Pole Placement Technique For Effective Design and Analysis of State Feedback Controller

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Abstract

The target of this paper is to examine the stability of control system. For further analysis compare the stability of the system with and without controller for the given system. The LTI system is subjected to linear state feedback and its response is studied under the application of some initial conditions. the stability of the system is discover using pole placement technique of the system to specified location in s-plane using appropriate linear state feedback is also discussed and implemented in MATLAB. For further analysis use the case study of inverted pendulum for pole placement technique

***Keywords**: stability, LTI system, MATLAB, pole placement, linear feedback control*

1.Introduction

In this model use pole placement method which is a method employed in feedback control system theory to place the closed-loop poles of a plant in pre-determined locations in the s-plane. Placing poles is desirable because the location of the poles corresponds directly to the eigen values of the system, which control the characteristics of the response of the system. The system must be considered controllable in order to implement this method.

A majority of the design technique in modern control theory is based on the state feedback configuration. that is, instead of using controller with fixed configuration in the forward or feedback path, control is achieved by feeding back the state variables through real constant gain.

Therefore In control theory there are two main methods of analyzing feedback systems: the transfer function (or frequency domain) method and the state space method. When the transfer function method is used, attention is focused on the locations in the s-plane where the transfer function (the poles) or zero (the zeroes).

Stability is an important aspect of control system. Since for any result with control system, its stability must be assured. Pole Placement Control gives effective and agreeable outcomes both for dynamic and steady state in close loop when applied to LTI System.

A.G.O.Mutumbara [1] provided the MATLAB implementation of the problem statement and its solution to verify and rectify the discussed algorithm in this paper. H. Seraji [2] provided the general method of controller design for multivariable system in which more than one control is provided. Kautsky, Nichols and Dooren [3] talked about the robust solution to pole placement issue through L.U. Decay (mathematical techniques). John W. Brewer[5] provides the analysis design and simulation of pole placement technique. K.Ogata [6] and M. Gopal [7] illuminated the rudiments of Modal control and plan investigation of a controller utilizing ideas of state space. Basics of linear feedback system can be seen in detailed in S. Barnett [9].. T. H.S. Abdelaziz and M. Valasek [10] altogether examined the pole placement issue and gave its immediate arrangement which is very like the Ackermann's formula. Timothy and Bona [11] talked about the fundamentals of controller plan and output response with the assistance of state space. Some more outcomes on pole placement utilizing various methodologies can be seen in [8] and [4].

In this further, discussed about the case study from which it introduce the Design and analysis of state feedback controller using pole placement technique. This can be better understanding by taking example as a case study of inverted pendulum. This paper has given the summed up strategy and procedure for tackling pole placement problem by given calculation which is considerably less iterative than different techniques accessible.

2. MODEL OF CLOSE LOOP SYSTEM TO FIND CONTROLLER

In this project consider the system which is linear time invariant system. For project assume the system is controllable as well as observable

As shown in fig.(1) Considering a linear time invariant system represented by following equations,

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$

$$y(t) = C\,x(t) + D\,u(t)$$

Here $x(t)$ = it is the column vector addresses the conditions of the system, $u(t)$ = it is the column m vector addressing control variables applied to the system.

A, B, C, D are genuine matrix having order (n*n), (n*m), (p*n) and (p*m) separately. The framework is thought to be totally controllable and observable.

Presently we apply linear state feedback to the above framework. That is each control variable is linear mix of n state factors and reference signal r. We get the square portrayal of the framework with such design as

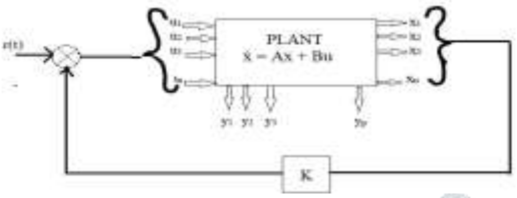


fig. 1

Block diagram representation of closed loop control system.

Assume that the reference signal isn't applied to the system i.e.

$r(t) = 0$, then we get the control variable $u(t)$ as:

$$u(t) = K\,x(t)$$

where K is the m*n real matrix and termed as feed-back matrix.

The overall system equation now becomes

$$\dot{x}(t) = (A + BK)\,x(t)$$

for this project limiting scope to m = 1, i.e. u(t) is scalar.

Presently the problem statemen for the project is to track down an appropriate worth of K to such an extent that the poles of the overall closed loop system are { $\lambda_1, \lambda_2, \ldots, \lambda_n$ } .and to improve the stability of the system and to improve the settling time of the system.

For this project Consider the system shown in Figure. The plant is given by $\dot{x} = Ax + Bu$

Where, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

The system uses the state feedback control u= −Kx. Let us choose the desired closed-loop poles at s =2 + j4 , s =2 - j4, s =-10 . Determine the state feedback gain matrix K

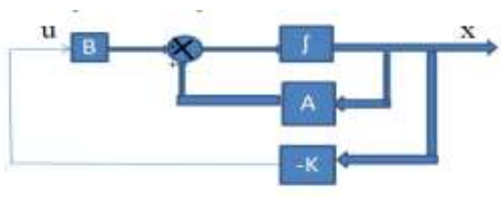


fig.2- Block diagram of system

2.1. POLE PLACEMENT PROBLEM BY MANUALLY AND BY USING MATLAB

- For finding general solution for pole placement problem.
- There are several methods of solution for pole placement problems, there three method of solving the pole placement problems, these are as follows.

1) Determination of Matrix K Using Transformation Matrix T.
2) Determination of Matrix K Using Direct Substitution Method
3) Determination of Matrix K Using Ackermann's Formula

- Now for comprehend the above techniques taking one issue and addressing by utilizing all the three strategy and check the appropriate response
- After that, compute same model in MATLAB utilizing Ackermann's formula.
- By addressing manually and in MATLAB compare that the appropriate response is same or not.

Example

a) **Solve the Example manually by using all three methods**

First, we need to check the controllability matrix of the m . Since the

$$M = [B\ AB\ A^2B] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 31 \end{bmatrix}$$

Hence, |M| = −1, and therefore, rank M=3 ,the system is completely state controllable and arbitrary pole placement is possible. Next, solve this problem and demonstrate each of the three methods .

Solution:-

1) **Method 1** : The first method is to Determination of Matrix K Using Transformation Matrix T , use Equation

$$K = [\alpha_n - a_n \quad \alpha_{n-1} - a_{n-1} \ldots\ldots\ldots \alpha_2 - a_2 \quad \alpha_1\, a_1]T^{-1}$$

The characteristic equation for the system is $sI - A$=0 ,therefore

$$K = [199 \quad 55 \quad 8]$$

2) **Method 2**: This method is to use Determination of Matrix K Using Direct Substitution Method. By defining the desired state feedback gain matrix K as

$$K = [\, k_1 \;\; k_2 \;\; k_3]$$

and equating |sI - A + BK| with the desired characteristic equation , we obtain

K = 199 55 8

3) **Method 3**: The third method is to use Determination of Matrix K Using Ackermann's Formula Ackermann's formula Referring equation of k, we obtain

$$K = [0\ \ 0\ \ldots\ldots 0\ 1][B\ \ AB\ \ A^2B]^{-1}\phi(A)$$

Since,

$$\phi(A) = A^3 + \alpha_1 A^2 + \alpha_2 A + \alpha_3 I$$

We obtain,

K = [199 55 8]

b) solve by ackermans formula using MATLAB

Consider the same system as shown fig.2 example. The system equation is

$$\dot{x} = Ax + Bu$$

Where, By using state feedback control it is desired to have the closed-loop poles at s= μ_i (i= 1, 2, 3), where μ_1 = -2+ j4 , μ_2 = -2-j4 , μ_3 = -10 Determine the state feedback-gain matrix K with MATLAB.

MATLAB Program

```
A=[0 1 0;0 0 1;-1 -5 -6];
B=[0;0;1];
J=[-2+j*4 -2-j*4 -10];
K=acker(A,B,J)

>> Programfork

K =

   199    55     8

fx >>
```

above MATLAB program is of command **acker** . and program shown below is of command **place** for finding controller K

MATLAB Program :

```
Command Window
>> A=[0 1 0;0 0 1;-1 -5 -6];
>> B=[0;0;1];
>> J=[-2+j*4 -2-j*4 -10];
>> K=place(A,B,J)

K =

   199.0000   55.0000    8.0000

fx >>
```

2.2. RESPONSE OF SYSTEM WITHOUT AND WITH USING CONTROLLER

Example Consider the system as shown in fig.2 above two example it is desired that this system have closed loop poles at

$$s = -2+j4\ ,\ s = -2 - j4\ ,\ s = -10$$

The necessary state feedback gain matrix K was obtained in above example as follows:

K = [199 55 8]

Using MATLAB, obtain the response of the system as :

1) Without using controller
2) With using controller

For the following initial condition

$$X(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

where t is the time duration which is,

t = 0:0.01:4;

and x1, x2, and x3 as follows:

x1 = [1 0 0]*x'; x2 = [0 1 0]*x'; x3 = [0 0 1]*x';

Solution:-

1) **MATLAB Program without using controller:-**

Response :-

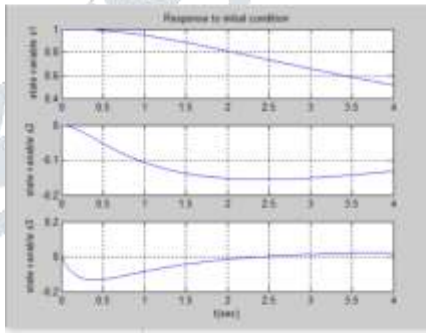


fig.3- response without using controller

2) **MATLAB Program with using controller**

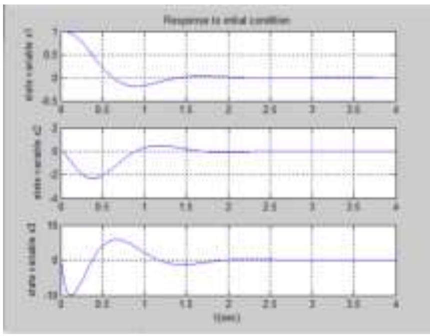


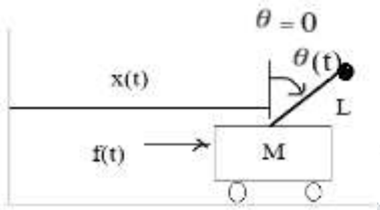
fig. 4 – response with using controller

To introduce the Design and analysis of state feedback controller using pole placement technique case study is shown below

3. DESCRIPTION OF THE CASE STUDY SYSTEM OF INVERTED PENDULUM

As an ordinary control system, the control of a changed pendulum is fabulous in testing and surveying modified control procedures. Its ubiquity gets partially from the way that it is shaky without control, that is, the pendulum will essentially fall over if the cart isn't moved to adjust it. Also, the elements of the system are nonlinear. The goal of the control system is to adjust the inverted pendulum by applying a force to the cart that the pendulum is appended to.

3.1.MATHEMATICAL MODELING OF THE SYSTEM



System Equations

Given parameters,

m	Mass of the inverted pendulum	0.1 kg
L	Length of the inverted pendulum	1 m
M	The mass of the cart	1 kg
θ(t)	The angular position of the pendulum from the vertical	
x(t)	The horizontal displacement of the cart	
f(t)	The applied force on the cart in the same direction as x(t)	
g	the acceleration due to gravity	9.8 m/s^2

Table no.1 Specification and values of constants

The equation for the system is shown below,

$$(M+m)\ddot{x}(t) + mL\ddot{\theta}(t)\cos\big(\theta(t)\big) - mL[\dot{\theta}(t)]^2\sin\big(\theta(t)\big) = f(t)$$

$$m\ddot{x}(t)\cos\big(\theta(t)\big) + mL\ddot{\theta}(t) - mg\sin\big(\theta(t)\big) = 0$$

- Linearizing the non-straight condition of movement of the system around θ=0 and
- Finding the related state space model of the system

System modeling

linearize the conditions about the in an upward direction up harmony position, θ = 0, and will accept that the system stays inside a little neighborhood of this balance.

Expecting f(t) to be the input to the system and x(t) and θ(t) to be the two output, let us infer a state-space portrayal of the system

The system is depicted by two second request differential conditions; henceforth, the request for the system is four. Thus,precisely four directly autonomous state-factors to depict the system. Let us take the state factors to be the angular position of the pendulum θ(t), the cart displacement x(t), the speed of the pendulum θ˙(t) and the carts velocity x˙(t).subjectively number the state factors as follows:

$$x_1(t) = \theta(t)$$

$$x_2(t) = x(t)$$

$$x_3(t) = \dot{\theta}(t)$$

$$x_4(t) = \dot{x}(t)$$

From above equation we get our first state equation as follows

$$\dot{x}_1(t) = x_3(t)$$

While the second state equation from above equation as follows

$$\dot{x}_2(t) = x_4(t)$$

The remaining state equation is as follows, $\dot{x}_3(t) = g\sin(x_1(t))/L - \dot{x}_4(t)\cos(x_1(t))/L$

$$\dot{x}_4(t) = \left[\frac{mL}{M+m}\right][\dot{x}_3(t)]^2\sin(x_1(t)) - [\frac{mL}{M+m}]\dot{x}_3(t)\cos(x_1(t)) + f(t)/(M+m)$$

The output equations are given by,

$$\theta(t) = x_1(t)$$

$$x(t) = x_2(t)$$

Note that because of the non-direct nature of the system, can't communicate the last two state-conditions in a structure with the end goal that every condition contains the time subordinate of just one state variable. Such a structure is called an unequivocal type of the state-conditions.

State Space model

we can linearize by assuming $\cos(\theta(t)) = \cos(x_1(t)) = 1$, $\sin(x_1(t)) = \sin(\theta(t)) = x_1(t)$ and $[\dot{\theta}(t)]^2 \sin(\theta(t)) = [\dot{x}_3(t)]^2 \sin(x_1(t)) = 0$. The linearized state equation of the system can be expressed in following matrix form,

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\\ \dot{x}_4(t)\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \frac{(M+m)g}{ML} & 0 & 0 & 0\\ \frac{-mg}{M} & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\\ x_4(t)\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -\frac{1}{ML}\\ \frac{1}{M}\end{bmatrix} f(t)$$

Output matrix is given below

$$\begin{bmatrix}\theta(t)\\ x(t)\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\\ x_4(t)\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix} f(t)$$

3.2 ANALYZING THE STABILITY, CONTROLLABILITY AND OBSERVABILITY CONDITION OF THE SYSTEM

Stability

In this issue r addresses the progression order of the carts position. The 4 states address the position and velocity of the cart and the point and precise velocity of the pendulum. The output y contains both the situation of the cart and the point of the pendulum.

The initial phase in planning a full-state feedback controller is to decide the open-circle shafts of the system. by using **poles=eig(A)** command in the MATLAB we can found out the poles which are,

Poles,
$P_1 = 0$
$P_2 = 0$
$P_3 = 3.2833$
$P_4 = -3.2833$

As displayed above, there is one right-half plane pole at 3.2833. This ought to affirm that the system is unstable in open loop. Or then again it tends to be displayed by pole zero mapping of the system as beneath using MATLAB command **pzmap(sys_ss)** the plot is shown below,

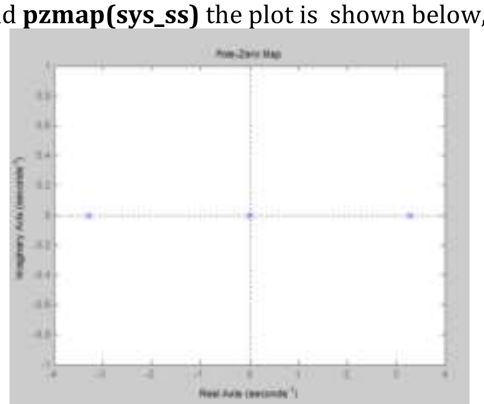


Fig. 5 Pole zero map of the system

- CONTROLLABILITY AND OBSERVABILITY

The controllability and observability of the system can calculate using MATLAB , by using command **ctrb** and **obsv** the system . from MATLAB program the given system is controllable and observable

3.3 DESIGNING FULL STATE FEEDBACK CONTROLLER

Planning a state feedback controller to balance out the system by further developing execution of the system utilizing **pole placement and linear quadratic regulator (LQR)** methods.

Here the primary object is to plan a controller with the goal that when a stage reference is given to the system, the pendulum ought to be displaced, yet at last re-visitation of nothing (for example vertical) and the cart should move to its new instructed position.

Design procedure:

- Checking if the pair (A,B) is controllable.
- Constructing conditions that will administer the controller elements.
- Placing the eigen values of the controller matrix in an desired situation by tracking down a self-assertive vector state input control gain vector K accepting that the entirety of the state variable are quantifiable . this can be cultivated utilizing both of the two techniques

1) pole placement method
2) LQR(linearQuadraticRegulation)method.

1. Using pole placement method

This method depends on the performance criteria, such as rise time, settling time, and overshoot used in the design.

The design requirements are,

- Settling time $T_{settling}$, for both outputs should be less than 1 seconds.
- %Overshoot, 5%, of the angle of the pendulum should be less than 20 (0.35 radians).

❖ Procedure-1
Given %OS = 5 ,$T_{settling}$=1sec calculating the parameters as follows:
$\zeta = 0.456$
$W_n = 8.7719$
$jw_d = 7.806$
So , the dominant poles are

$-\zeta w_n \pm jw_n\sqrt{1-\zeta^2}$ = -3.995 + j7.806 and -3.995 - j7.806

❖ Procedure -2
Using MATLAB software, the following are the gain vectors for different sets of desired poles.

➢ Test -1

K_1 = [-92.9841 -10.2041 -24.2449 -12.2449]
Desired poles_1 = [-1+i; -1-i; -5+5*i; -5-5*i]

by making the remaining poles 2 and 3 times faster than the real part of the dominant poles.

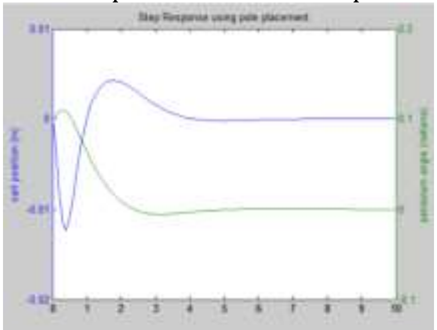


Fig. 31 Step Response using Pole Placement for test 1

- Test-2

K_2 = [1.0e+03 *(-2.9192 -2.2959 -0.5707 -0.5357)]
Desired poles_2 = [-7.5+7.5*i; -7.5-7.5*i; -10+10*i; -10-10*i]

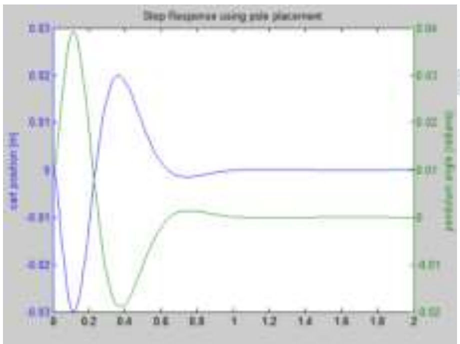


Fig. 32 Step Response using Pole Placement for test 2

As can be seen from the individual plots of the system step response for each determined gain vectors according to the desired pole location, System plan necessities are fulfilled in every one of the two tests. Furthermore, the system response will in general be quicker when the genuine pole go farther to left side from the genuine piece of the dominant posts.

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2.Using Linear Quadratic Regulator(LQR)

The Linear Quadratic Regulator (LQR) is a well-known method that provides optimally controlled feedback gains to enable the closed-loop stable and high performance design of systems. The **linear quadratic regulation** method is use for determining the state-feedback control gain matrix *K.*
This approach is to place the pole locations so that the closed-loop system optimizes the cost function given by $J_{QR} = \int_0^\infty [x(t)^T Q x(t) + u(t)^T R u(t)]dt$

Use the MATLAB commands **place or acker.** Another choice is to utilize the lqr order which returns the ideal controller gain accepting a linear plant, quadratic expense capacity, and reference equivalent to zero.

The MATLAB work lqr permits to pick two boundaries, R and Q, which will adjust the overall significance of the control exertion (u) and error (deviation from 0), individually, in the expense work that you are attempting to improve. The least difficult case is to accept R=1,and Q=C′C. The expense work relating to this R and Q places equivalent significance on the control and the state factors which are yields (the pendulum's point and the truck's position). Basically, the lqr technique considers the control of the two output. For this situation, it is quite simple to do. The controller can be tuned by changing the nonzero components in the Q grid to accomplish a positive reaction. To observe the structure of Q, we use MATLAB command $\boldsymbol{Q} = \boldsymbol{C}' * \boldsymbol{C}$ and we get the matrix Q as follows,

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The element in the (1,1) position of Q represents the weight on the cart's position and the element in the (2,2) position represents the weight on the pendulum's angle. The input weighting R will remain at 1.

For that we are using some MATLAB command for finding the response.by using the value of time and r as follows we see a response.
t=0:0.01:10;
response is as follows,

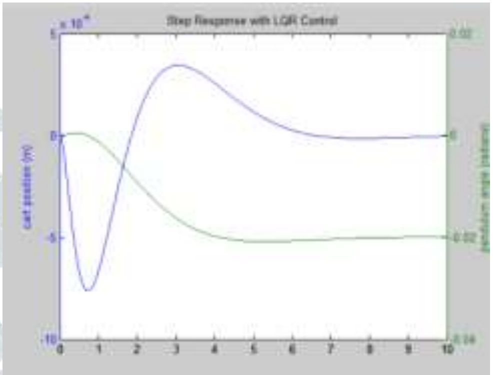

Fig. 33 Step Response with LQR control

The curve in green addresses the pendulum's angle in radians, and the curve in blue addresses the carts situation in meters. As displayed in this plot isn't good. The pendulum overshoot and carts undershoot show up fine, however their settling times need improvement and the truck's undershoot time should be diminished. Return to the MATLAB program and change the Q grid to see it can improve reaction. To track down that expanding the (1,1) and (2,2) components makes the settling and rise times go down, and brings down the point the pendulum moves. As such, putting more weight on the mistakes at the expense of expanded control effert u.

Replacing the value of $Q(1,1)$ = 80 and $Q(2,2)$=400 in MATLAB program we will produce the new response and new value of K &Q.

These are as follows $Q = \begin{bmatrix} 80 & 0 & 0 & 0 \\ 0 & 400 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

then,

K =[-101.7119 -20.0000 -32.0490 -19.2653]

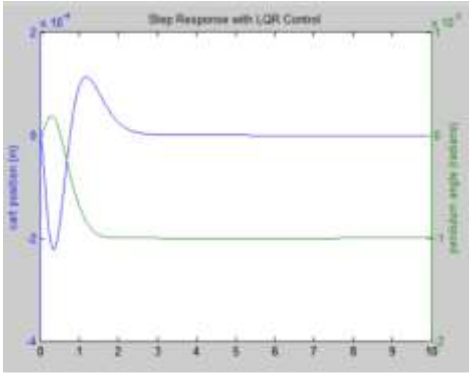

Fig.34 Step response with LQR control by changing Q matrix

The reason this weighting was chosen is just satisfies the transient design requirements. Increasing the magnitude of Q more would make the tracking error smaller ,but would require greater control force. the above LQR design method has brought a good stability to the stability and fulfilled the design requirement in good manner. It is seen that that if the values of the elements Q increased even higher, you could improve the response even more control effort generally corresponds to greater cost (more energy, larger actuator, etc.).

Adding precompensation

The controller have planned so far meets our transient necessities, however presently it should address the consistent state error. As opposed to the next plan strategies, where feedback the output and contrast it with the reference input to register a mistake, with a full-state input controller which taking care of back the entirety of the states. It need to process what the consistent state worth of the states ought to be, duplicate that by the picked gain K, and utilize another worth as "reference" for registering the info. This should be possible by adding a steady addition $\bar{N}$ after the reference. The schematic beneath shows this relationship:

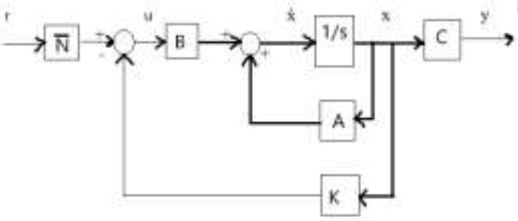

Fig.35 Block diagram of system by adding precompensator

Then, the above equations becomes $\dot{x} = (A - BK)x + B\bar{N}r$

$y = Cx$

We can find this $\bar{N}$ factor by using the MATLAB command **Nbar = rscale(sys_ss, K).** In this specific problem, the $c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ should be modified to a new one **C_n=[0 1 0 0] .**

Therefore, having K from the previous LQR design and scaling the reference input with following result, the system response are plotted below. For that the MATLAB program we use the command

C_n=[0 1 0 0]; sys_ss = ss(A,B,C_n,0); Nbar = rscale(sys_ss,K) Therefore Nbar= -20.0000

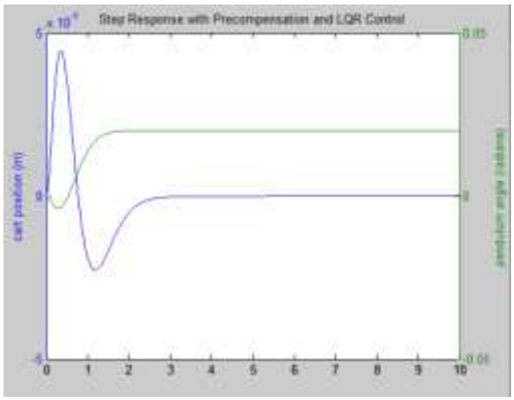

Fig.35 Step Response with precompensation and LQR control

As can be seen from the above plot, all design necessities are fulfilled. And furthermore, it very well may be obviously seen that the system execution turns out to be preferable and smooth over the one found utilizing the past controller planned by LQR.

CONCLUSION

From both the graphs of response to initial condition without and with controller as shown above .it is observed that for the system without controller which was initialized by some initial condition it response to initial condition of system is far from the point of equilibrium and the system is not reaching to the stability mode. whereas the system with controller the response to initial condition and to the point of equilibrium hence the system gets stable in less time. thus from stability point of view system with controller is useful for the stability of system.it is important to note that the matrix K is not unique for a given system but depend on the desired close-loop location which determines most of the parameters of the system. To improve stability of system and for better performance pole placement technique is used in the system

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