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A note on SVD and QR-decomposition.

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Abstract : In this paper we are going to present some of the properties of SVD & QR-decomposition and its applications. Here we have presented some of the applications of concepts and techniques from Singular Value decomposition to the world of science and technology. We also highlight some of the unique properties of QR and Singular Value decomposition-based technique.

***IndexTerms* - SVD, QR-decomposition, machine learning**

I. INTRODUCTION

Linear Algebra is a major part of mathematics which deals with vectors, matrices, and linear transforms. It also plays an important role in machine learning. The process of machine learning could be divided into four sections. In first section, we decide what is the problem that we are going to solve, such as model to identify airplane or bird, to identify whether the emails are spam or non-spam, model to identify whether the tumor cells are cancerous or non-cancerous, model to improvise client experience by routing calls into many fields so that clients problems could be sorted out with the right expertise, model to predict if a loan will charge off after the duration of the loan, model to predict price of a house based on different features or predictors, and so on. Second section is where we handle the data available for constructing the model. It includes feature extraction, handling missing data and categorical data, encoding class labels, normalization and standardization of features, feature engineering, dimensionality reduction, data partitioning into training, validation, and testing sets, etc. Thirdly, we choose the suitable model or algorithm that we would like to use. Some of the common machines learning algorithms are linear regression, logistic regression, KNN, SVM, K-means, Monte Carlo simulation, time series analysis, etc. In final section, the machine learning algorithm or model is implemented to start improving the client experience or increasing productivity, etc.

Contemporary Statistics has been outlined using the inscription of linear algebra and modern statistical methods make use the tools of linear algebra. Machine learning techniques are also outlined in same procedure, using the inscriptions and tools which are given from linear algebra. Traditional methods of machine learning also use the linear algebra techniques, such as linear regression through QR-decomposition, singular value decomposition and least squares (linear algebra techniques). Principal component analysis (PCA) has been evolved from linear algebra and statistics. So, linear algebra is one of the most important tools in machine learning. In many areas like space science, medicine, agriculture, economics, social sciences, engineering and Technology, etc.; there have been implementation of computer aided decision-making systems. We can see tremendous increase in demand and popularity of computer aided decision making and hence research and development in this field has gain its own momentum. Principal component analysis (PCA) is one of important machine learning algorithm used for dimensionality reduction and it could be applied in pattern classification and data representation areas. Here a QR method and SVD based PCA has been presented in this paper.

It is quit challenging to model data with many features, moreover datasets perhaps has many columns say hundreds and thousands or more. It is hard to know which features of the data are relevant and which are not. Methods for automatically reducing the number of columns of a dataset are called dimensionality reduction, and perhaps the most popular method is called the principal component analysis or PCA for short. So, in machine learning, PCA is used to projections of high dimensional data for training models and data visualization. A matrix factorization technique from linear algebra is the central idea of PCA. PCA is the foundation stone, dimensionally reduction technique for probability and statistics frequently used in machine learning and data science applications where you reveal the low dimension patterns.

SVD is linear algebra technique to decompose a matrix into three matrices of orthogonal components with which optimal sub rank approximations. Suppose we have a real valued matrix A of order $l \times m$ with rank n, where $n \leq m \leq l$. SVD is a matrix splitting to make certain subsequent matrix computations easier. SVD is define as

$$A = U\Sigma V^T$$

where A is the real $l \times m$ matrix that we wish to factorize, U is an $l \times l$ matrix, Σ is an $l \times m$ diagonal matrix, and $V^T$is an $m \times m$ matrix. The most beautiful features about SVD is that the left -singular vectors of the original matrix A is the columns of the U matrix, the right-singular vectors are the columns of V and the diagonal elements of Σ matrix is the singular values of the main matrix A [1].

Suppose $U^T AV = \Sigma$ be the SVD of A and if $\sigma_k \neq 0$ and $\sigma_{k+1} = 0$,were U, V are orthogonal matrices then $X = A^T A = V\Sigma U^T U\Sigma V^T = V\Sigma^2 V^T$ and $Y = AA^T = U\Sigma V^T V\Sigma U^T = U\Sigma^2 U^T$. The diagonal elements of the Σ matrix is the eigen values of the matrices X, Y. Here the nullity of A is the span of $v_{k+1}, v_{k+2}, \dots, v_n$ and if $A^\dagger$ is equal to $[v_1, v_2, \dots, v_k] diag\left[\frac{1}{\sigma_1}, \frac{1}{\sigma_2}, \dots, \frac{1}{\sigma_k}\right][u_1, u_2, \dots, u_k]^T$, then $x = A^\dagger b$ is a least square solution of $Ax = b$. Also, the matrix A could be reconstructed by the formula $A = [\sigma_1 u_1 {v_1}^T] + [\sigma_2 u_2 {v_2}^T] + \cdots + [\sigma_k u_k {v_k}^T]$, using U, Σ and V^T i.e.,

$$A = U\Sigma V^T = [\sigma_1 u_1, \sigma_2 u_2, \dots, \sigma_k u_k, 0, \dots, 0][v_1, v_2, \dots, v_n]^T$$

$$= \sum_{j=1}^{k} \sigma_j u_j v_j^T = [u_1, u_2, \dots, u_k]\, diag[\sigma_1, \sigma_2, \dots, \sigma_k][v_1, v_2, \dots, v_n]^T$$

And so, data matrix $A = \sum_{i=1}^{n} u_i \sigma_i v_i^T$ where u_i are the linearly independent columns and v_i^T are linearly independent rows of the data matrix A i.e., eigen vectors of a correlation matrices of AA^T and $A^T A$. We can truncate at rank r if we are having lots of negligible small singular values. So SVD is used to convert experimental data into a well-organized non redundant, noise free to extract hidden information [2].

Singular value decomposition (SVD) is an algebraic tool that factorized the data matrix which provides a foundation for almost all the data methods that are numerically stable. We know that SVD offers significant properties in data science and engineering particularly in data science and signal processing. SVD is one of the most important techniques to decompose matrix used in data science and many allied areas which is numerically quite stable. It is used to reduce low rank approximations to data matrices. Also, it is used to solve system of equations $Ax = c$ of non-square matrices. The most important uses of SVD are SVD based principal component analysis (PCA). It is very stable technique where high dimensional datasets are decomposed into its most statistically illustrative factors. SVD based PCA has been applied to a wide variety of problems in data science and engineering. Many analytical and numerical results of the fast Fourier transform (FFT) can be generalized using SVD. However, SVD is more generic, and data driven technique.

2 QR and SVD based PCA method:

In this method we use a rectangular matrix $A \in R^{m\times n}$(where $m \gg n$) to determine the Eigen value decomposition (EVD) of $\Sigma_A = AA^T$. Let us consider the rank of $\Sigma_A \in R^{m\times m}$ be r, where $1 \leq r < n$. The rectangular matrix A is decomposed into $Q_1 \in R^{m\times r}$ which is orthogonal and $R_1 \in R^{r\times n}$, upper triangular matrix using QR decomposition as

$$A = Q_1 R_1 \text{(i)}$$

Since, $\Sigma_A = AA^T$ (ii)

$\Rightarrow \Sigma_A = Q_1 R_1 {R_1}^T {Q_1}^T$ using (i) in (ii)

Now we factor ${R_1}^T$ using singular value decomposition.

$${R_1}^T = U_1 M_1 V^T \qquad \text{(iii)}$$

where $U_1 \in R^{n\times r}$ and $V \in R^{r\times r}$ is a diagonal matrix and $M_1 \in R^{r\times r}$ is a diagonal matrix which consists only of Eigen values. Now (iii) in (ii) implies,

$$\Sigma_A = Q_1 V M_1 {U_1}^T U_1 M_1 V^T {Q_1}^T$$

$$\Sigma_A = Q_1 V M_1^2 V^T {Q_1}^T$$

$$\Sigma_A = Q_1 V \Lambda V^T {Q_1}^T \text{ where } \Lambda = M_1^2$$

We can show that $(Q_1 V)^T (Q_1 V) = V^T {Q_1}^T Q_1 V = V^T (I) V = I$,

The transform $Q_1 V$ is orthogonal matrix and it also diagonalizes matrixΣ_A . $Q_1 V$ is an eigenvector matrix and Λ is an Eigen value matrix of Σ_A . Let $V_{r_1} \in R^{r\times r_1}$, $1 \le r_1 \le r$ be a matrix containing r_1 column vectors of V corresponding to the largest r_1 diagonal entries of M_1, then the PCA transform would be $\Omega = Q_1 V_{r_1} \in R^{m\times r_1}$ [3].

3 Conclusion

Signal processing and machine learning techniques play a vital role in dealing with engineering and biomedical problems. One of the most challenging tools in machine learning is to develop effective tools for analyzing the datasets. SVD and QR & SVD based technique is a very useful and very efficient tool among the analysis techniques. SVD is very efficient method and could be able to determine best sub-rank approximations by decomposing given matrix into orthogonal dominant and subdominant subspaces effectively [1]. So, using the largest singular value one can easily determine the low rank approximation. We can represent a sum of rank one matrix from a given matrix by using singular value decomposition. We can truncate the matrix A to a specific rank k matrix by using SVD and store the approximated A_k of the matrix in place of the entire matrix A [10]. This technique is often used in watermarking, noise reduction, image compression, etc. SVD can be used to extract mechanical meaningful signals from rotating machine from the data. For example, in [4][5][6][7][9], SVD is used to detect the frequency allocation of the fault by decomposing the covariance matrix of the stator current. SVD is also used for diagnosing and fault features extraction of diesel engine crankshaft bearings effectively. High resolution radar is widely implemented in the fields of military, remote sensing, air traffic control, measuring ocean surface waves, detection of speeding traffic, satellite. SVD helps improving the computational complexity by removing the redundant and noisy information for high resolution range profile target recognition. SVD technique is also used to enhance the visual quality of subtle organs as well as segmenting capabilities from MR images [8]. Singular value decomposition (SVD) could be used to complete the missing data by selecting one having minimal rank. Even though lots of works have been done on SVD, SVD in image processing, fault detection, medical imaging is still in early stage. As there are many unexplored properties of SVD, our aim is to highlight those characteristics of SVD and gives a contribution for future research challenges. Briefly speaking SVD is a very efficient technique which is fast and easy to implement. It provides a practical solution to fault detection, image compression and recognition problems. We can compare the analytical result of SVD and QR & SVD techniques using the above parameters and determine which of them has more efficiency in terms of dimensionally reduction, computational complexity, computation of Eigen value decomposition and extracting signals from rotating machine from the data.

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