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Random Fixed Point Theorem for Contractive Mappings

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Abstract: In this paper, we prove the results of existence of random common fixed point and its uniqueness for a pair of random mappings under weakly contractive condition .

Keywords: Polish space, Random fixed point theorem, Weakly contractive mapping. .

1. Introduction

Random fixed point theory has receive much attention in recent years and it is needed for the study of various classes of random equations. The study of random fixed point theorems was initiate by the Prague school of probabilistic in the 1950s. The interest in this subject enhanced after publication of the survey paper of Bharucha Reid [6].

Obtaining the existence and uniqueness of fixed points for the self -maps of a metric space by altering distances between the points with the use of a control function is an interesting aspect in the classical fixed point theory. In this direction, Khan et al. [10] introduced a new category of fixed point problems for a single self-map with the help of a control function that alters the distance between two points in a metric space which they called an altering distance function. However, similar type of function was already in use in the fixed point theory under the title function and the details may be found in Dhage [7].

Definition 1.1. [Dhage [7]] A function $\phi : [0, \infty) \to [0, \infty)$ is called a function if it is a continuous and monotone nondecreasing function satisfying ϕ (0) = 0.

There do exist *D*function useful in the fixed point theory and applications and commonly *D* used functions are $\phi(r) = kr$ and $\psi(r) = \dfrac{Lr}{K + r}$ The *D* functions ϕ and ψ are respectively used in the fixed point theory for linear and nonlinear contraction mappings in metric spaces (cf. Dhage [7] and the references cited therein).

Definition 1.2. (Weakly contractive mapping): Let X be a metric space. A mapping $T : X \to X$ is called weakly contractive if for each x, y € X,

where : ϕ $[0, \infty) \to [0, \infty)$ is positive on $(0, \infty)$ and $\phi(0) = 0$.

In fact, Alber and Guerre-Delabriere [2] assumed an additional condition on ϕ that is lim $\phi(t) = \infty$. But Rhoades [11] obtained the result noted in following theorem without using this particular assumption.

Theorem 1.1. (Rhoades [11]) *If $T : X \to X$ is a -weakly contractive mapping, where (X,d) is a complete metric space, then T has a unique fixed point.*

It may be observed that though the function ϕ has been defined in the same way as the *D* - function, the way it has been used ha Theorem 2.1 is completely different from the use of *D*- function.

Definition 1.3. A self mapping *T* of a metric space *(X, d)* is said to be weakly contractive with respect to a self mapping *S*: *X* $\rightarrow$ *X*, if for each *x,y* € *X*,

$$d(Tx,Ty) \leq d(Sx,Sy) - \psi(d(Sx,Syj),$$

where ψ: [0, ∞) $\rightarrow$ [0, ∞) is a continuous and nondecreasing function such that ψ is positive on (0, ∞), ψ (0) = 0 and lim ψ *(t)* = ∞

Theorem 1.2. [1]Let (X, d) be a complete metric space, _: [0, +∞ $t \rightarrow \infty$) be an altering distance function, and T : X ® X be a self-mapping which satisfies the following inequality:

$\phi(d(Tx, Ty)) \leq c\phi(d(x, y))$ (1:1)

for all x, y Î X and for some 0 <c < 1. Then, T has a unique fixed point.
Letting _(t) = t in Theorem 1.2, we retrieve immediately the Banach contraction principle. In 1997, Alber and Guerre-Delabriere [2] introduced the concept of weak contractions in Hilbert spaces. This concept was extended to metric spaces in [3].

2. Random Common Fixed Point Theorem For Generalized Weakly Contractions

let *(X,d)* be a polish space, i.e., a separable complete metric space and (Ω , A) be a measurable space (i.e., A is σ algebra of subsets of Ω). A function $\xi : \Omega \rightarrow X$ is said to be a A measurable if for any open subsets B of *X*, $\xi^{-1}(B)$ € A.

A mapping S : Ω x *X* $\rightarrow$ *X* is said to be a random map if and only if for each fixed *x* € *X*, the mapping S (. *x*) : $\Omega \rightarrow X$ is measurable. A random map S : Ω x *X* $\rightarrow$ *X* is continuous if for each ω, €, Ω , the mapping S(Ω , ·) : *X* $\rightarrow$ *X* is continuous. A measurable mapping $\xi : \Omega . \rightarrow X$ is a random fixed point of the random map S : Ω x *X* $\rightarrow$ *X* if and only if S $(\omega, \zeta(\omega)) = \xi(\omega)$ for each ω, €, Ω
,

Definition 2.1. A measurable mapping $\xi : \Omega \rightarrow K$, is said to be a random common fixed point of random operators $S : \Omega \text{ x } K \rightarrow$ and $T : \Omega \text{ x } K \rightarrow K$ if for each $\omega \in \Omega(\omega) = S(\omega, \xi(\omega)) = T(\omega, \xi(\omega))$.
In [6], Choudhury introduced the concept of a generalized altering distance function for three variables. In the following we generalized this notion for five variables.

Definition 2.2. A function : ϕ [0, ∞) $\rightarrow$ [0, ∞) is said to be a generalized altering distance function if the following conditions are satisfied:

(i) ϕ is continuous,

(ii) ϕ is monotone increasing for every variables, and

(iii) ϕ (x_1, x_2, x_3, x_4) = 0 if and only if $x_1, x_2, x_3, x_4 = 0$

we prove a random common fixed point theorem for a pair of mappings.

Theorem 2.1 *Let X be a separable metric space and K be a nonempty Polish subspace of X. Let S,T:* Ω x *K* $\rightarrow$ *K be two continuous self maps satisfying for each* ω, €, Ω ,

$$\begin{aligned} \psi(d(S(\omega,x)S(\omega,y))) \leq \phi_1(&d(x(\omega),y(\omega)),d(x(\omega),T(\omega,x)),d(y(\omega),S(\omega,y)),\\ &d(x(\omega),S(\omega,y)),d(y(\omega),T(\omega,x)) \;) \qquad (2.1)\\ -\phi_2\big(&d(x(\omega),y(\omega)),d(x(\omega),T(\omega,x))\,d(y(\omega),S(\omega,y)),\\ &d(x(\omega),T(\omega,y)),d(y(\omega),T(\omega,x)) \;\big) \end{aligned}$$

for each x, y, $\in$, K *where* ϕ_i (i=1,2) *are generalized D-functions and the function* ψ *is defined by* ϕ_i (x,x,x,x,x) *Then there exists a measurable mapping* $\xi:\Omega\to K$ *Such that* $\xi(\omega)=S(\omega,\xi(\omega))=T(\omega,\xi(\omega))$

Proof. $\xi_0:\Omega\to K$ be a measurable but fixed mapping in *K*, we get

$$\xi_1(\omega)=T(\omega,\xi_0(\omega)) \quad \text{and } =\xi_2(\omega)=S(\omega,\xi_1(\omega))$$

Similarly, we get

$$\xi_3(\omega)=T(\omega,\xi_2(\omega)) \quad \text{and } =\xi_4(\omega)=S(\omega,\xi_3(\omega))$$

Inductively, we construct a sequence of measurable maps $\{\xi_n\}$ from Ω to *K* such that

$$\xi_{2n+1}(\omega)=S(\omega,\xi_{2n}(\omega)) \quad \text{and } =\xi_{2n+2}(\omega)=T(\omega,\xi_{2n+1}(\omega)) \qquad (2.2)$$

Since *S* and *T* are continuous, by a result of Himmelberg [9], $\{\xi_n\}$ is a measurable sequence. First we will prove that

$$d(\xi_n(\omega),\xi_n(\omega))\le d(\xi_{n-1}(\omega),(\omega),\xi_n(\omega))$$

Consider, the following estimate:

$$\begin{aligned}
&\psi(d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))\\
&\le\psi(d(T(\omega,\xi_{2n}(\omega)),S(\omega,\xi_{2n+1}(\omega)))\\
&\le\phi_1(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n}(\omega),T(\omega,\xi_{2n+1}(\omega),S(\omega,\xi_{2n+1}(\omega))),\\
&d(\xi_{2n}(\omega),S(\omega,\xi_{2n+1}(\omega))),d(\xi_{2n+1}(\omega),T(\omega,\xi_{2n}(\omega)))\\
&-\phi_2(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n}(\omega),T(\omega,\xi_{2n}(\omega))),d(\xi_{2n+1}(\omega),S(\omega,\xi_{2n+1}(\omega))),\\
&d(\xi_{2n}(\omega),S(\omega,\xi_{2n+1}(\omega))),d(\xi_{2n+1}(\omega),T(\omega,\xi_{2n}(\omega))),\\
&=\phi_1(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)),\\
&d(\xi_{2n}(\omega),\xi_{2n+2}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+1}(\omega))),\\
&-\phi_2(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)),\\
&d(\xi_{2n}(\omega),\xi_{2n+2}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+1}(\omega))),\\
&\le\phi_2(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+1}(\omega)),\\
&d(\xi_{2n}(\omega),\xi_{2n+2}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))),\\
&-\phi_2(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega),\\
&d(\xi_{2n}(\omega),\xi_{2n+1}(\omega))+d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))),
\end{aligned}$$

If

$$d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))\ \rangle, d(\xi_{2n}(\omega),\xi_{2n+1}(\omega))$$

then,

$$\begin{aligned}
&\psi(d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)))\langle\phi_1(d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)),\\
&d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)), \qquad (2.4)\\
&\psi(d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))),
\end{aligned}$$

which is a contradiction. Since 0i is monotone increasing for all variables and

$$\phi_2[d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))]\ne 0$$

whenever

$$d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))$$

So, we have

$$d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)) \le d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)) \qquad (2.5)$$

for all n = 0, 1, ... Putting $x=\xi_{2n}(\omega), y=\xi_{2n-1}(\omega)$ in (2.1) we have

$$\psi(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega))$$
$$=\psi(d(T(\omega,\xi_{2n-1}(\omega)),d(S(\omega,\xi_{2n}(\omega)))))$$
$$\le\phi_1(d(\xi_{2n-1}(\omega),\xi_{2n}(\omega)),d(\xi_{2n-1}(\omega),T(\omega,\xi_{2n-1}(\omega))),d(\xi_{2n}(\omega),S(\omega,\xi_{2n}(\omega))),$$
$$d(\xi_{2n+1}(\omega),S(\omega,\xi_{2n}(\omega))),d(\xi_{2n}(\omega),\xi_{2n,}T(\omega,\xi_{2n-1}(\omega))))$$
$$-\phi_2(d(\xi_{2n-1}(\omega),\xi_{2n}(\omega)),d(\xi_{2n-1}(\omega),T(\omega,\xi_{2n-1}(\omega))),d(\xi_{2n}(\omega),S(\omega,\xi_{2n}(\omega))),$$
$$d(\xi_{2n-1}(\omega),S(\omega,\xi_{2n}(\omega))),d(\xi_{2n}(\omega),\xi_{2n,}T(\omega,\xi_{2n-1}(\omega))))$$
$$=\phi_2(d(\xi_{2n-1}(\omega),\xi_{2n}(\omega)),d(\xi_{2n-1}(\omega)),d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),$$
$$d(\xi_{2n-1}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n}(\omega),\xi_{2n}(\omega))$$
$$-\phi_2(d(\xi_{2n-1}(\omega),\xi_{2n}(\omega)),d(\xi_{2n-1}(\omega)),d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),$$
$$d(\xi_{2n-1}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n}(\omega),\xi_{2n}(\omega))$$

By similar arguments, we have

$$d(\xi_{2n+2}(\omega),\xi_{2n+3}(\omega)) \le d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)) \qquad (2.7)$$

for all $n \in$ N. From (2.5) and (2.7) we obtain

$$d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)) \le d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)) \qquad (2.8)$$

for all $n \in$ N. From (2.3) and (2.8), we have for all integers $n \ge 0$

$$\psi(d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))) \le \phi_1(d(\xi_{2n,}\xi_{n+1}(\omega))),-\phi_2(d(\xi_{2n}(\omega),\xi_{n+1}(\omega)))$$

or, equivalently,

$$\phi_2(d(\xi_{n+1}(\omega),\xi_{2n+2}(\omega))),\le\phi_1(d(\xi_n(\omega),\xi_{n+1}(\omega)))-\phi_1(d(\xi_n(\omega),\xi_{n+1}(\omega))),$$

Summing up from (2.8), we obtain

$$\sum_{n=0}^{\infty}\phi(d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))) \le \phi_1(d(\xi_{0,}(\omega),\xi_1(\omega))),\prec\infty$$

This implies,

$$\phi_2(d(\xi_n(\omega),\xi_{n+1}(\omega))),\to 0 \text{ as } n\to\infty \qquad (2.9)$$

Again, from (2.8), the sequence $\{d(\xi_n(\omega),\xi_{n+1}(\omega))\}$ is monotone non-increasing and bounded. Hence there exists a real number $r(\omega)\ge 0$ such that,

$$\lim_{n\to\infty} d(\xi_n(\omega),\xi_{n+1}(\omega)) = r(\omega)$$

Then, by continuity of , ϕ, from (2.9), we obtain ϕ_2 (r(ω))) = 0 which imphes that by the property of function ϕ, , we have $r(\omega)$ = 0. Thus,

$$\lim_{n\to\infty} d(\xi_n(\omega),\xi_{n+1}(\omega)) = 0 \qquad (2.10)$$

Now we claim that $\{\xi_n(\omega)\}$ is a Cauchy sequence in K. If possible, let $\{\xi_n(\omega)\}$ is not a Cauchy sequence then there exists ε > 0 for which we can find subsequences $\{\xi_n(\omega)\}$ and $\{\xi_n(\omega)\}$ with ni > mi >i such that

$$d(\xi_n(\omega),\xi_{ni}(\omega))\angle=\varepsilon \qquad (2.11)$$

Further we can choose *ni* corresponding *mi*, in such a way that it is smallest integer with *ni* > *mi* satisfying

$$d(\xi_{ni}, \xi_{ni-1}(\omega)) \angle = \varepsilon \qquad (2.12)$$

Using (2.1 1), (2.12) and the triangle inequality, we have

$$\begin{aligned} \varepsilon &\le d(\xi_{mi}(\omega), \xi_{ni}(\omega)) \\ &\le d(\xi_{mi}(\omega), \xi_{ni-1}(\omega)) + d(\xi_{ni-1}(\omega), \xi_{ni-1}(\omega)) \qquad (2.13) \end{aligned}$$

$$\varepsilon + d(\xi_{ni-1}(\omega), \xi_{ni}(\omega))$$

Letting $i \to \infty$ and using (2. 10),

$$\lim_{n \to \infty} d(\xi_{mi}(\omega), \xi_{ni}(\omega)) = \varepsilon \qquad (2.\ 14)$$

Again, from the triangle inequality we get

$$\begin{aligned} d(\xi_{mi}(\omega), \xi_{ni}(\omega)) &\le d(\xi_{mi}(\omega), \xi_{mi-1}(\omega)) + d(\xi_{mi-1}(\omega), \xi_{ni-1}(\omega)) \\ &\quad + d(\xi_{ni-1}(\omega), \xi_{ni}(\omega)) \qquad (2.15) \\ d(\xi_{mi-1}(\omega), \xi_{ni-1}(\omega)) &\le d(\xi_{mi-1}(\omega), \xi_{mi-1}(\omega)) + d(\xi_{mi}(\omega), \xi_{ni}(\omega)) \\ &\quad + d(\xi_{ni}(\omega), \xi_{ni-1}(\omega)) \end{aligned}$$

Letting i — > oo and using the inequalities (2.10) and (2.14), we obtain

$$\lim_{n \to \infty} d(\xi_{mi-1}(\omega), \xi_{ni-1}(\omega)) = \varepsilon \qquad (2.16)$$

Setting $x = \xi_{mi}(\omega)$ and $y = \xi_{ni}(\omega)$ in (2. 1), we obtain

$$\begin{aligned} &\psi(d(\xi_{mi-1}(\omega), \xi_{ni-1}(\omega)) \\ &= \psi(d(T(\omega, \xi_{mi}(\omega)), S(\omega, \xi_{ni}(\omega))) \qquad (2.17) \\ &\le \phi_1((d(\xi_{mi}(\omega), \xi_{ni}(\omega)), d(\xi_{mi}(\omega), T(\omega, \xi_{mi}(\omega))), d(\xi_{ni}(\omega), S(\omega, \xi_{ni}(\omega))) \\ &d(\xi_{mi}(\omega), S(\omega, \xi_{ni}(\omega))), d(\xi_{ni}(\omega), T(\omega, \xi_{mi}(\omega))))) \\ &-\phi_2((d(\xi_{mi}(\omega), \xi_{ni}(\omega)), d(\xi_{mi}(\omega), T(\omega, \xi_{mi}(\omega))), d(\xi_{ni}(\omega), S(\omega, \xi_{ni}(\omega))), \\ &d(\xi_{mi}(\omega), S(\omega, \xi_{ni}(\omega))), d(\xi_{ni}(\omega), T(\omega, \xi_{mi}(\omega))))) \end{aligned}$$

Letting $i \to \infty$ in (2.17) and using the inequalities (2.2), (2.1 1) and (2.12), we obtain

$$\begin{aligned} \psi(\omega) &\le \lim_{i \to \infty} \psi(d(T(\omega, \xi_{mi}(\omega)), S(\omega, \xi_{ni}(\omega)))) \\ &\le \lim_{i \to \infty} \phi_1(d(\xi_{mi}(\omega), \xi_{ni}(\omega)), d(\xi_{mi}(\omega), T(\xi_{mi}(\omega))), d(\xi_{ni}(\omega), S(\omega))), \\ &\qquad d(\xi_{mi}(\omega), S(\omega, \xi_{ni}(\omega))), d(\xi_{ni}(\omega), T(\omega, \xi_{ni}(\omega))) \\ &-\lim_{i \to \infty} \phi_2(d(\xi_{mi}(\omega), \xi_{ni}(\omega)), d(\xi_{mi}(\omega), T(\omega, \xi_{mi}(\omega))), d(\xi_{ni}(\omega), S(\omega, \xi_{ni}))), \\ &\qquad d(\xi_{mi}(\omega), S(\omega, \xi_{ni}(\omega))), d(\xi_{ni}(\omega), T(\omega, \xi_{ni}(\omega))) \qquad (2.18) \\ &= \lim_{i \to \infty} \phi_1(d(\xi_{mi}(\omega), \xi_{ni}(\omega)), d(\xi_{mi}(\omega), \xi_{mi+1}(\omega)), d(\xi_{ni}(\omega), \xi_{ni+1}(\omega)), \\ &\qquad d(\xi_{mi}(\omega), \xi_{ni+1}(\omega)), d(\xi_{ni}(\omega), \xi_{ni+1}(\omega))) \\ &-\lim_{i \to \infty} \phi_2(d(\xi_{mi}(\omega), \xi_{ni}(\omega)), d(\xi_{mi}(\omega), \xi_{mi+1}(\omega)), d(\xi_{ni}(\omega), \xi_{ni+1}(\omega)), \end{aligned}$$

$$d(\xi_{mi}(\omega),\xi_{ni+1}(\omega)),d(\xi_{ni}(\omega),\xi_{mi+1}(\omega)))$$

Using inequalities (2.10), (2.12) and (2.14), we have

$$\psi(\varepsilon)\le\phi_1(\varepsilon,0,0,0,0)-\phi_2(\varepsilon,0,0,0,0)\prec\phi_1(\varepsilon)$$

Since ϕ_1 is monotone increasing in its variables and by the property ϕ_2 that

$$\phi(t_1,t_2,t_3,t_4,t_5)=0 \quad \text{if and only if} \quad t_1=t_2=t_3=t_4=t_5.$$

Thus we arrive at a contradiction as $\varepsilon > 0$.

Hence $\{\xi_{ni}(\omega)\}$ is Cauchy sequence in *K*, there exists $\xi:\Omega\to K$ such that $\xi_{ni}(\omega)\to\xi(\omega)$ for all $\omega \in \Omega$. We show that $\xi(\omega)$ is random common fixed point of *S* and *T*.

$$\mathrm{T}(\omega,\xi(\omega))=\lim_{i\to\infty}\mathrm{T}(\omega,\xi_{n2}(\omega))=\lim_{i\to\infty}\xi_{2n+1}(\omega)=\xi(\omega)$$

Similarly, we can prove $\xi(\omega) = S(\omega))\ \xi(\omega)$. Hence, $T(\omega,\xi(\omega))=\xi(\omega)=S(\omega,\xi(\omega))$ and consequently $\xi(\omega)$ is common fixed point of S(ω) S and *T i.e.*.

Finally, we prove the uniqueness of the common random fixed point ξ of S and *T*. Let $\xi(\omega)$ and $\xi(\omega)$ be two random fixed points of S and *T* i.e.

$$S(\omega,\xi(\omega))=\xi(\omega))=\mathrm{T}(\omega,\xi(\omega))$$

and

$$T(\omega,\xi(\omega))=\xi(\omega)=\mathrm{S}(\omega,\xi(\omega))$$

for each $\omega \in \Omega$ Using inequah'ty *(2.1)*, we have

$$\begin{aligned}
&\psi(d(\xi(\omega),\xi(\omega)))=\psi(d(T(\omega,\xi(\omega)),T(\omega,\xi(\omega))))\\
&\le\phi_1(d(\xi(\omega),\xi(\omega)),d(\xi(\omega),T(\omega,\xi(\omega)))),d(\xi(\omega),T(\omega,\xi(\omega))),\\
&d(\xi(\omega),T(\omega,\xi(\omega))),d(\xi(\omega),T(\omega,\xi(\omega)))),\\
&-\phi_2(d(\xi(\omega),\xi(\omega)),d(\xi(\omega),T(\omega,\xi(\omega))),d(\xi(\omega),T(\omega,\xi(\omega))),\\
&d(\xi(\omega),T(\omega,\xi(\omega))),d(\xi(\omega),T(\omega,\xi(\omega)))), \qquad (2.19)\\
&=\phi_1(d(\xi(\omega),\xi(\omega)),0,0,d(\xi(\omega),\xi(\omega)),0,d(\xi(\omega)\xi(\omega))),\\
&-\phi_2(d(\xi(\omega),\xi(\omega)),0,0,d(\xi(\omega),\xi(\omega)),0,d(\xi(\omega)\xi(\omega))),\\
&\angle\phi_1(d(\xi(\omega),\xi(\omega))
\end{aligned}$$

which is possible only when $\xi(\omega)=\xi(\omega)$ since ϕ_1 is monotone increasing in all its variables and $\phi(t_1,t_2,t_3,t_4,t_5)\le 0$ if at least one of t_1,t_2,t_3,t_4,t_5 is nonzero. Hence, $\xi(\omega)$ is the unique random common fixed point of S and T i.e., S $(\omega,\xi,(\omega))=\xi(\omega)=T(\omega,\xi(\omega))$ for all $\in \Omega$.

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