JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

BAYESIAN ESTIMATION OF THE UNKNOWN PARAMETER AND RELIABILITY OF THE EXPONENTIAL DISTRIBUTION WITH A NON-NATURAL CONJUGATE PRIOR

Randhir Singh

Department of Statistics, Ewing Christian College, Prayagraj, India.

Email address:

dr.singh.ecc@gmail.com

ABSTRACT

This paper deals with the Bayesian estimation of unknown parameter θ and reliability of exponential distribution. The estimation has been performed for type II censored samples. In the Bayesian setup estimates of θ and the reliability of the distribution have been obtained by taking Maxwell's distribution as the prior distribution for the unknown parameter θ which is a Non-Natural Conjugate prior distribution for θ and four different types of loss functions. On the part of loss functions, the Squared Error Loss Function (SELF), DeGroot Loss Function (DLF), Minimum Expected Loss (MELO) Function and Exponentially Weighted Minimum Expected Loss (EWMELO) Function have been considered. Bayes Risks of Bayes estimators corresponding to four loss functions have also been obtained.

Keywords: Exponential Distribution, Maxwell's distribution, Reliability, Type II Censoring Bayes Estimator, Squared Error Loss Function (SELF), DeGroot Loss Function (DLF), Minimum Expected Loss (MELO) Function and Exponentially Weighted Minimum Expected Loss (EWMELO) Function. Bayes Risk.

1.Introduction

A continuous random variable X is said to have exponential distribution with parameter θ ,if its probability density function is given by,

$$f(x/\theta) = \begin{cases} \theta e^{-\theta x}, \text{if } x > 0, \theta > 0 \\ 0, \text{Otherwise} \end{cases} \quad (1.1)$$

For a specified mission time t, $t > 0$, the reliability of the distribution, denoted by, $R(t/\theta) = P(X \geq t)$,is given by,

$$R(t/\theta) = e^{-\theta t} \quad (1.2)$$

In this paper, Bayesian procedure has been adopted to obtain estimates of θ and $R(t/\theta)$. In Bayesian framework, estimation has been performed under the assumption Maxwell's distribution as the prior density for θ given as under:

$$\pi(\theta)=\begin{cases} 4\,\pi^{-\frac{1}{2}}\,\alpha^{-\frac{3}{2}}\,\theta^2 e^{-\frac{\theta^2}{\alpha}}, \text{if } \theta > 0\,, \alpha > 0 \\ 0, \text{otherwise} \end{cases} \quad (1.3)$$

This p. d. f. is an Non-Natural Conjugate p. d. f. for θ ,as the posterior p. d. f. does not have the form given by (1.3). Bayesian estimation with Non-Natural Conjugate p. d. f. has been considered by Bhattacharya and Kumar (1986) using Inverse Gaussian distribution as the prior distribution for θ. Bhattacharya and Tyagi (1988) have used truncated normal distribution as the prior distribution for θ.

The case of 'Attribute Life Testing' has also been considered as in Bhattacharya (1967), wherein life testing and reliability estimation in the Bayesian framework was introduced for the first time. Martz and Waller (1982) also contains some details of this concept.

The loss functions considered are as under:

1.The Squared Error Loss Function (SELF): In this case, the loss function denoted by $L(\theta, \delta)$,

is given by,

$$L(\theta, \delta) = (\theta - \delta)^2 \quad (1.4)$$

This loss function is symmetric and unbounded. It suffers from the drawback of giving equal weights to underestimation as well as to overestimation.

2. DeGroot Loss Function (DLF): In this case

$$L(\theta, \delta) = \delta^{-2}(\theta - \delta)^2 \qquad (1.5)$$

This loss function, introduced by DeGroot (2005), is asymmetric. It gives more weight to underestimation than to overestimation.

3.Minimum Expected Loss (MELO) Function: In this case,

$$L(\theta, \delta) = \theta^{-2}(\theta - \delta)^2 \qquad (1.6)$$

This loss function is asymmetric and bounded. In this case weight due to underestimation and overestimation is changed by a factor θ^{-2} as compared to the SELF. This loss function was used by Tummala and Sathe (1978) for estimating reliability of certain life time distribution and by Zellner (1979) for estimating functions of parameters in econometric models.

4. Exponentially Weighted Minimum Expected Loss (EWMELO) Function

$$w(\theta, \delta) = \theta^{-2} e^{-a\theta}(\theta - \delta)^2 \quad (1.7)$$

This loss function is asymmetric and bounded. In this case weight due to underestimation and overestimation is changed by a factor $e^{-a\theta}$ as compared to the MELO and by a factor $\theta^{-2}e^{-a\theta}$ as compared to the SELF.

This type of loss function was used by the author (1997) for the first time in his work for D.Phil. SELF, MELO and EWMELO were used by Singh, the author, (1999) in the study of reliability of a multicomponent system and (2010) in Bayesian Estimation of the mean and distribution function of Maxwell's distribution. Recently, the author again used these loss functions in Bayesian estimation of function of the unknown parameter θ for the Modified Power Series Distribution (MPSD) (2021) ,for estimating Loss and Risk Functions of a continuous distribution (2021),for estimating moments and reliability of Geometric distribution. In addition to these loss functions, the author has used Degroot loss function while estimating the unknown parameter and reliability of Burr Type XII distribution (2021) and for Weibexpo distribution (2021).

2.BAYESIAN ESTIMATION

Let $X_1, X_2, X_3, \ldots X_n$ be a random sample of size n and $X_{(1)} < X_{(2)} < X_{(3)} < \ldots < X_{(n-1)} < X_{(n)}$ be the order statistic corresponding to this random sample having p. d. f given by (1.1). In case of type II censoring, n items are placed on test and the test is terminated after first 'r' (r pre-specified) failures. Thus, only r ordered observed values of $X_{(1)} < X_{(2)} < X_{(3)} < \ldots < X_{(r-1)} < X_{(r)}$ are recorded. For observed values $x_{(1)} < x_{(2)} < x_{(3)} < \ldots < x_{(r-1)} < x_{(r)}$,the likelihood function, when α is known ,denoted by $L(\theta)$,is given by,

$$L(\theta) = k\theta^r e^{-\theta t_r} \qquad (2.1)$$

Where, k is function of n, r and $x_{(i)}$, i=1,2…r and does not contain θ.

$$t_r = \sum_{i=1}^{r} x_{(i)} + (n-r)x_{(r)} \qquad (2.2)$$

t_r is an observed value of the statistic T_r given by,

$$T_r = \sum_{i=1}^{r} X_{(i)} + (n-r)X_{(r)}$$

The posterior distribution of θ,denoted by $\pi(\theta / t_r)$,is given by.

$$\pi(\theta / t_r) = \frac{L(\theta)\pi(\theta)}{\int_0^\infty L(\theta)\pi(\theta)d\theta}$$

Or,

$$\pi(\theta / t_r) = \begin{cases} \frac{\theta^{(r+2)} e^{-\left(\theta t_r + \frac{\theta^2}{\alpha}\right)}}{K}, & \text{if } \theta > 0, \alpha > 0 \\ 0, & \text{otherwise} \end{cases} \qquad (2.3)$$

Where,

$$K = \Gamma(r+3)(2\alpha^{-1})^{-\frac{(r+3)}{2}} e^{\left(\frac{t_r^2}{8\alpha^{-1}}\right)} D_{-(r+3)}\left(\frac{t_r}{\sqrt{(2\alpha^{-1})}}\right) \qquad (2.4)$$

and $D_{-\nu}(.)$ is the Parabolic Cylinder Function as given in Abramowitz and Stegun (1964) and has the mathematical representation as under:

$$D_{-\nu}(z) = \frac{e^{-\frac{z^2}{4}}}{\Gamma(\nu)} \int_0^\infty t^{(\nu-1)} e^{-zt-\frac{t^2}{2}} dt \qquad (2.5)$$

Provided $\text{Re } \nu > 0$

1.Under the Squared Error Loss Function given by, $L(\theta, \delta) = (\theta - \delta)^2$, the Bayes estimate of θ,denoted by $\hat{\theta}_B$, is given by,

$$\hat{\theta}_B = E(\theta / t_r) = \int_0^\infty \theta\pi(\theta / t_r)d\theta = \int_0^\infty \frac{\theta^{(r+3)} e^{-\left(\theta t_r + \frac{\theta^2}{\alpha}\right)}}{K} d\theta = \frac{(r+3)(2\alpha^{-1})^{-\frac{1}{2}} D_{-(r+4)}(u_r)}{D_{-(r+3)}(u_r)}$$

So,

$$\hat{\theta}_B = \frac{(r+3)\left(2\alpha^{-1}\right)^{-\frac{1}{2}} D_{-(r+4)}(u_r)}{D_{-(r+3)}(u_r)} \qquad (2.6)$$

Where,

$$u_r = \frac{t_r}{\sqrt{(2\alpha^{-1})}} \qquad (2.7)$$

2.Under the DeGroot Loss Function given by ,$L(\theta,\delta) = \delta^{-2}(\theta-\delta)^2$, the Bayes estimate of θ,denoted by $\hat{\theta}_{DG}$, is given by,

$$\hat{\theta}_{DG} = \frac{E(\theta^2/t_r)}{E(\theta/t_r)} = \frac{\int_0^\infty \theta^{(r+4)} e^{-(\theta t_r + \frac{\theta^2}{\alpha})} d\theta}{\int_0^\infty \theta^{(r+3)} e^{-(\theta t_r + \frac{\theta^2}{\alpha})} d\theta} = \frac{(r+4)\left(2\alpha^{-1}\right)^{-\frac{1}{2}} D_{-(r+5)}(u_r)}{D_{-(r+4)}(u_r)}$$

So,

$$\hat{\theta}_{DG} = \frac{(r+4)\left(2\alpha^{-1}\right)^{-\frac{1}{2}} D_{-(r+5)}(u_r)}{D_{-(r+4)}(u_r)} \qquad (2.8)$$

3.Under the Minimum Expected Loss (MELO) Function, given by, $L(\theta,\delta) = \theta^{-2}(\theta-\delta)^2$

,the Bayes estimate of θ,(also known as the Minimum Expected Loss (MELO) Estimate, denoted by $\hat{\theta}_M$, is given by,

$$\hat{\theta}_M = \frac{E(\theta^{-1}/t_r)}{E(\theta^{-2}/t_r)} = \frac{\int_0^\infty \theta^{(r+1)} e^{-(\theta t_r + \frac{\theta^2}{\alpha})} d\theta}{\int_0^\infty \theta^{r} e^{-(\theta t_r + \frac{\theta^2}{\alpha})} d\theta} = \frac{(r+2)\left(2\alpha^{-1}\right)^{-\frac{1}{2}} D_{-(r+2)}(u_r)}{D_{-(r+1)}(u_r)}$$

So,

$$\hat{\theta}_M = \frac{(r+2)\left(2\alpha^{-1}\right)^{-\frac{1}{2}} D_{-(r+2)}(u_r)}{D_{-(r+1)}(u_r)} \qquad (2.9)$$

4.Under the Exponentially Weighted Minimum Expected Loss (EWMELO) Function, given by, $L(\theta,\delta) = \theta^{-2}e^{-a\theta}(\theta-\delta)^2$, the Bayes estimate of θ ,known as the Exponentially Weighted Minimum Expected Loss (MELO) Estimate, denoted by $\hat{\theta}_{EW}$, is given by,

$$\hat{\theta}_{EW} = \frac{E(\theta^{-1}e^{-a\theta}/t_r)}{E(\theta^{-2}e^{-a\theta}/t_r)} = \frac{\int_0^\infty \theta^{-1} e^{-a\theta} \pi(\theta/t_r) d\theta}{\int_0^\infty \theta^{-2} e^{-a\theta} \pi(\theta/t_r) d\theta} = \frac{(r+2)\left(2\alpha^{-1}\right)^{-\frac{1}{2}} D_{-(r+2)}(v_r)}{D_{-(r+1)}(v_r)}$$

So,

$$\hat{\theta}_{EW} = \frac{(r+2)\left(2\alpha^{-1}\right)^{-\frac{1}{2}} D_{-(r+2)}(v_r)}{D_{-(r+1)}(v_r)} \qquad (2.10)$$

Where,

$$v_r = \frac{t_r + a}{\sqrt{(2\alpha^{-1})}} \qquad (2.11)$$

The Bayes risk of a Bayes estimator $\hat{\theta}$ of θ,corresponding to a given loss function $L(\theta,\delta)$ is given by,$B(\hat{\theta}) = E\{L(\theta,\hat{\theta})\}$.Bayes risks of Bayes estimators corresponding to four loss functions considered are in the table as follows:

2.1 BAYES RISKS OF VARIOUS BAYES ESTIMATES OF θ

S.No.	Loss Function	Bayes Estimate	Bayes Risk
1.	SELF	$\hat{\theta}_B = \frac{(r+3)(2\alpha^{-1})^{-\frac{1}{2}}D_{-(r+4)}(u_r)}{D_{-(r+3)}(u_r)}$	$B(\hat{\theta}_B) = \frac{(r+3)(r+4)(2\alpha^{-1})^{-1}D_{-(r+4)}(u_r)}{D_{-(r+3)}(u_r)} - (\hat{\theta}_B)^2$
2.	DLF	$\hat{\theta}_{DG} = \frac{(r+4)(2\alpha^{-1})^{-\frac{1}{2}}D_{-(r+5)}(u_r)}{D_{-(r+4)}(u_r)}$	$B(\hat{\theta}_{DG}) = 1 - \frac{(r+3)\{D_{-(r+4)}(u_r)\}^2}{(r+4)D_{-(r+3)}(u_r)D_{-(r+5)}(u_r)}$
3.	MELO	$\hat{\theta}_M = \frac{(r+2)(2\alpha^{-1})^{-\frac{1}{2}}D_{-(r+2)}(u_r)}{D_{-(r+1)}(u_r)}$	$B(\hat{\theta}_M) = 1 - \frac{(r+1)\{D_{-(r+2)}(u_r)\}^2}{(r+2)D_{-(r+1)}(u_r)D_{-(r+3)}(u_r)}$
4.	EWMELO	$\hat{\theta}_{EW} = \frac{(r+2)(2\alpha^{-1})^{-\frac{1}{2}}D_{-(r+4)}(v_r)}{D_{-(r+3)}(v_r)}$	$B(\hat{\theta}_{EW}) = 1 - \frac{(r+1)\{D_{-(r+2)}(v_r)\}^2}{(r+2)D_{-(r+1)}(u_r)D_{-(r+3)}(v_r)}$

5.Under the Squared Error Loss Function, the Bayes estimate of $R(t/\theta) = e^{-t\theta}$, denoted by $\hat{R}_B$, is given by,

$$\hat{R}_B = E\{R(t/\theta)/t_r\} = \int_0^\infty R(t/\theta)\pi(\theta/t_r)d\theta = \int_0^\infty \frac{e^{-t\theta}\,\theta^{(r+2)}e^{-(\theta t_r+\frac{\theta^2}{\alpha})}}{K}d\theta = \frac{D_{-(r+3)}(w_r)e^{\left(\frac{t(t+2t_r)}{8\alpha^{-1}}\right)}}{D_{-(r+3)}(u_r)}$$

So,

$$\hat{R}_B = \frac{D_{-(r+3)}(w_r)e^{\left(\frac{t(t+2t_r)}{8\alpha^{-1}}\right)}}{D_{-(r+3)}(u_r)} \quad (2.12)$$

Where,

$$w_r = \frac{t_r+t}{\sqrt{(2\alpha^{-1})}} \quad (2.13)$$

6. Under the DeGroot Loss Function the Bayes estimate of $R(t/\theta) = e^{-t\theta}$, denoted by $\hat{R}_{DG}$, is given by,

$$\hat{R}_{DG} = \frac{E[\{R(t/\theta)\}^2/t_r]}{E\{R(t/\theta)/t_r\}} = \frac{\int_0^\infty\{R(t/\theta)\}^2\pi(\theta/t_r)d\theta}{\int_0^\infty R(t/\theta)\pi(\theta/t_r)d\theta} = \frac{\int_0^\infty e^{-2t\theta}\,\theta^{(r+2)}e^{-(\theta t_r+\frac{\theta^2}{\alpha})}d\theta}{\int_0^\infty e^{-t\theta}\,\theta^{(r+2)}e^{-(\theta t_r+\frac{\theta^2}{\alpha})}d\theta} = \frac{D_{-(r+3)}(y_r)e^{\left(\frac{t(3t+2t_r)}{8\alpha^{-1}}\right)}}{D_{-(r+3)}(w_r)}$$

So,

$$\hat{R}_{DG} = \frac{D_{-(r+3)}(y_r)e^{\left(\frac{t(3t+2t_r)}{8\alpha^{-1}}\right)}}{D_{-(r+3)}(w_r)} \quad (2.14)$$

Where,

$$y_r = \frac{t_r+2t}{\sqrt{(2\alpha^{-1})}} \quad (2.15)$$

7. Under the Minimum Expected Loss (MELO) Function, the Bayes estimate of $R(t/\theta) = e^{-t\theta}$, denoted by $\hat{R}_M$, is given by,

$$\hat{R}_M = \frac{E(\theta^{-2}R(t/\theta)/t_r)}{E(\theta^{-2}/t_r)} = \frac{\int_0^\infty \theta^{-2}R(t/\theta)\pi(\theta/t_r)d\theta}{\int_0^\infty \theta^{-2}\pi(\theta/t_r)d\theta} = \frac{\int_0^\infty \theta^{-2}e^{-t\theta}\,\theta^{(r+2)}e^{-(\theta t_r+\frac{\theta^2}{\alpha})}d\theta}{\int_0^\infty \theta^{-2}\theta^{(r+2)}e^{-(\theta t_r+\frac{\theta^2}{\alpha})}d\theta} = \frac{D_{-(r+1)}(w_r)e^{\left(\frac{t(t+2t_r)}{8\alpha^{-1}}\right)}}{D_{-(r+1)}(u_r)}$$

So,

$$\hat{R}_M = \frac{D_{-(r+1)}(w_r)e^{\left(\frac{t(t+2t_r)}{8\alpha^{-1}}\right)}}{D_{-(r+1)}(u_r)} \quad (2.16)$$

8. Under the Exponentially Weighted Minimum Expected Loss (EWMELO) Function, the Bayes estimate of $R(t,\theta) = e^{-\theta t}$,denoted by $\hat{R}_{EW}$, is given by,

$$\hat{R}_{EW} = \frac{E(\theta^{-2}e^{-a\theta}R(t/\theta)/t_r)}{E(\theta^{-2}e^{-a\theta}/t_r)} = \frac{\int_0^\infty \theta^{-2}e^{-(t+a)\theta}\,\theta^{(r+2)}e^{-(\theta t_r+\frac{\theta^2}{\alpha})}d\theta}{\int_0^\infty \theta^{-2}\theta^{(r+2)}e^{-a\theta}e^{-(\theta t_r+\frac{\theta^2}{\alpha})}d\theta} = \frac{D_{-(r+1)}(z_r)e^{\left(\frac{t(t+2t_r+2a)}{8\alpha^{-1}}\right)}}{D_{-(r+1)}(v_r)}$$

So,

$$\hat{R}_{EW} = \frac{D_{-(r+1)}(z_r)e^{\left(\frac{t(t+2t_r+2a)}{8\alpha^{-1}}\right)}}{D_{-(r+1)}(v_r)} \quad (2.17)$$

Where,

$$z_r = \frac{t_r+t+a}{\sqrt{(2\alpha^{-1})}} \quad (2.18)$$

The Bayes risk of a Bayes estimator $\hat{R}$ of $R(t/\theta)$,corresponding to a given loss function $L(\theta,\delta)$ is given by,$B(\hat{R}) = E\{L(R(t/\theta),\hat{R})\}$.Bayes risks of Bayes estimators corresponding to four loss functions considered are in the table as follows:

2.2 BAYES RISKS OF VARIOUS BAYES ESTIMATES OF $R(t/\theta)$

S.No.	Loss Function	Bayes Estimate	Bayes Risk
1.	SELF	$\hat{R}_B = \frac{D_{-(r+3)}(w_r)e^{\left(\frac{t(t+2t_r)}{8\alpha^{-1}}\right)}}{D_{-(r+3)}(u_r)}$	$B(\hat{R}_B) = \frac{D_{-(r+3)}(y_r)e^{\left(\frac{4t(t+t_r)}{8\alpha^{-1}}\right)}}{D_{-(r+3)}(u_r)} - (\hat{R}_B)^2$
2.	DLF	$\hat{R}_{DG} = \frac{D_{-(r+3)}(y_r)e^{\left(\frac{t(3t+2t_r)}{8\alpha^{-1}}\right)}}{D_{-(r+3)}(w_r)}$	$B(\hat{R}_{DG}) = 1 - \frac{\{D_{-(r+3)}(w_r)\}^2 e^{\left(\frac{-t^2}{4\alpha^{-1}}\right)}}{D_{-(r+3)}(u_r)D_{-(r+3)}(y_r)}$
3.	MELO	$\hat{R}_M = \frac{D_{-(r+1)}(w_r)e^{\left(\frac{t(t+2t_r)}{8\alpha^{-1}}\right)}}{D_{-(r+1)}(u_r)}$	$B(\hat{R}_M) = \frac{2\alpha^{-1})}{(r+1)(r+2)D_{-(r+3)}(u_r)}\{D_{-(r+1)}(y_r)e^{\left(\frac{4t(t+t_r)}{8\alpha^{-1}}\right)} - \frac{\{D_{-(r+1)}(w_r)\}^2 e^{\left(\frac{2t(t+2t_r)}{8\alpha^{-1}}\right)}}{D_{-(r+1)}(u_r)}\}$
4.	EWMELO	$\hat{R}_{EW}$	$B(\hat{R}_{EW}) =$

		$=\frac{D_{-(r+1)}(z_r)e^{\left(\frac{t(t+2t_r+2a)}{8\alpha^{-1}}\right)}}{D_{-(r+1)}(v_r)}$	$\frac{2\alpha^{-1})}{(r+1)(r+2)D_{-(r+3)}(u_r)}\{D_{-(r+1)}(s_r)e^{\left(\frac{(a+2t)(2t+2t_r+a)}{8\alpha^{-1}}\right)} - \frac{\{D_{-(r+1)}(z_r)\}^2 e^{\left(\frac{2t(t+2t_r+2a)}{8\alpha^{-1}}\right)}}{D_{-(r+1)}(v_r)}\}$ $s_r=\frac{t_r+2t+a}{\sqrt{(2\alpha^{-1})}}$

3.ATTRIBUTE LIFE TESTING

In this case n units from the population specified by (1.1) are placed on a life test and after a fixed duration T,the number of survivors, say s, $(0 \le s \le n)$are recorded. The exact failure times are not available even for the failed units. The likelihood function, denoted by $L_A(\theta)$,in this case, is given by,

$$L_A(\theta)=\binom{n}{s}e^{-sT\theta}(1-e^{-T\theta})^{n-s} \quad (3.1)$$

Where, $s=0,1,2\dots n$. The posterior distribution of θ,in this case ,denoted by $\pi_*(\theta/T)$,is given by.

$$\pi_*(\theta/T)=\frac{L_A(\theta)\pi(\theta)}{\int_0^\infty L_A(\theta)\pi(\theta)d\theta}$$

Or,

$$\pi_*(\theta/T)=\begin{cases}\frac{\theta^2 e^{-(\theta sT+\frac{\theta^2}{\alpha})}(1-e^{-T\theta})^{n-s}}{K_1}, & \text{if } \theta>0, \alpha>0 \\ 0, \text{otherwise}\end{cases} \quad (3.2)$$

Where,

$$K_1=\sum_{j=0}^{n-s}\binom{n-s}{j}(-1)^{n-s-j}\{2(2\alpha^{-1})^{-\frac{3}{2}}e^{\left(\frac{A_j^2}{8\alpha^{-1}}\right)}D_{-3}\left(u_{A_j}\right)\} \quad (3.3)$$

$$A_j=(n-j)T \quad (3.4)$$

$$u_{A_j}=\frac{A_j}{\sqrt{(2\alpha^{-1})}} \quad (3.5)$$

Bayes estimates of $R(t/\theta)=e^{-t\theta}$,under various loss functions as mentioned above are given in the following table :

3.1 BAYES ESTIMATES OF R(t/ θ)FOR ATTRIBUTE LIFE TESTING

S.No.	Loss Function	Bayes Estimate
1.	SELF	$\widehat{R}_{AB}=\frac{\sum_{j=0}^{n-s}\binom{n-s}{j}(-1)^{n-s-j}e^{\left(\frac{B_j^2}{8\alpha^{-1}}\right)}D_{-3}(u_{B_j})}{\sum_{j=0}^{n-s}\binom{n-s}{j}(-1)^{n-s-j}e^{\left(\frac{A_j^2}{8\alpha^{-1}}\right)}D_{-3}(u_{A_j})}$, $B_j=A_j+t$, $u_{B_j}=\frac{B_j}{\sqrt{(2\alpha^{-1})}}$

2.	DLF	$\widehat{R}_{ADG} = \frac{\sum_{j=0}^{n-s}\binom{n-s}{j}(-1)^{n-s-j} e^{\left(\frac{C_j^2}{8\alpha^{-1}}\right)} D_{-3}\left(u_{C_j}\right)}{\sum_{j=0}^{n-s}\binom{n-s}{j}(-1)^{n-s-j} e^{\left(\frac{B_j^2}{8\alpha^{-1}}\right)} D_{-3}\left(u_{B_j}\right)}, B_j = A_j + t, u_{B_j} = \frac{B_j}{\sqrt{(2\alpha^{-1})}}$ $C_j = A_j + 2t, u_{C_j} = \frac{C_j}{\sqrt{(2\alpha^{-1})}}$
3.	MELO	$\widehat{R}_{AM} = \frac{\sum_{j=0}^{n-s}\binom{n-s}{j}(-1)^{n-s-j} e^{\left(\frac{B_j^2}{8\alpha^{-1}}\right)} D_{-1}\left(u_{B_j}\right)}{\sum_{j=0}^{n-s}\binom{n-s}{j}(-1)^{n-s-j} e^{\left(\frac{A_j^2}{8\alpha^{-1}}\right)} D_{-1}\left(u_{A_j}\right)},$
4.	EWMELO	$\widehat{R}_{AEW}$ $= \frac{\sum_{j=0}^{n-s}\binom{n-s}{j}(-1)^{n-s-j} e^{\left(\frac{Q_j^2}{8\alpha^{-1}}\right)} D_{-1}\left(u_{Q_j}\right)}{\sum_{j=0}^{n-s}\binom{n-s}{j}(-1)^{n-s-j} e^{\left(\frac{P_j^2}{8\alpha^{-1}}\right)} D_{-1}\left(u_{P_j}\right)}, Q_j = B_j + a, P_j = A_j + a,$ $u_{P_j} = \frac{P_j}{\sqrt{(2\alpha^{-1})}}, u_{Q_j} = \frac{Q_j}{\sqrt{(2\alpha^{-1})}}$

References

[1] Abramowitz, M and Stegun,I.(1964): Handbook of Mathematical Functions; U. S. Bureau of Standards, Washington D.C.

[2] B. Epstein and M. Sobel, (1953): “Life Testing”, JASA 48,pp.486-501

[3] Bhattacharya, S.K. (1967): “Bayesian Approach to Life Testing and Reliability Estimation” JASA 62, pp.48-62

[4] Bhattacharya, S.K. & Kumar, S. (1986): “Bayesian life estimation with an inverse Gaussian prior”, South African Statist. J. 20,37-43.

[5] Bhattacharya, S.K. &Tyagi, R.K. (1988): “Bayesian reliability estimation with truncated normal prior”, Calcutta Statistical Association Bulletin,Vol.37.September & December Nos.147-148.

[6] DeGroot, M.H. (2005), Optimal Statistical Decisions, Vol. 82, John Wiley & Sons

[7] Singh, Randhir. (1997), D.Phil Thesis (Unpublished), Department of Mathematics and Statistics, University of Allahabad, Allahabad (INDIA).

[8] Singh, Randhir .(1999), “Bayesian Analysis of a Multicomponent System” ,Proceedings of NSBA-TA, 16-18 Jan.1999,pp.252-261.Editor -Dr. Rajesh Singh. The conference was organised by the Department of Statistics, Amrawati University, Amrawati-444602. Maharastra,India.

[9] Singh, Randhir .(2010), “Simulation Aided Bayesian Estimation for Maxwell’s Distribution”, Proceedings of National Seminar on Impact of Physics on Biological Sciences (August,26,2010),held by the Department of Physics, Ewing Christian College, Prayagraj,India,pp.203-210; ISBN No.: 978-81-905712-9-6

[10] Singh Randhir .(2021), “On Bayesian Estimation of Loss and Risk Functions. Science Journal of Applied Mathematics and Statistics”, Vol.9, No.3,2021, pp.73-77.doi: 10.11648/j.sjams.20210903.11

[11] Singh, Randhir. (2021):"On Bayesian Estimation of Function of Unknown Parameter of Modified Power Series Distribution". International Journal of Innovative Science and Research Technology ISSN No:2456-2165, Volume 6,Issue 6, June-2021,pp.861-864 www.ijirst.com

[12] Singh, Randhir. (2021):" Bayesian Estimation of Function of Unknown Parameters of Some Particular Cases of Modified Power Series Distribution". Journal of Emerging Technologies and Innovative Research (JETIR), ISSN No:2349-5162, Volume 8,Issue 7, pp.d673-d681 www.jetir.org

[13] Singh, Randhir. (2021): Bayesian Estimation of Moments and Reliability of Geometric Distributtion, Quest Journals : Journal of Research in Applied Mathematics,Volume7~Issue 7(2021) pp:19-25. www.questjournals.org

[14] Singh, Randhir. (2021): Bayesian and Classical Estimation of Parameter and Reliability of Burr Type XII Distribution. Journal of Emerging Technologies and Innovative Research (JETIR), ISSN No:2349-5162, Volume 8,Issue 9, pp.a546-a556 www.jetir.org

[15] Singh, Randhir. (2021):" Bayesian and Classical Estimation of Parameter and Reliability of Weibexpo Distribution". International Journal of Innovations in Engineering Research and Technology and Innovative Research (IJIERT), ISSN(E):2394-3696, Volume 8,Issue 9, pp.131 -139 www.ijiert.org

[16].Tummala ,V.M.& Sathe ,P.T. (1978) MELO Estimators of Reliability and Parameters of certain Life time distributions ,IEEE Transactions on Reliability,Vol.R-27,No .4 pp 283-285

[17].Zellner ,A. & Park ,S.B. (1979) Minimum Expected Loss (MELO) Estimators of functions of parameters and structural coefficients of econometric models ; JASA ,74,pp185-193.