



Turbulence Modeling: Augmenting the Molecular Viscosity with an Eddy Viscosity

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Abstract

This paper attempts to study Turbulence modeling and the construction and use of a mathematical model to predict the effects of turbulence augmenting the molecular viscosity with an eddy viscosity. Turbulent flows are commonplace in most real life scenarios, including the flow of blood through the cardiovascular system, the airflow over an aircraft wing, the re-entry of space vehicles, besides others. In spite of decades of research, there is no analytical theory to predict the evolution of these turbulent flows. The equations governing turbulent flows can only be solved directly for simple cases of flow. For most real life turbulent flows, CFD simulations use turbulent models to predict the evolution of turbulence. These turbulence models are simplified constitutive equations that predict the statistical evolution of turbulent flows. Solving for any kind of fluid flow problem — laminar or turbulent — is computationally intensive. Relatively fine meshes are required and there are many variables to solve for. Ideally, you would have a very fast computer with many gigabytes of RAM to solve such problems, but simulations can still take hours or days for larger 3D models. Therefore, we want to use as simple a mesh as possible, while still capturing all of the details of the flow.

We can establish that for the flat plate (and for most flow problems), the velocity field changes quite slowly in the direction tangential to the wall, but quite rapidly in the normal direction, especially if we consider the buffer layer region. This observation motivates the use of a boundary layer mesh. Boundary layer meshes (which are the default mesh type on walls when using our physics-based meshing) insert thin rectangles in 2D or triangular prisms in 3D at the walls. These high-aspect-ratio elements will do a good job of resolving the variations in the flow speed normal to the boundary, while reducing the number of calculation points in the direction tangential to the boundary. When solving a model using low Reynolds number wall treatment, check the dimensionless distance to cell center (also generated by default). This value should be of order unity everywhere for the algebraic models and less than 0.5 for all two-equation models and the ν_2 -f model. If it is not, then refine the mesh in these regions.

Key words: Democracy, politics, participation, Islam, India, Muslims, General Elections.

Introduction

Flow separation has become the subject of interest for many years due to its importance in practical engineering problems especially for lifting structures or airfoils. Unfortunately, these lifting devices often attain optimum performance at the condition of "onset of separation" which implies that separation phenomena must be well understood if the analysis is aimed at practical applications. The separation process can be explained by the change of pressure gradient occurring on the airfoil surface. It happens when the angle of incidence of the incoming flow increases, which results in an augmentation of the pressure gradient especially on the suction side of the airfoil. This causes flow detachment resulting in a significant loss of the aerodynamic performance. Rhie and Chow described that this condition involves a strong interaction between the viscous flow and the inviscid flow outside of the boundary layer. On the other hand, in the linear lift region where the flow is attached, the interaction is significantly weaker.

This indicates that viscous effects are important and need to be considered for the flow experiencing a large adverse pressure. Wind turbines often operate at over-rated wind speed or pitch-fault conditions. These issues increase the local angle of attack and pressure gradient occurring on the surface. As a result, stronger separation is expected to occur. These phenomena become more severe if the inboard blade area is considered because the local inflow angle is usually very high due to limitation of the twist angle by structural constraints. Furthermore, as the rotor size increases vastly nowadays, thicker airfoils need to be used from the inboard area extending further outboard longer than more conventional rotors which even enhance the adverse pressure gradient occurring on the blade sections. It is clear that the rotor becomes more vulnerable towards flow separation, and this implies that accurate predictions of flow separation are important for wind turbine aerodynamics. Due to complexity of the issue, one effective way to bring the physics into numerical approach is to employ a large scale computation using the Navier-Stokes equations. These computational fluid dynamics methods have been applied in many engineering problems involving the prediction of flow over wind turbines. However, due to the lack of robust CFD methods, most CFD studies were done focusing only on the validation of the CFD codes rather than the physical phenomena of the flow. It is already well known that the Reynolds

Averaged Navier - Stokes method, commonly used to treat the flow turbulence, is inaccurate for massively separated flows. The maximum lift coefficient and post stall behaviour of airfoils were reported not to be sufficiently captured. Applying eddy-resolving models like Large Eddy Simulations or Detached Eddy Simulations has been shown to improve the situation. It has been demonstrated that RANS had difficulty in predicting the stall characteristics of some airfoils, while the LES based simulations were able to deliver accurate predictions. However, due to the 3D nature of eddies, computations should be carried out in three dimensional configuration. For this purpose, two dimensional simulations can not be used and this can be quite an issue if a lot of computations need to be performed like in generating airfoil databases for preparation of the Blade Element Momentum model. It is always possible to develop the airfoil databases using DES only for the high angle of attack cases where separation presents while the standard RANS approach or even XFOIL can be applied for the lower angle of attack. However, the grids should be sufficiently small to resolve the inertial scales of turbulence. The problem arises when this approach is used for the inboard wind turbine airfoil because the relative thickness of the airfoil is greater than 35%. As a consequence, no linear region presents and separation occurs for the whole range of the angle of attack. In the URANS approach, the turbulent closure

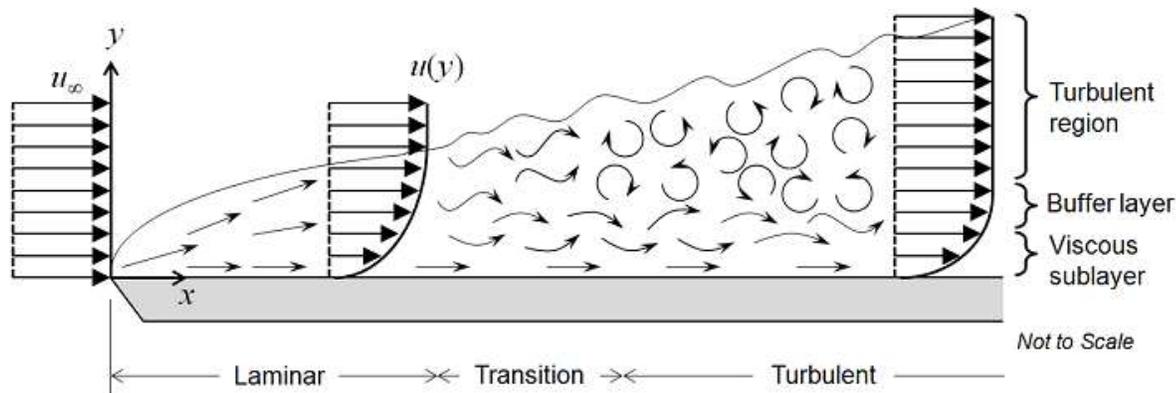
is modeled by assuming the "eddy" viscosity as a function of the "laminar" viscosity. In fact, actually eddy viscosity is not physical and is generated based on a simple relation called Boussinesq hypothesis.

Objective:

This paper intends to explore and analyze Turbulence modeling as key topic in most CFD simulations. Virtually all engineering applications are turbulent and hence require a turbulence model derived from Linear eddy-viscosity models, Reynolds stress transport models, Detached eddy simulations and other hybrid models

Turbulence Modeling

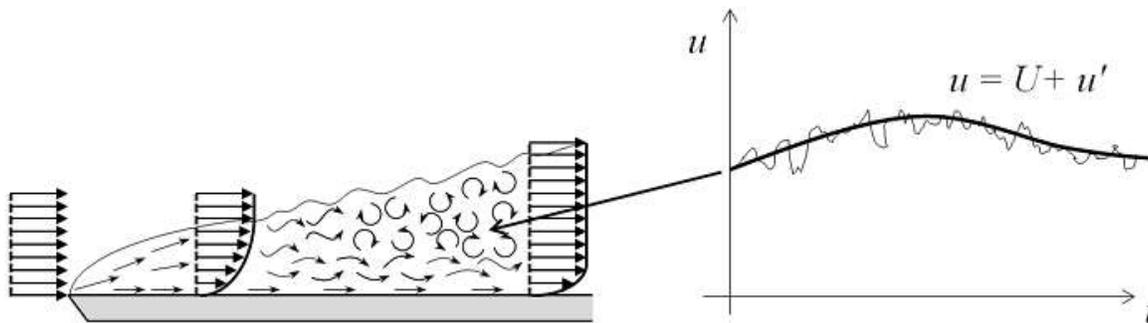
Consider the fluid flow over a flat plate, as shown in the figure below. The uniform velocity profile hits the leading edge of the flat plate, and a laminar boundary layer begins to develop. The flow in this region is very predictable. After some distance, small chaotic oscillations begin to develop in the boundary layer and the flow begins to transition to turbulence, eventually becoming fully turbulent.



The transition between these three regions can be defined in terms of the Reynolds number, $Re = \rho v L / \mu$, where ρ is the fluid density; v is the velocity; L is the characteristic length (in this case, the distance from the leading edge); and μ is the fluid's dynamic viscosity. We will assume that the fluid is *Newtonian*, meaning that the viscous stress is directly proportional, with the dynamic viscosity as the constant of proportionality, to the shear rate. This is true, or very nearly so, for a wide range of fluids of engineering importance, such as air or water. Density can vary with respect to pressure, although it is here assumed that the fluid is only weakly compressible, meaning that the Mach number is less than about 0.3. The weakly compressible flow option for the fluid flow interfaces in COMSOL Multiphysics neglects the influence of pressure waves on the flow and pressure fields.

In the laminar regime, the fluid flow can be completely predicted by solving Navier-Stokes equations, which gives the velocity and the pressure fields. Let us first assume that the velocity field does not vary with time. An example of this is

outlined in The Blasius Boundary Layer tutorial model. As the flow begins to transition to turbulence, oscillations appear in the flow, despite the fact that the inlet flow rate does not vary with time. It is then no longer possible to assume that the flow is invariant with time. In this case, it is necessary to solve the time-dependent Navier-Stokes equations, and the mesh used must be fine enough to resolve the size of the smallest eddies in the flow. Such a situation is demonstrated in the Flow Past a Cylinder tutorial model. Note that the flow is unsteady, but still laminar in this model. Steady-state and time-dependent laminar flow problems do not require any modules and can be solved with COMSOL Multiphysics alone.



As the flow rate — and thus also the Reynolds number — increases, the flow field exhibits small eddies and the spatial and temporal scales of the oscillations become so small that it is computationally unfeasible to resolve them using the Navier-Stokes equations, at least for most practical cases. In this flow regime, we can use a Reynolds-averaged Navier-Stokes (RANS) formulation, which is based on the observation that the flow field (u) over time contains small, local oscillations (u') and can be treated in a time-averaged sense (U). For one- and two-equation models, additional transport equations are introduced for turbulence variables, such as the turbulence kinetic energy (k in k - ϵ and k - ω).

In algebraic models, algebraic equations that depend on the velocity field — and, in some cases, on the distance from the walls — are introduced in order to describe the turbulence intensity. From the estimates for the turbulence variables, an eddy viscosity that adds to the molecular viscosity of the fluid is calculated. The momentum that would be transferred by the small eddies is instead translated to a viscous transport. Turbulence dissipation usually dominates over viscous dissipation everywhere, except for in the viscous sublayer close to solid walls. Here, the turbulence model has to continuously reduce the turbulence level, such as in low Reynolds number models. Or, new boundary conditions have to be computed using wall functions.

Eddy-Viscosity Hypothesis

One of the most significant contributions to turbulence modelling was presented in 1877 by Boussinesq, . His idea is based on the observation that the momentum transfer in a turbulent flow is dominated by the mixing caused by large energetic turbulent eddies. The Boussinesq hypothesis assumes that the turbulent shear stress depends linearly on the

mean rate of strain, as in a laminar flow. The proportionality factor is the *eddy viscosity*. The Boussinesq hypothesis for Reynolds averaged incompressible flow] can be written as

$$\tau_{ij}^R = -\rho \widetilde{v'_i v'_j} = 2\mu_T \bar{S}_{ij} - \frac{2}{3}\rho K \delta_{ij},$$

where \bar{S}_{ij} denotes the Reynolds-averaged strain-rate tensor , cf. also Eq.], K is the turbulent kinetic energy $\overline{v'_i v'_i}$],, and μ_T stands for the eddy viscosity. Unlike the molecular viscosity μ , the eddy viscosity μ_T represents no physical characteristic of the fluid, but it is a function of the local flow conditions. Additionally, μ_T is also strongly affected by flow history effects.

In the case of the compressible Favre- and Reynolds-averaged Navier-Stokes equations , the Boussinesq eddy-viscosity hypothesis reads

$$\tau_{ij}^F = -\bar{\rho} \overline{v''_i v''_j} = 2\mu_T \tilde{S}_{ij} - \left(\frac{2\mu_T}{3}\right) \frac{\partial \tilde{v}_k}{\partial x_k} \delta_{ij} - \frac{2}{3}\bar{\rho} \tilde{K} \delta_{ij},$$

where \tilde{S}_{ij} and \tilde{K} are the Favre-averaged strain rate and turbulent kinetic energy, respectively. Note the similarity to Eq. . The term $\bar{\rho} K \delta_{ij}$ in Eqs. and is required in order to obtain the proper trace of τ_{ij}^R or τ_{ij}^F . This means that we must have

$$\tau_{ii}^R = -2\rho K \quad \tau_{ii}^F = -2\bar{\rho} \tilde{K}$$

in the case of $\bar{S}_{ii}=0$ or $\tilde{S}_{ii}=0$, in order to fulfil the relations Eq. or for the turbulent kinetic energy. However, the term $\bar{\rho} K \delta_{ij}$ is often neglected, particularly in connection with simpler turbulence models .

The approximation, which is commonly used for the modelling of the turbulent heat-flux vector, is based on the classical Reynolds analogy . Hence, we may write

$$\bar{\rho} \widetilde{v''_j h''} = -k_T \frac{\partial \tilde{T}}{\partial x_j}$$

with the *turbulent thermal conductivity coefficient* k_T being defined as

$$k_T = c_p \frac{\mu_T}{Pr_T}.$$

In Equation, c_p denotes the specific heat coefficient at constant pressure and Pr_T is the turbulent Prandtl number. The turbulent Prandtl number is in general assumed to be constant over the flow field .

By applying the eddy-viscosity approach to the Reynolds-averaged form of the governing equations or Eq. , the dynamic viscosity coefficient μ in the viscous stress tensor Eq. or Eq. is simply replaced by the sum of a laminar and a turbulent component, i.e.,

$$\mu = \mu_L + \mu_T.$$

The laminar viscosity μ_L is computed, for example, with the aid of the Sutherland formula . Furthermore, according to the Reynolds analogy given by Eq. , the thermal conductivity coefficient k in Eq. or Eq. is evaluated as

$$k = k_L + k_T = c_p \left(\frac{\mu_L}{Pr_L} + \frac{\mu_T}{Pr_T} \right).$$

The eddy-viscosity concept of Boussinesq is, at least from the engineering point of view, very attractive since it requires “only” the determination of μ_T $\rho K \delta_{ij}$ in Eq. or is either obtained as a by-product of the turbulence model or is simply omitted]. Once we know the eddy viscosity μ_T , we can easily extend the Navier-Stokes equations or to the simulation of turbulent flows by introducing averaged flow variables and by adding μ_T to the laminar viscosity. Therefore, Boussinesq's approach became the basis for a large variety of first-order turbulence closures. However, there are applications for which the Boussinesq hypothesis is no longer valid p. 214 or p. 111]:

- flows with sudden change of mean strain rate,
- flows with significant streamline curvature,
- flows with rotation and stratification,
- secondary flows in ducts and in turbomachinery,
- flows with boundary layer separation and reattachment.

The limitations of the eddy-viscosity approach are caused by the assumption of equilibrium between the turbulence and the mean strain field, as well as by the independence on system rotation. The results can be notably improved by using appropriate correction terms in the turbulence models , . Further increased accuracy of predictions can be achieved through the application of non-linear eddy-viscosity models

Computational mesh and numerical methods

Two dimensional numerical simulations have been carried out using a CFD code, FLOWer, from the German Aerospace Center employing the Unsteady Reynolds-Averaged Navier-Stokes approaches. During the last years, the code was continuously extended at the Institute of Aerodynamics and Gas Dynamics - University of Stuttgart for wind turbine applications. The numerical procedure of the FLOWer code is based on structured meshes. The spatial discretization scheme used in the present study is a central cell-centered finite volume formulation because it provides high robustness and is well-suited for parallel applications. The method employs the Jameson-Schmidt-Turkel approach for flux computations. The scheme is second order accurate in space on smooth meshes. The method utilizes central space discretization with artificial viscosity and explicit hybrid 5-stage RungeKutta time-stepping schemes. Dual time-stepping according to Jameson with second-order accuracy in time, multi-grid level 3 and the implicit residual smoothing with variable coefficients

Conclusion

Turbulence is the apparent chaotic motion of fluid flows. Fluid flows can be laminar, when they are regular and flow in an orderly manner. When the speed or characteristic length of the flow is increased, the convective forces in the flow overcome the viscous forces of the fluid and the laminar flow transitions into a turbulent one. **Eddy viscosity** A coefficient relating the average shear stress within a turbulent flow of water or air to the vertical gradient of velocity. The eddy viscosity depends on the fluid density and distance from the river bed or ground surface. The concept of eddy viscosity is fundamental to the von Karman—Prandtl description of the velocity profile in turbulent flow, and is important in determining rates of evaporation or cooling by wind

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