MAGNETO HYDRODYNAMIC FREE CONVECTIVE FLOW ANALYSIS IN A VERTICAL POROUS MICROCHANNEL WITH HEAT GENERATION/ABSORPTION

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Abstract:

This study is devoted to analyse the MHD natural convection flow in a vertically placed microchannel. The channel is filled with porous medium and the flow is analysed with the generation and absorption of heat.The governing nonlinear coupled equations are tackled using the finite difference method. The important flow parameters such as velocity and temperature, skin friction and rate of heat transfer of the flow are calculated by considering the velocity slip and temperature jump conditions at the walls of microchannel. The study highlights that rarefaction parameter, fluid-wall interaction and heat generation in the flow affects the flow significantly and enhances the fluid velocity and due to the retardation force raised by the porous medium the fluid velocity shows the declining nature.

Keywords: Porous medium, Heat generation/absorption, Velocity slip, Temperature jump.

1. Introduction

The study of flow in microchannel is an important area of research from past two decades due to its new applications in micro fluidic flow systems such as biomedical instruments, micro heat exchangers, solar energy systems, micro coolers and many micro engineering instruments and electronic devices. Knudsen number distinguishes the fluid flow in micro and macro-channel. It is defined as the ratio of the mean free path length of the fluid to the characteristic length of the fluid domain. The magneto hydrodynamic phenomenon has received more attention because of its applications in applied science and engineering hence we can see a vast literature in this area of research. The basic field equations, jump conditions and constitutive equations of simple micro fluent are discussed by Cemal [1]. Tuckerman and Pease [2] investigated the high performance forced cooling of planer integrated circuits. Philips [3] presented a heat sink model and experimentally studied the microchannel heat sink and the applications of it. The fully developed laminar flow in a vertical channel by considering the both free and forced convection is examined by Tao [4] studied. Single phase forced-flow convection of water or methanol through microchannel is experimentally studied by Wang and Peng [5].

Mala et al. [6] considered a microchannel between two parallel plates at constant and equal temperatures and studied the effects of EDL at the solid-liquid interface on liquid flow and heat transfer. Ming and Chong [7] had given the detail explanation about the microchannels and the forces acting on the micro electro mechanical systems. Liqing et al. [8] studied the electrokinetic effect in microchannel and the magnitude of the additional flow resistance caused by it. Dongqing [9] reviewed theoretical models of the electrokinetic effects on pressure driven flow in microchannel. Buonomo and Manca [10, 12] numerically investigated the transient natural convection and natural convection in parallel-plate vertical microchannel. The effect of rarefaction and fluid wall interaction on steady fully developed natural convection in a vertical microchannel is studied by Chen and Weng [11]. Sparrow and Cess [13] analysed that action of buoyancy and induced magnetic force results in the natural convection heat transfer. The free and forced convection magnetohydrodynamic flow in a vertical channel is studied by Umavathi and Malashetty [14]. Jha et al. [15, 16] considered the steady natural convection flow of viscous, incompressible, electrically conducting fluid in a vertical parallel plate microchannel and studied the effect of influence of magnetic field and suction/injection parameter on the flow. Raptis and Kafousias [17] considered a steady two dimensional flow and investigated the flow by considering an electrically conducting fluid through a porous medium with the effect of magnetic field.

Vafai [18] reported some important results about the convection in porous media. The magnetohydrodynamic flow in a porous channel is considered and the heat transfer is studied by Hayat and Abbas [19]. Umavathi et al. [20] considered a vertical channel filled with the porous medium and investigated the fully developed laminar mixed convective flow in that channel in the presence of heat generation and heat absorption effect. Aina and Malgwi [21] considered the MHD natural convective flow of conducting fluid in an inclined micro-porous-channel investigated the influence of transverse magnetic field and suction/injection parameter. Jha et al. [22] studied the natural convection of fully developed hydromagnetic steady flow in a vertical micro porous channel and given an exact solution for it. Shashikumar et al. [23] considered the casson fluid flow through a porous microchannel with the effect of MHD in the presence of thermal radiation and studied the heat transfer and entropy generation. Umavathi [24] constructed a flow model for natural convection in a vertical channel filled with porous medium and studied the flow in the presence of heat source/sink.

Molla et al. [25] studied the two dimensional natural convection flow of viscous incompressible fluid with the effect of heat generation/absorption. Pop [26] analysed the mixed convection in a vertical channel with the effect of heat generation in the presence of first order chemical reaction. Umavathi and Chamka [27] examined the mixed convective flow of a viscous incompressible fluid in an infinite vertical plate with the influence of heat source or sink. Patil et al. [28] investigated the laminar mixed convection flow of a fully developed hydromagnetic flow in a vertical channel in the presence of electrical conductivity and under the effect of heat generation/absorption. Malvandi et al. [29] theoretically investigated the nano particle migration effects on mixed convection of alumina/water nanofluid in a vertical microchannel with heat source/sink with asymmetric heating wall. Zaidi and Ahmad [30,31] studied the fully developed natural convection of electrically conducting fluid in an inclined microchannel in the presence of transverse magnetic field and internal heat generation. Gireesha et al. [32,33] considered the casson fluid in the presence of magnetic field in an annular microchannel and analysed the flow with the combined effect of heat generation/absorption parameter and porous parameter. Shobha et al. [33] studied the mixed convection in a channel with the chemical reaction effect and solved using shooting technique and results highlights that variable nature of viscosity-thermal conductivity enhances the velocity-temperature of the fluid.

The aim of the present study is to investigate the effect of porous medium and heat generation/absorption parameter on the magneto hydrodynamic natural convection in a vertical microchannel. The effects of fluid wall interaction parameter, wall ambient temperature, rarefaction parameter, Hartman number, porous medium and heat generation/absorption parameter on the fluid velocity, rate of heat transfer and velocity distribution on the walls of the microchannel.

2. Mathematical Analysis

A vertical microchannel formed by using two parallel plates and considered a fully developed steady natural convective flow of viscous, incompressible, electrically conducting fluid under the effect of transverse magnetic field and porous medium. The flow is influenced by the generation/absorption of heat. The x-axis is parallel and is opposite in direction to the gravitational force and Y-axis is perpendicular to the vertical microchannel. The distance between two plates of the microchannel is b. The walls of the microchannel are at temperature T_1 and T_0, the flow configuration is displayed in figure 1.

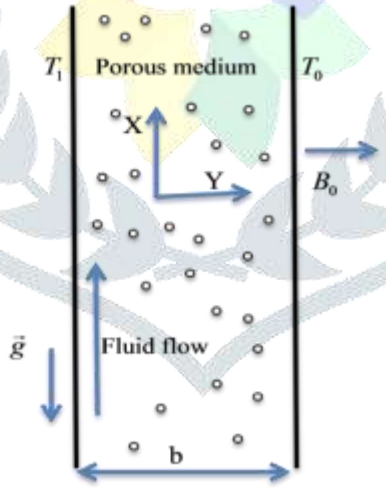


Figure 1: Flow configuration

The equations which govern the flow equation of motion and energy are considered in the dimensional form as,

$$\upsilon\frac{d^2U}{dY^2}+g\beta_T(T-T_0)-\frac{\sigma_e B_0^2}{\rho_0}U-\frac{\upsilon}{K}U=0 \tag{1}$$

$$\alpha_1\frac{d^2T}{dY^2}+\frac{\sigma_e B_0^2}{\rho_0 C_P}U^2+\frac{\upsilon}{C_P K}U^2\pm\frac{Q(T-T_0)}{\rho_0 C_P}=0 \tag{2}$$

The dimensional boundary conditions are

$$U\Big|_{y=0} = \frac{2-\sigma_v}{\sigma_v}\lambda\frac{dU}{dY}\Big|_{y=0}, \quad U\Big|_{y=1} = \frac{2-\sigma_v}{\sigma_v}\lambda\frac{dU}{dY}\Big|_{y=1},$$

$$T\Big|_{y=0} = T_2 + \left(\frac{2-\sigma_t}{\sigma_t}\right)\cdot\left(\frac{2\gamma_s}{\gamma_s+1}\right)\cdot\left(\frac{\lambda}{\Pr}\right)\cdot\frac{dU}{dY}\Big|_{y=0},$$

$$T\Big|_{y=1} = T_1 - \left(\frac{2-\sigma_t}{\sigma_t}\right)\cdot\left(\frac{2\gamma_s}{\gamma_s+1}\right)\cdot\left(\frac{\lambda}{\Pr}\right)\cdot\frac{dU}{dY}\Big|_{y=1}. \tag{3}$$

following are the dimensionless quantities:

$$Y = \frac{y}{b}, \quad \theta = \frac{(T-T_0)}{(T_1-T_0)}, \quad U = \frac{u}{U_0}, \quad M^2 = \frac{\sigma_e B_0^2 b^2}{\mu}, \quad Gr = \frac{g\beta(T_1-T_0)b^2}{\nu U_0}$$

$$\sigma^2 = \frac{b^2}{K}, \quad \phi = \frac{Qb^2}{K}, \quad Br = \frac{\mu U_0^2}{K\Delta T}. \tag{4}$$

where $U_0 = \frac{\rho g\beta(T_1-T_2)b^2}{\mu}$, $K = \alpha\rho_0 c_p$

The dimensionless form of the governing equations using (4) in the presence of velocity slip and temperature jump under Boussinesq's approximation are:

Dimension less form of equations 1, 2 are,

$$\frac{d^2u}{dy^2} - (M^2+\sigma^2)u + Gr\theta = 0 \tag{5}$$

$$\frac{d^2\theta}{dy^2} + Br(M^2+\sigma^2)u^2 \pm \phi\theta = 0 \tag{6}$$

the dimensionless boundary conditions are:

$$u(0) = \beta_v K_n \frac{du}{dy}\Big|_{y=0}, \quad u(1) = -\beta_v K_n \frac{du}{dy}\Big|_{y=1} \tag{7}$$

$$\theta(0) = \xi + \beta_v K_n l_n \frac{d\theta}{dy}\Big|_{y=0}, \quad \theta(1) = 1 - \beta_v K_n l_n \frac{d\theta}{dy}\Big|_{y=1} \tag{8}$$

where: $\beta_v = \frac{2-\sigma_v}{\sigma_v}$, $\beta_t = \left(\frac{2-\sigma_t}{\sigma_t}\right)\times\left(\frac{2\gamma_s}{\gamma_s+1}\right)\times\left(\frac{1}{\Pr}\right)$, $K_n = \frac{\lambda}{b}$, $l_n = \frac{\beta_t}{\beta_v}$, $\xi = \frac{T_2-T_0}{T_1-T_0}$.

Here kn, ln, Pr, γ_s, σ_t,σ_v,ξ, b is Knudsen number, fluid-wall interaction parameter, Prandtl number, ratio of specific heats, tangential momentum coefficient and thermal accommodation coefficients.

The two important properties of the fluid flowing in microchannel are rate of heat transfer and skin friction.

The rate of heat transfer which is expressed as the nusseltnumber (Nu) is

$$Nu = \frac{qb}{\left(T_1 - T_0\right)K} = \frac{d\theta(y)}{dy} \tag{9}$$

Therefore:

$$Nu_0 = \left.\frac{d\theta}{dy}\right|_{y=0} \tag{10}$$

$$Nu_1 = \left.\frac{d\theta}{dy}\right|_{y=1} \tag{11}$$

the skin friction (τ) on the microchannel walls is

$$\tau_0 = \left.\frac{du}{dy}\right|_{y=0} \tag{12}$$

$$\tau_1 = \left.\frac{du}{dy}\right|_{y=1} \tag{13}$$

3. Results and Discussion:

The solution for the problem effect of porous medium on MHD natural convective flow in a vertical microchannel with heat generation/absorption is obtained numerically and solved using Finite difference method. The velocity profile is discussed under the effect of different parameters by using the graphs. The present work is done in the range $0 \le \beta vKn \le 0.1, 0 \le l_n \le 3,\ 0 \le M \le 2,\ 0 \le \sigma \le 2,\ 1 \le \phi \le 5.$ The numerical values for the calculation and to draw the graphs of the different parameters which influences the flow are taken as $\beta vKn = 0.05, M = 2, \sigma = 2, l_n = 1.667, Gr = 1, Br = 1, \xi = 1, \phi = 5.$

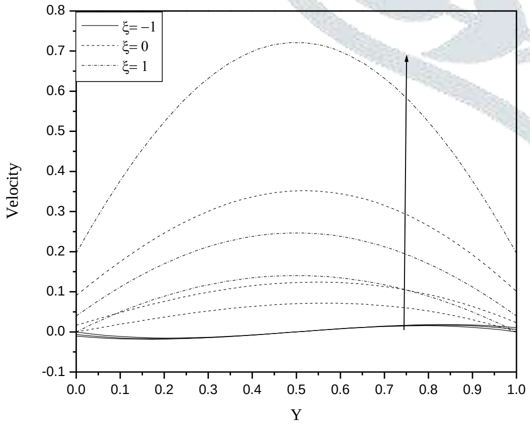


Fig 2: Impact of rarefaction parameter on the velocity.

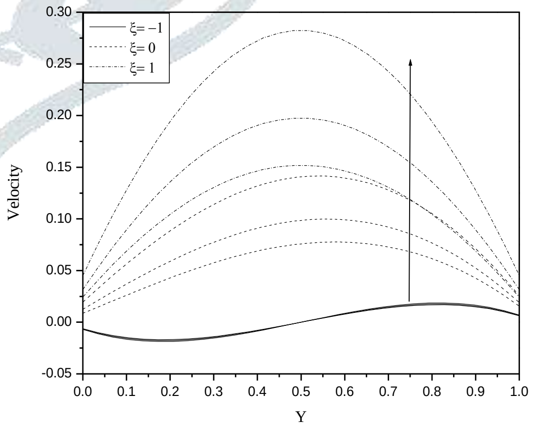


Fig 3: Impact of fluid-wall interaction parameter on the velocity.

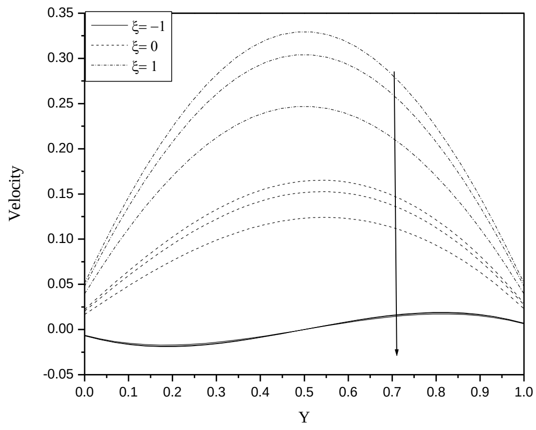


Fig 4: Impact of porous parameter on the velocity.

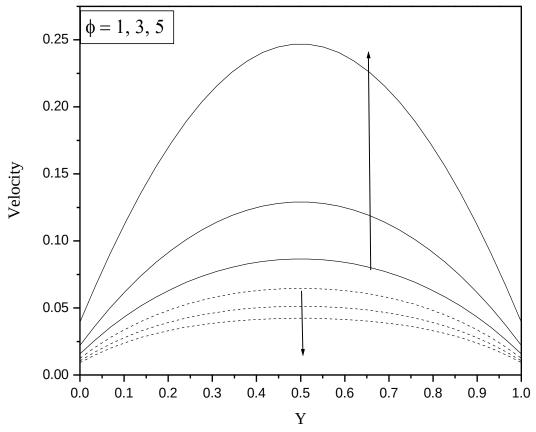


Fig 5: Impact of ϕ on the velocity.

Figure 2 presents the velocity profile for rarefaction parameter $\beta vKn = 0, 0.05, 0.1$ and wall ambient temperature ξ=-1, 0, 1. by the results it is observed that as the rarefaction parameter increases the fluid velocity increases. This is due to the reduction in retardation force, as the ambient temperature increases it influences the rarefaction parameter and this results in the increase in the velocity of the fluid. Fig 3 is the graph of velocity profile for fluid-wall interaction parameter ln=0, 1, 2 and ambient temperature ξ=-1, 0, 1. As the interaction parameter increases there is increase in the temperature jump and wall ambient temperature influences the fluid-wall interaction this results in the increase in the velocity, from the result it is clear that as the fluid-wall interaction increases velocity of the fluid increases.

Fig 4 illustrates the effect of wall ambient temperature and porous medium on the velocity profile. Porous parameter σ=1, 1.5, 2 decreases the fluid velocity. As porous parameter increases there is an increase in the resistance for the fluid flow in the channel this accounts for the decrease in the velocity of the fluid. Figure 5 presents the graph of effect of heat generation/absorption parameter ϕ=1, 3, 5 on the fluid velocity. As there is an increase in the heat generation, there is an increase in the conduction of the fluid particles this results in the increase in the fluid velocity and also it is found that as there an increase in the heat absorption this reduces the movement of the fluid particles and this results in the decrease in the velocity of the fluid.

Figure 6 and figure 7 presents the effect of porous medium on the Skin friction. There is a significant effect of porous medium on the skin friction. The skin friction is obtained for ξ=-1, 0, 1 and σ=1, 1.5, 2. As the value of the porous parameter increases and as the fluid move away from the continuum regime there is a decrease in the value of the skin friction and skin friction increases at y=1.it is observed that magnitude of the skin friction is high in case of y=0 when compared the case y=1.

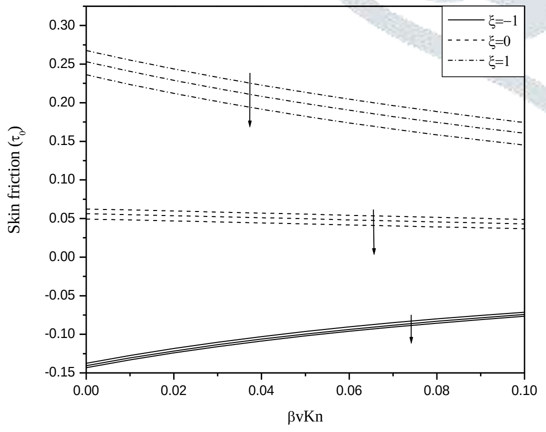


Fig 6: Impact of σ on skinfriction at y=0.

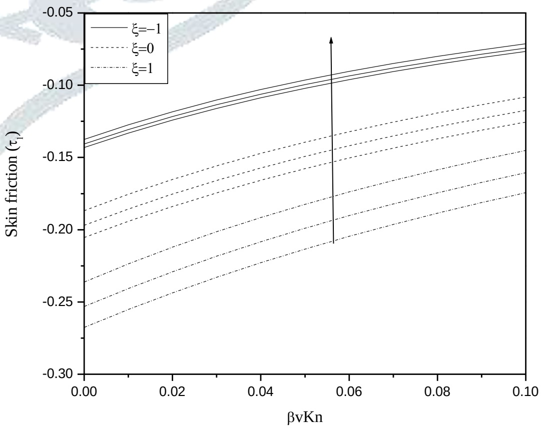
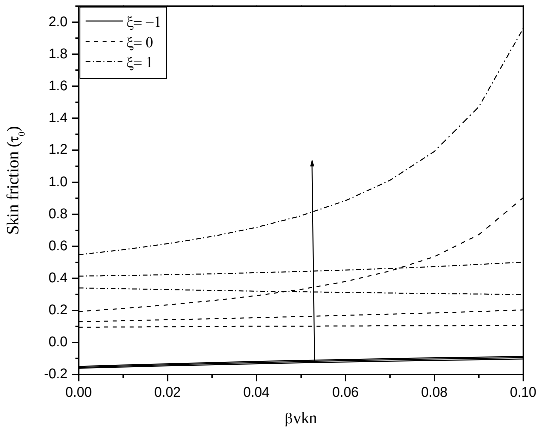


Fig 7: Impact of σ on skinfriction at y=1.



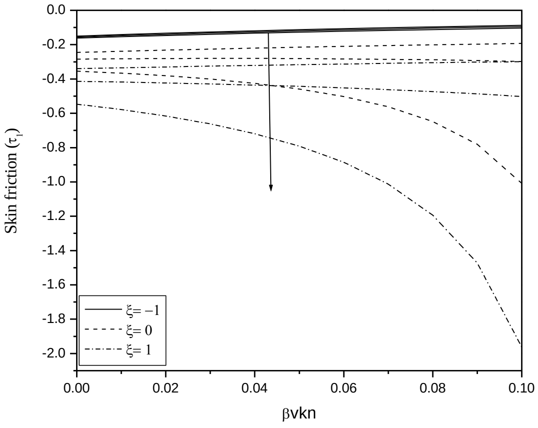
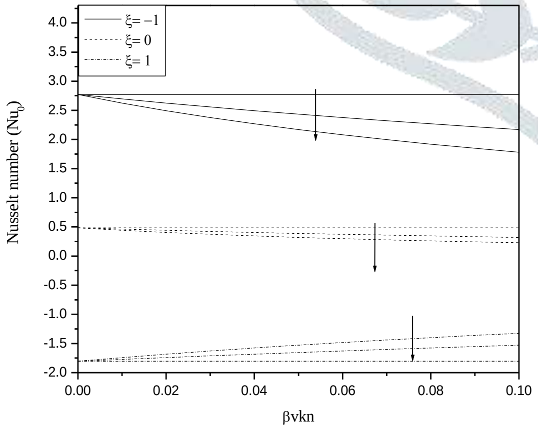


Fig 8: Impact of ϕ on skinfriction at y=0. Fig 9: Impact of ϕ on skinfriction at y=1.

Figure 8 and figure 9 presents the graph of effect of heat generation/absorption parameter on the skin friction. As the value of heat generation increases and as the fluid moves away from the continuum regime the skin friction increases at y=0 and the magnitude of the skin friction is high for the highest value of the wall ambient temperature. At y=1 there is a decrease in the motion of fluid particles this results to decrease in the fluid velocity and thus there is decrease in velocity of the fluid as increase in the heat absorption parameter.

Figure 10 and Figure 11 is the graph of nusselt number for different values of fluid-wall interaction parameter. The nusselt number is obtained for ξ=-1, 0, 1 and ln=0, 1, 2. As the value of fluid-wall interaction parameter and the wall ambient temperature increases there is a decrease in the heat transfer at both the walls of the microchannel. Figure 12 and figure 13 is the graph of nusselt number variation for different values of porous medium at y=0 and at y=1 respectively. The rate of heat transfer is obtained for ξ=-1, 0, 1 and σ=1, 1.5,2 as the value of porous parameter and the wall ambient temperature increases there is a decrease in nusselt number at both the walls of the microchannel this is due to the resistance force experienced by the fluid due to the presence of porosity in the medium. The magnitude of the nusselt number is comparatively high in case of y=1 compare to nusselt number at y=0.

Figure 14 and Figure 15 is the graph of nusselt number variation for different values of heat generation at y=0 and at y=1 respectively. The rate of heat transfer is obtained for ξ=-1, 0, 1 and ϕ=1, 3, 5 as the value of heat generation and the wall ambient temperature increases the rate of heat transfer increases at y=0 and decreases at y=1 and the magnitude of the nusselt number is comparatively high in case of y = 0 than the nusselt number at y =1.



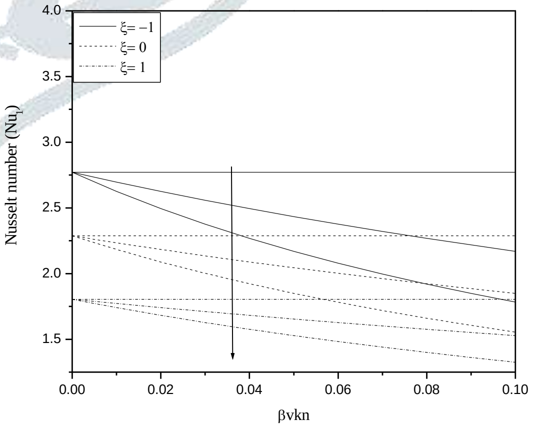
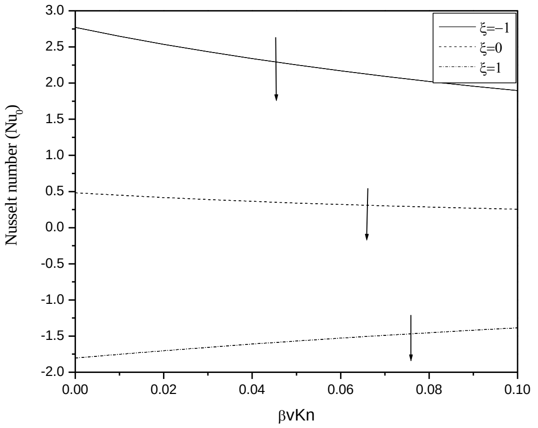


Fig 10: Impact of ln on nusselt number at y=0. Fig 11: Impact of ln on nusselt number at y=1.



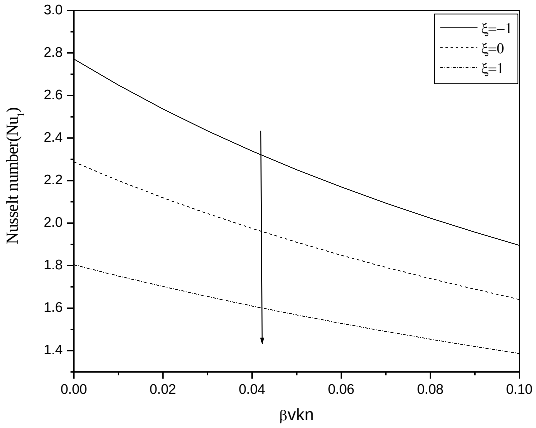
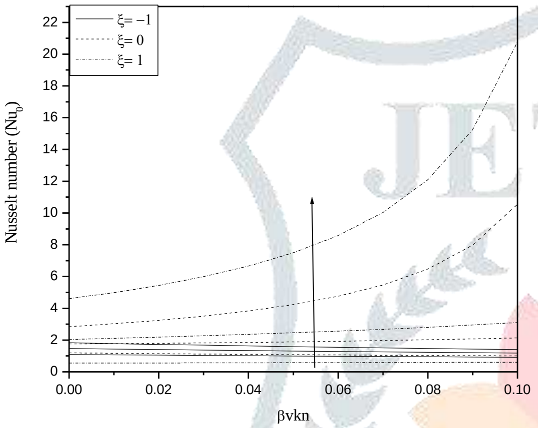


Fig 12: Impact of σ on nusselt number at y=0.

Fig 13: Impact of σ on nusselt number at y=1.



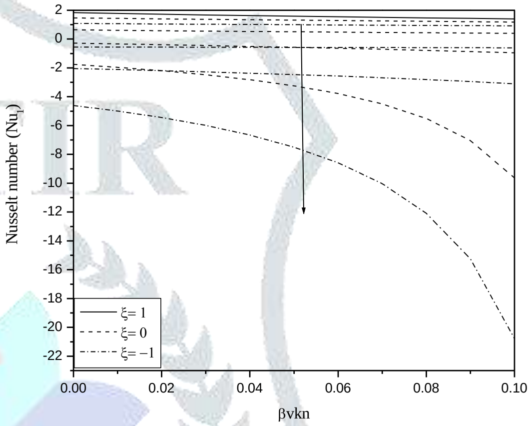


Fig 14: Impact of ϕ on nusselt number at y=0.

Fig 15: Impact of ϕ on nusselt number at y=1.

4. Conclusions:

This study investigates the problem of effect of porous medium on the MHD natural convective flow in a vertical microchannel with heat generation/absorption parameter. The effect of fluid-wall interaction parameter, wall ambient temperature, rarefaction parameter, Hartman number, heat generation/absorption parameter and porous medium on the velocity and skin friction and nusselt number is studied and analysed

- The wall ambient temperature, fluid-wall interaction parameter, rarefaction parameter and heat generation parameter enhances the fluid velocity and porous medium and heat absorption parameter suppresses the fluid velocity.
- The porous parameter decreases the skin friction at y=0 and increases skin friction at y=1,this is due to the resistance effect applied by the porous medium to the fluid flow.
- Heat generation parameter increases the skin friction at y=0 and decreases the skin friction at y=1.
- The fluid-wall interaction parameter and the porous parameter decrease the rate of heat transfer at both the walls of the microchannel.
- Heat generation in the fluid decreases, the rate of heat transfer at the wall y=0 and it increases the rate of heat transfer at y=1.

Nomenclature :

g acceleration due to gravity(m s^{-2})

u Fluid velocity along x-direction.

l_n Fluid-wall interaction parameter

b Channel width(m)

Kn Knudsen number

M Hartman number

Nu Nusselt number

Gr Grashouf number

Br Brinkmen number

ϕ Dimensionless heat generation/absorption parameter

T Fluid temperature (K)

θ Dimensionless temperature.

Nu_0 Nusselt number at y=0

Nu_1 Nusselt number at y=1

σ Porous parameter

βvKn Rarefaction parameter

Greek letters:

λ Mean free path of molecules

τ Skin friction

τ_0 Skin friction at y=0

τ_1 Skin friction at y=1

ξ Wall ambient temperature

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