APPLICATION OF INVERSE LAPLACE TRANSFORMATIONABSTRACT

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Abstract

The Laplace transformation is a mathematical tool which is used in the solving of differential equations by converting it from one from in to another from. Regularly it is effective in solving linear differential equations either ordinary or partial . The Laplace transformation is used in solving the time domain function by converting it Into frequency domain function. Laplace transformation makes it easier to solve the differential problem in engineering application and make differential equations simple to solve. In this paper we will discuss how to find the inverse Laplace transformation through Heaviside is expansion formula.

Keywords: Laplace transformation, Differential equations.

1. INTRODUCTION

The Laplace transformation is applied in different areas of science, Engineering and technology . The Laplace transformation is applicable in so many fields. The Laplace transformation was primary used and named after by plerre simon laplace was a French Mathematics, statics, Physics and astronomy. He contributed seriously to physical mechanics, by converting the previous geometrical analysis to one based on calculus , which opened up application of his formulas to a wider range of problems . it is effective in solving linear differential equation either ordinary or partial. It reduces an ordinary differential equation into algebraic equation.

If F(P)= : $\frac{UCP)}{VCP)}, where\ Ucp)\$\ Vcp)\ do\ not$

Keep common factor and the degree of numerator is less than the degree of denominator. If VCP) has different roots, than we find by this theorem, inverse Laplace transformation.

HEAVISIDE'S EXPANSION FORMULA
If UCP) & VCP) be polynomials in P where the degree of Numerator is less than the degree of denominators. If VCP) has n different roots
C1, C2, C3 ………………………. Cn, Then

$$\mathrm{L}^{-1}\left\{\frac{UCP)}{VCP)}\right\} = \sum_{i=1}^{n} \frac{UCC)}{VCCi)} \text{e tci}$$

Proof:
Since V(P) has n different roots, it factorize into n linear factors and hence by partial fractions, we have

$$\frac{UCP)}{\mathrm{VCP)}} = \frac{C1}{\mathrm{P-C1}} + \frac{C2}{\mathrm{P-C2}} + \frac{C3}{\mathrm{P-C3}} \quad \frac{Ci}{\mathrm{P-Ci}} \text{ or } + \frac{Cn}{\mathrm{P-Cn}}$$

Multiplying both sides by (P –Ci),

We get,

$$\mathrm{Ci{=}lim}\frac{UCP)}{VCP)}\ (P-ci)$$
$$p-Ci$$

$$\text{Ci}=\lim Ucp)\frac{(p-ci)}{VCP)}$$
$$p-Ci$$
$$\text{Ci}=\lim Ucp)\frac{lim}{P-ci}\frac{(p-ci)}{VCP)}$$
$$p-Ci$$
$$\text{Ci}=\frac{UCCi)}{V1\ cci)}$$
$$\text{Hence, }\frac{\text{Ucp)}}{VCP)}=\sum_{i=1}^{n}\frac{UCC)}{V1CCi)}\frac{1}{(p-ci)}$$
$$\text{L}^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\sum_{i=1}^{n}\frac{UCCi)}{V1CCi)}\text{L}-1\left\{\frac{1}{(p-ci)}\right\}$$
$$\text{L}^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\sum_{i=1}^{n}\frac{UCCi)}{V1CCi)}\text{etci}$$

(A) We have to find

$$\text{L}^{-1}\left\{\frac{107p2-q7p+203}{P3+6p2+11p-6}\right\}$$

Here $U(p)=107p^2-97p+203$

$V(p)=p^3+6p^2+11p-6$

$V'(p)=3p^2-12p+11$

To find roots of

$V(p)=p^3+6p^2+11p-6=0$

Then roots are

$C1=1,\ C2=2,\ C3=3$

Now

Since

$U(p)=107p^2-97p+203$

Then

$V'(p)=2,\ V'(2)=-1,\ V'(3)=2$

Now, by Heaviside's Expansion formula

$$\text{L}^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\sum_{i=1}^{n}\frac{UCCi)}{V1CCi)}\text{etci}$$
$$\text{L}^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\frac{UCc1)}{V1Cc1)}\text{e}^{tc1}+\frac{UCc2)}{V1Cc2)}\text{e}^{tc2}+\frac{UCc3)}{V1Cc3)}\text{e}^{tc3}$$
$$\text{L}^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\frac{UC1)}{V1C1)}\text{e}^{t}+\frac{UC2)}{VC2)}\text{e}^{2t}+\frac{UC3)}{V1C3)}\text{e}^{3t}$$
$$\text{L}^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\frac{213}{2}\text{e}^{t}+\frac{437}{-1}\text{e}^{2t}+\frac{875}{2}\text{e}^{3t}$$
$$\text{L}^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\frac{213}{2}\text{e}^{t}-437\text{e}^{-2t}+\frac{875}{2}\text{e}^{3t}$$

Hence,

$$\text{L}^{-1}\left\{\frac{107p2-q7p+203}{p3+6p2+11p-6}\right\}=\frac{213}{2}\text{e}^{t}-437\text{e}^{2t}+\frac{875}{2}\text{e}^{3t}$$

(A) Find

$$\mathrm{L}^{-1}\ \left\{\frac{21p2+10p-41}{p4+2p3-5p2-6p}\right\}$$

Let,

U(p)=21p^2+10p-41
V(p)=p^4+2p^3-5p^2-6p
V'(p)=4p^3+6p^2-10p-6

To find roots of

V(p)=p^4+2p^3-5p^2-6p=0

The roots are

C1=0, C2=-1, C3=2, C4=-3

Now, Since

U(p)=21p^2+10p-41
U(0)=-41, U(-1)=-30, U(2)=63, U(-3)=118

And, Since

V'(p)=V'(p)=4p^3+6p^2-10p-6
V'(0)=-6, V'(-1)=6, V'(2)=30, V'(-3)=-48

Now by Heaviside's Expansion formula

$$\mathrm{L}-1\left\{\frac{UCP)}{VCP)}\right\}=\sum_{i=1}^{n}\frac{UCCi)}{V1CCi)}\mathrm{etci}$$

Or

$$\mathrm{L}^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\frac{UCc1)}{V1Cc1)}\mathrm{e}^{tc1}+\frac{UCc2)}{V1Cc2)}\mathrm{e}^{tc2}+\frac{UCc3)}{V1Cc3)}\mathrm{e}^{tc3}$$

$$\mathrm{L}^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\frac{UCo)}{V1Co)}+\frac{UC-1)}{V-1C-1)}\mathrm{e}^{-t}+\frac{UC2)}{V1C2)}\mathrm{e}^{2t}+\frac{UC-3)}{V1C-3)}\mathrm{e}^{-3t}$$

$$\mathrm{L}^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\frac{41}{6}-\frac{30}{6}\mathrm{e}^{-t}-\frac{63}{30}\mathrm{e}^{2t}-\frac{118}{48}\mathrm{e}^{-3t}$$

$$\mathrm{L}^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\frac{41}{6}-5\mathrm{e}^{-t}-\frac{21}{10}\mathrm{e}^{2t}-\frac{59}{24}\mathrm{e}^{-3t}$$

Hence

$$\mathrm{L}^{-1}\left\{\frac{21p2+10p-41}{p4+2p3-5p2-6p}\right\}=\frac{41}{6}-5\mathrm{e}^{-t}-\frac{21}{10}\mathrm{e}^{2t}-\frac{59}{24}\mathrm{e}^{-3t}$$

(C)

$$\mathrm{L}^{-1}\ \left\{\frac{7p2+11p+13}{p3-p2+4p-4}\right\}$$

Let

U(p)= 7p^2+11p+13

V(p)=p^3-p2+4p-4

$$V'(p)=3p^2-2p+4$$

To find roots of

$$V(p)=p^3-p^2+4p-4=0$$

Then roots are

$$C1=1,\ C2=2,\ C3=-2$$

Now since

$$U(p)=7p^2+11p+13$$

Then U(1)=31,U(2)=63,U(-2)=19

And since

$$V'(p)=3p^2-2p+4$$

$$V'(1)=5,\ V'(2)=12,\ V'(-2)=20$$

Now by Heaviside's Expansion formula

$$L^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\sum_{i=1}^{n}\frac{UCCi)}{V1CCi)}\text{etci}$$

Or

$$L^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\frac{UCc1)}{V1Cc1)}e^{tc1}+\frac{UCc2)}{V1Cc2)}e^{tc2}+\frac{UCc3)}{V1Cc3)}e^{tc3}$$

$$L^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\frac{UC1)}{V1C1)}et+\frac{UC2)}{V1C2)}e^{2t}+\frac{UC-2)}{V1C-2)}e^{-2t}$$

$$L^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\frac{31}{5}et+\frac{63}{12}e^{2t}+\frac{19}{20}e^{-2t}$$

$$L^{-1}\left\{\frac{UCP)}{VCP)}\right\}=\frac{31}{5}et+\frac{21}{4}e^{2t}+\frac{19}{20}-^{2t}$$

Hence

$$L^{-1}\left\{\frac{7p2+11p+13}{p3-p2+4p-4}\right\}=\frac{31}{5}et+\frac{21}{4}e^{2t}+\frac{19}{20}e^{-2t}$$

Conclusion

The main purpose of this paper is to give brief ided to find the inverse Laplace Transformation when numerator and denominator do not keep common factor and degree of numerator is less than the degree of denominator . The Primary use of Laplace Transformation is converting a time domain function . Laplace Transformation is a very useful mathematical tool to make simpler complex problems in the area of stability and control.

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