y}\right)^2+\left(\frac{W_{i,j-1}-W_{i,j-1}}{\Delta Y}\right)^2\right]-Q\theta_{i,j} \tag{9}$$

$$\frac{\phi'_{i,j}-\phi_{i,j}}{\Delta\tau}+U_{i,j}\frac{\phi_{i,j}-\phi_{i-1,j}}{\Delta X}+V_{i,j}\frac{\phi_{i,j}-\phi_{i,j-1}}{\Delta Y}=\frac{1}{S_c}\left[\frac{\phi_{i,j+1}-2\phi_{i,j}+\phi_{i,j-1}}{(\Delta Y)^2}\right]+S_r\left[\frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i,j-1}}{(\Delta Y)^2}\right] \tag{10}$$

The boundary conditions with the finite difference are given as follows:

$\tau>0,\ U_{i,L}=0,\ W_{i,L}=0,\ \theta_{i,L}=1,\ \phi_{i,L}=1$ at $L=0$

$U_{i,L}\to 0,\ W_{i,L}\to 0,\ \theta_{i,L}\to 0,\ \phi_{i,L}\to 0$ at $L\to\infty$

Here the subscripts i and j designate the grid points with X and Y coordinates respectively.

RESULTS AND DISCUSSION

To study the physical condition of the developed mathematical model, the constant state numerical values have been computed for the non-dimensional primary velocity (U), Secondary velocity (W), Temperature (θ) and concentration (ϕ) distributions inside the boundary layer. The steady state solution has been obtained at the dimensionless time τ=15.The effect of Magnetic parameter (M), Hall parameter (m), Prandtl number (P_r), Grashof number (G_r), modified Grashof number (G^*_r), Eckert number (E_c), Radiative parameter (Q_r), Schmidt number (S_c), Heat source parameter ($\widetilde{Q}$), Soret number (S_r), Joule Heating parameter (J_h) and casson fluid parameter (β) are not shown.

Mesh sensitivity test: To attain the appropriate mesh space for m and n, the computation have been carried out for 3 distinct mesh spaces such as (m, n) = (60, 60), (80, 80), (100, 100) as shown in the figure 2. The curves are smooth for all mesh spaces and suggests a negligible modifications among the curves. Thus the mesh size (m, n) = (60, 60) has been considered.

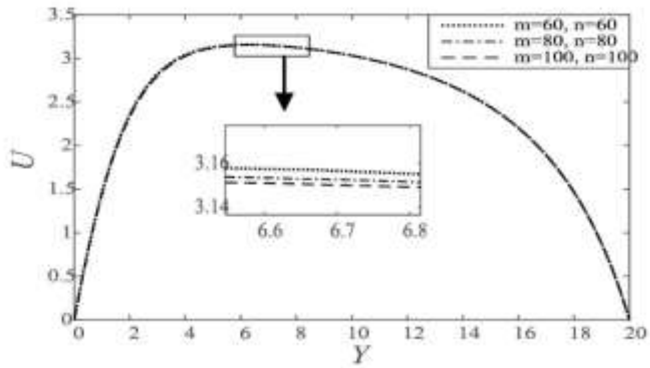


FIGURE 2. Mesh Sensitivity for Primary Velocity

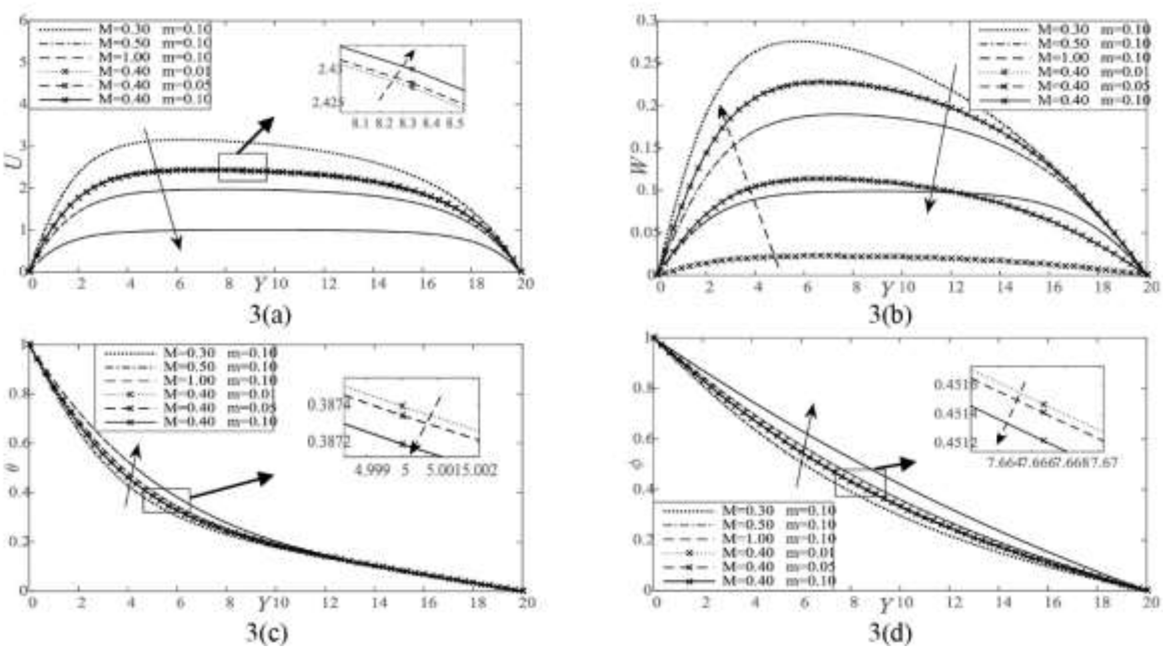


Figure 3: Effect of Magnetic parameter (M) and Hall parameter (m) on (a) Primary velocity; (b) Secondary velocity; (c) Temperature and (d) Concentration distributions; where, G_r =1.00 G^*_r =1.00, E_c=0.01, P_r =0.30, Q_r=0.05, S_c=0.10, S_r=0.10, $\widetilde{Q}$=0.05 and β=1.00 at time $\boldsymbol{\tau}$=20 (Steady state)

Fig. 3 shows that the primary and secondary velocities both decreases with the increase of (M). On the other hand, the primary and secondary velocities both increases with the increase of (m) while the temperature and concentration distributions both decreases with the increase of (m).

CONCLUSION

The results are discussed for different values of important parameters as magnetic parameter (M) and Hall parameter (m). The effects of other parameters are not shown. From the above results and discussion following conclusions are made.

1.The primary and secondary velocities both decrease with the increase of (M) while both primary and secondary velocity increase with the increase of (m).

2.The temperature and concentration distributions both increases with the increase of (M) while both decreases with the increase of (m).

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