JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

Phantom behavior of Bianchi Type-III cosmological models in *f(T)* gravity

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Abstract:

The behavior of equation of state parameter and energy density for dark energy in *f(T)* gravity has been investigated. Here we propose two *f(T)* models and observe transition between phantom and non-phantom phase.

Keywords: Bianchi type III Universe, *f(T)* theory of gravity, Equation of state parameter, Hybrid expansion law.

[1] Introduction:

One of the greatest cosmological mysteries till today is the origin of structure in the universe. At large scale, the universe is homogeneous and isotropic and it is in accelerating phase suggests the current day observations (Gasperini 2003). Despite the success of general relativity, the interest in theories beyond it, generically known as 'modified gravity' theories has grown substantially in recent decades.

Amongst the modified theories, *f(R)* theory of gravity has been extensively investigated by many authors (Capozzilello *et al.* 2005; Nojiri *et al.* 2006; Nojiri and Odintsov 2007). The *f(R)* gravity is also considered to be helpful in describing the evolution of the universe. It is the modification of general theory of relativity which is proposed by Einstein. *f(R)* theory was first proposed by Buchdahl in 1970. The *f(R)* theory of gravity provides a very natural unification of the early time inflation and late time acceleration. The presence of late time cosmic acceleration of the universe in *f(R)* gravity was explained by Carroll *et al.* (2004). Generalization of *f(R)* gravity is *f(R,T)* gravity which is developed by Harko *et al*. (2011). This theory is based upon the coupling of matter and geometry. Several problems have been considered by multiple authors in the *f(R,T)* theory of gravity (Sharif and

Zubair 2012a, 2014a, 2014b; Singh and Sharma 2014; Hossienkhani *et al*. 2014; Shamir and Raza 2015; Sahoo and Sivakumar 2015).

As an alternative to *f(R)* theories (Bengochea and Ferraro 2009) models based on modified teleparallel gravity were proposed, namely *f(T)* theories, in which the torsion is responsible for the late accelerated expansion and the field equations are always of second order which are simpler than *f(R)* theories. *f(T)* theories are generalizations of the action of teleparallel gravity where *f(T)* is an arbitrary function of the torsion scalar. *f(T)* models use the Weitzenbock connections which has only torsion and responsible for the accelerated expansion of the universe (Myrzakulov 2011). This feature of *f(T)* gravity has led to a rapid increasing interest in the literature. Momein *et al.* (2018) have discussed the new exact model of anisotropic star in *f(T)* theory of gravity. Recently, Cai *et al.* (2020) studied the model-independent reconstruction of *f(T)* gravity from Gaussian processes.

In order to understand in a better way the observed small amount of anisotropy in the universe, many authors have studied Bianchi Type Models. During the formation of large scale structure these models have also been used to examine the role of certain anisotropic sources. Exact solutions of general theory of relativity for homogeneous space-time belong to Bianchi Type Models, as is widely known (Roy Choudhari 1979). Bianchi type-III cosmological model in presence of dark energy have been studied in general relativity by numerous authors. Singh *et al*. (2007) has investigated a model with variable G and Λ in presence of perfect fluid by assuming a conservation law of energy-momentum tensor. Adhav *et al*. (2009) obtained an exact solution the vacuum Brans-Dicke field equations for the metric tensor of spatially homogeneous anisotropic Bianchi type-III model.

Under the premise of the hybrid expansion law, Raut *et al.* (2016) investigated anisotropic and homogeneous Bianchi Type-I space-time for the interaction between dark matter and holographic dark energy (HEL). Reddy *et al.* (2018) used the hybrid expansion law to study the modified holographic Ricci dark energy model in the modified theory of gravitation. Yadav *et al.* (2015) looked at the possibility of the existence of the LRS Bianchi-I dark energy model in the *f (R, T)* gravity by hybrid expansion law and found that it produces a time-dependent DP, indicating that the Universe is transitioning from an early decelerating phase to the current accelerating phase. Bianchi dark energy cosmological models with hybrid expansion laws were examined by Santhi *et al.* (2016). For LRS Bianchi Type-II space-time filled with dark matter and anisotropic modified Ricci dark energy, Das and Sultana (2015) used the hybrid expansion law to find an accurate solution to Einstein's field equations. By studying the hybrid expansion law (HEL) for the average scale factor, Mahanta and Sarma (2017) investigated the anisotropic Bianchi Type-VI_0 metric filled with dark matter and anisotropic ghost dark energy.

Motivated by the above discussions, in the present paper, we propose to investigate Bianchi type-III universe with EoS parameter and energy density in the framework of *f(T)* theory of gravity. The paper follows the following structure: In Section 1 the introduction has been given. In Section 2, *f(T)* formalism is given.

Section 3 is devoted to the solution of field equations. Physical properties of models are discussed in section 4. Conclusions are summarized in section 5.

[2] A brief review of $f(T)$ gravity:

In teleparallelism the dynamical object is the vierbein field $h_i x^{\mu}$, $i = 0, 1, 2, 3$. Each vector h_i is described by its components h_i^{μ}, $u = 0, 1, 2, 3$ in a coordinate basis.

The vierbein vector fields are related with the metric through

$$g_{\mu\nu} = \eta_{ij} h_{\mu}^{i} h_{\nu}^{j}, \tag{1}$$

where $h_i h_j = \eta_{ij}$ and $\eta_{ij} = diag(1,-1,-1,-1)$ is the Minkowski metric for the tangent space.

The matrix h_i^{μ} is inversible i.e. there exist a matrix h_i^{μ} satisfying

$$h_{\mu}^{i} h_{j}^{\mu} = \delta_{j}^{i}, \quad h_{\mu}^{i} h_{i}^{\nu} = \delta_{\mu}^{\nu} . \tag{2}$$

The torsion $T^{\rho}{}_{\mu\nu}$ and contorsion $K^{\mu\nu}{}_{\rho}$ tensors are defined by

$$T^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\nu\mu} - \Gamma^{\rho}{}_{\mu\nu} = h_i^{\rho} (\partial_{\mu} h_{\nu}^{i} - \partial_{\nu} h_{\mu}^{i}), \tag{3}$$

$$K^{\mu\nu}{}_{\rho} = -\frac{1}{2}\left(T^{\mu\nu}{}_{\rho} - T^{\nu\mu}{}_{\rho} - T_{\rho}{}^{\mu\nu}\right). \tag{4}$$

The teleparallel Lagrangian density is described by the torsion scalar *T*, defined as

$$T = S_{\rho}{}^{\mu\nu} T^{\rho}{}_{\mu\nu} . \tag{5}$$

where

$$S_{\rho}{}^{\mu\nu} = \frac{1}{2}\left(K^{\mu\nu}{}_{\rho} + \delta_{\rho}^{\mu} T^{\theta\nu}{}_{\theta} - \delta_{\rho}^{\nu} T^{\theta\mu}{}_{\theta}\right). \tag{6}$$

The action of *f(T)* gravity is given by (Bamba *et al*. 2011)

$$I = \frac{1}{16\pi G}\int d^4xe(T + f(T) + L_m)$$

where T is the torsion scalar, $f(T)$ is a general differentiable function of the torsion, L_m corresponds to the matter Lagrangian and $e = \sqrt{-g}$.

Varying the action with respect to the vierbein field h_i^μ, we obtain (Bengochea and Ferraro 2009)

$$\left[e^{-1}\partial_\mu\left(eS_i{}^{\mu\nu}\right) - h_i^\lambda T^\rho{}_{\mu\lambda}S_\rho{}^{\nu\mu}\right](1 + f(T)) + S_i{}^{\mu\nu}\partial_\mu(T)f_{TT} + \frac{1}{4}h_i^\nu[T + f(T)] = \frac{1}{2}k^2h_i^\rho T_\rho^\nu, \quad (7)$$

where $S_i^{\mu\nu} = h_i^\rho S_\rho^{\mu\nu}$, $k^2 = 8\pi G$, $f_T \equiv \frac{df}{dT}$.

[3] Solutions of the field equations:

We consider the homogeneous and anisotropic space time described by Bianchi type-III metric of the form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2\alpha x}dy^2 - C^2dz^2 , \quad (8)$$

where A, B and C are scale factors and functions of cosmic time t and $\alpha \neq 0$ is a constant.

The tetrad components are obtained by using equations (1) and (8) as

$$h_\mu^i = diag(1, A, Be^{-\alpha x}, C) , \quad h_i^\mu = diag(1, A^{-1}, B^{-1}e^{\alpha x}, C^{-1}) . \quad (9)$$

On substituting Equations (3) and (6) in equation (5) the torsion scalar T for Bianchi Type-III has the following form

$$T = -2\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right) \quad (10)$$

The anisotropy parameter of expansion, average scale factor and mean Hubble parameter are given by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2,$$

$$R = (ABC)^{\frac{1}{3}} ,$$

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) , \tag{11}$$

where H_i are the directional Hubble parameters in x, y and z direction respectively given by,

$$H_1 = \frac{\dot{A}}{A} , \; H_2 = \frac{\dot{B}}{B}, \; H_3 = \frac{\dot{C}}{C}.$$

The energy momentum tensor for perfect fluid is defined as

$$T_\rho^\nu = diag(\rho_M, -P_M, -P_M, P_M) \tag{12}$$

Where ρ_M and P_M are the energy density and the pressure of matter inside the universe.

For the values of $i = 0 = \nu$ and $i = 1 = \nu$, the field equations are obtained as follows

$$T + f(T) - 4\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}\right)(1+f(T)) = 2k^2\rho_M \tag{13}$$

$$\begin{aligned} &2\left(\frac{\dot{A}\dot{B}}{AB}+2\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right)(1+f(T)) \\ &-4\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\left[\left(\frac{\ddot{A}}{A}-\frac{\dot{A}^2}{A^2}\right)\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}\right)\left(\frac{\dot{C}}{C}+\frac{\dot{A}}{A}\right)+\left(\frac{\ddot{C}}{C}-\frac{\dot{C}^2}{C^2}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\right]f_{TT} - (T+f(T)) \\ &= 2k^2P_M \end{aligned} \tag{14}$$

Using the Hybrid expansion law,

$$V = \left(t^a e^{bt}\right)^{\frac{3}{c}} , \tag{15}$$

where $a > 0,\; b > 0$ and $c \geq 0$ are constants.

Using the above condition, the anisotropic parameter is obtained as

$$\Delta = 0 \tag{16}$$

The isotropic behavior of the expanding Universe for $\Delta=0$ has obtained here. On using this condition the above field equation takes the form,

$$H^2 = \frac{-8\pi G\rho_M}{9}+\frac{f}{18}+\frac{Tf_T}{9} \tag{17}$$

$$\left(H^2\right)' = \frac{4\pi G P_M}{3(1+f_T)} + \frac{T+f(T)}{12(1+f_T)} + \frac{a}{3ct^2} \tag{18}$$

Here it can be noted that for $T+f(T)=T$ the above equations reduces to field equations as,

$$H^2 = \frac{-8\pi G}{9}(\rho_M + \rho_{DE}) \tag{19}$$

$$\left(H^2\right)' = \frac{4\pi G}{3}(P_M + P_{DE}) + \frac{T}{12} + \frac{a}{3ct^2} \tag{20}$$

We have assumed here non-relativistic matter with pressure zero i.e. $P_M = 0$.

On comparing Eq.(17) with Eq.(19) and Eq.(18) with Eq.(20) the energy density and pressure of the effective dark energy can be described as

$$\rho_{DE} = \frac{-1}{16\pi G}(f + 2Tf_T) \tag{21}$$

$$P_{DE} = \frac{3}{4\pi G}\left(\frac{f(T) - Tf_T}{12(1+f_T)}\right) \tag{22}$$

The EoS parameter for dark energy is obtained by dividing Eq.(22) by Eq.(21) as

$$\omega_{DE} = -\left[\frac{f(T) - Tf_T}{(1+f_T)(f + 2Tf_T)}\right] \tag{23}$$

The energy conservation equations corresponding to dark matter and dark energy reads as

$$\dot{\rho}_M + 3H\rho_M = 0,$$

$$\dot{\rho}_{DE} + 3H\rho_{DE}(1+\omega_{DE}) = 0$$

[4] Physical properties of the model:

In this section we construct two *f(T)* models.

[4.1] MODEL-I

We consider the *f(T)* model of the type (Yang 2011a)

$$f(T) = T + \eta T_0 \frac{\left(T^2 + T_0^2\right)}{1 + \left(\frac{T^2}{T_0^2}\right)^{\lambda}} \tag{24}$$

where η and λ are the positive constants.

The corresponding EoS parameter for above model is given as

$$\omega_{DE} = -\frac{\eta\left(\frac{T}{T_0}\right)^{2\lambda-1}}{K\left(1+\left(\frac{T}{T_0}\right)^{2\lambda}\right)}\left[1 - 2\lambda\left(1 - \frac{\left(\frac{T}{T_0}\right)^{2\lambda}}{1+\left(\frac{T}{T_0}\right)^{2\lambda}}\right)\right] \tag{25}$$

where

$$K = \left[2 + \frac{2\eta\lambda\left(\frac{T}{T_0}\right)^{2\lambda-1}}{1+\left(\frac{T}{T_0}\right)^{2\lambda}}\left\{1 - \frac{\left(\frac{T}{T_0}\right)^{2\lambda}}{1+\left(\frac{T}{T_0}\right)^{2\lambda}}\right\}\right]\left[3 + \frac{\eta\left(\frac{T}{T_0}\right)^{2\lambda-1}}{1+\left(\frac{T}{T_0}\right)^{2\lambda}}\left\{1 + 4\lambda\left(1 - \frac{\left(\frac{T}{T_0}\right)^{2\lambda}}{1+\left(\frac{T}{T_0}\right)^{2\lambda}}\right)\right\}\right]$$

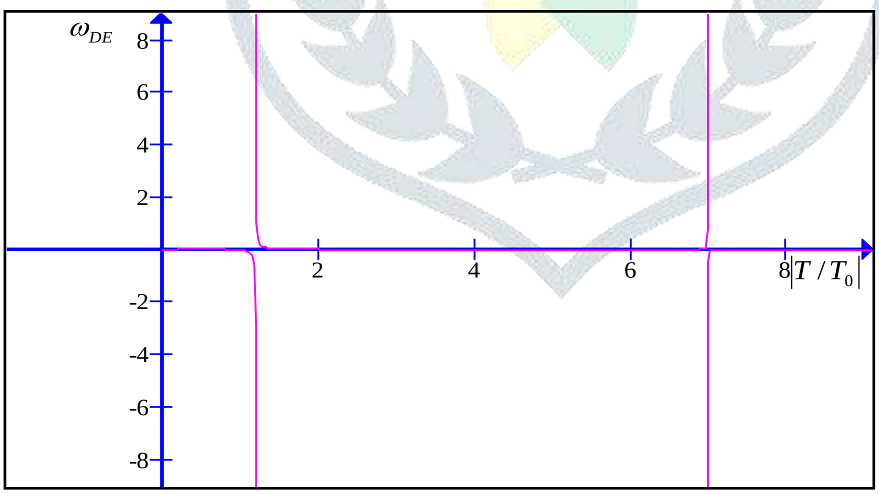

Fig.1 Variation of ω_{DE} vs $|T/T_0|$ for $\eta = 3$ and $\lambda = 2$

Fig.1 shows the graphical representation of ω_{DE} vs $|T/T_0|$. It can be seen that, first the universe is in non-phantom phase but as $|T/T_0|$ increases it crosses the phantom divide line and enters in phantom phase. After sometime ω_{DE} release from phantom phase, again crosses the phantom divide line and enters in non-phantom phase and remains there for some time.

Equating Eq.(15) in Eq.(10) we get a torsion scalar which is a function of redshift z.

$$T = \frac{-6\dot{Z}^2}{(1+Z)^2} \quad , \quad T_0 = -6\dot{Z}^2 \tag{26}$$

By using Eq.(26) in Eq.(25) ω_{DE} in terms of z is obtained as

$$\omega_{DE} = -\frac{\eta(1+Z)^{-2(2\lambda-1)}}{J\left(1+(1+Z)^{-4\lambda}\right)}\left[1-2\lambda\left(1-\frac{(1+Z)^{-4\lambda}}{1+(1+Z)^{-4\lambda}}\right)\right] \tag{27}$$

where

$$J = \left[2+\frac{2\eta\lambda(1+Z)^{-2(2\lambda-1)}}{1+(1+Z)^{-4\lambda}}\left\{1-\frac{(1+Z)^{-4\lambda}}{1+(1+Z)^{-4\lambda}}\right\}\right]\left[3+\frac{\eta(1+Z)^{-2(2\lambda-1)}}{1+(1+Z)^{-4\lambda}}\left\{1+4\lambda\left(1-\frac{(1+Z)^{-4\lambda}}{1+(1+Z)^{-4\lambda}}\right)\right\}\right]$$

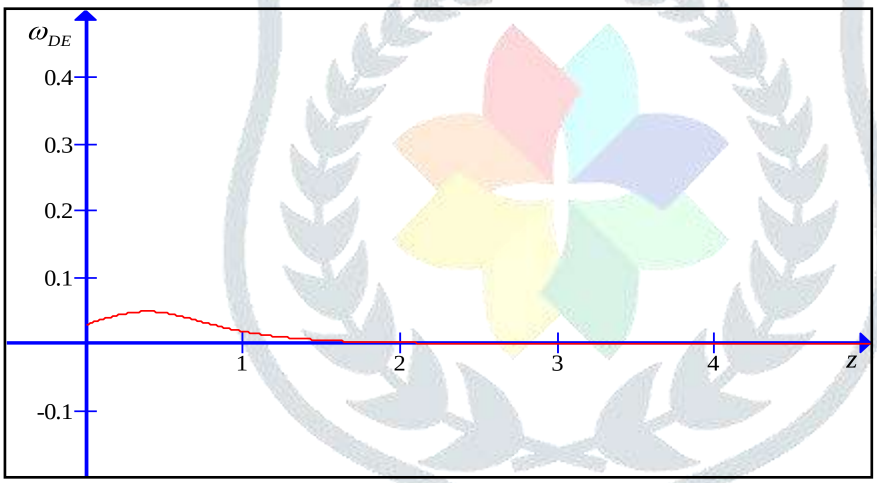

Fig.2 Variation of ω_{DE} vs z for $\eta = 3$ and $\lambda = 2$

Fig.2 shows the variation of ω_{DE} with z. It is noted that ω_{DE} lies in the non-phantom phase and as z increases it converges to zero. Here dominance of matter over DE is observed.

Inserting Eq.(26) in Eq.(21) the corresponding parameter $\overset{(*)}{\rho}_{DE}(z)$ takes the form

$$\overset{(*)}{\rho}_{DE} = \frac{3\dot{Z}^2(1+Z)^{-2}}{8\pi G\overset{(0)}{\rho}_{DE}}\left[3+\frac{\eta(1+Z)^{-2(2\lambda-1)}}{1+(1+Z)^{-4\lambda}}\left(1+4\lambda\left(1-\frac{(1+Z)^{-4\lambda}}{1+(1+Z)^{-4\lambda}}\right)\right)\right] \tag{28}$$

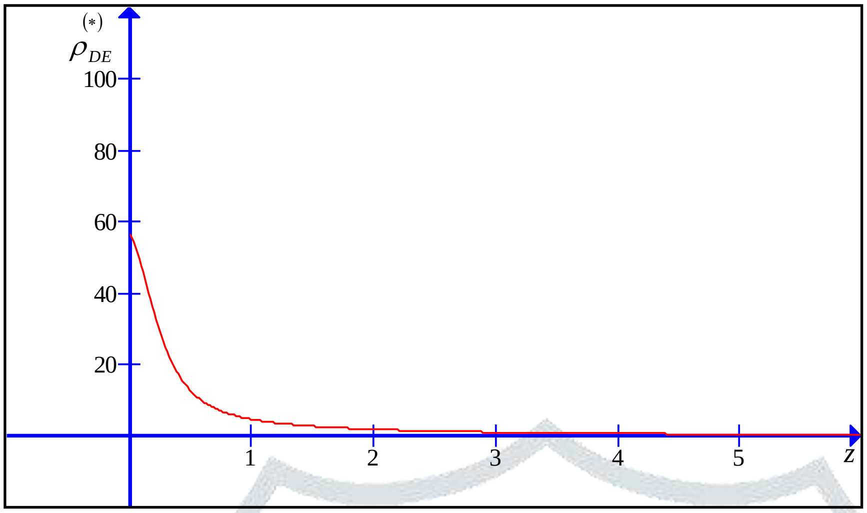

Fig.3 Variation of $\overset{(*)}{\rho}_{DE}$ vs *z* for $\eta = 3$ and $\lambda = 2$

Variation of $\overset{(*)}{\rho}_{DE}$ vs *z* is shown in fig.3. It is seen that $\overset{(*)}{\rho}_{DE}$ is in non-phantom phase and converges to zero.

[4.2] MODEL- II

Now we assume the *f(T)* model in the form (Yang 2011a,Yang 2011b)

$$f(T) = T - \eta T_0\left[\left(1 + \frac{T^2}{T_0^2}\right)^{-\lambda} - 1\right] \tag{29}$$

ω_{DE} can be written as

$$\omega_{DE} = \frac{1}{L}\left[\frac{-\eta T_0}{T}\left(\left(1 + \frac{T^2}{T_0^2}\right)^{-\lambda} - 1\right) - 2\eta\lambda\left(\frac{T}{T_0}\right)\left(1 + \frac{T^2}{T_0^2}\right)^{-\lambda-1}\right] \tag{30}$$

where

$$L = \left[2 + 2\eta\lambda\left(\frac{T}{T_0}\right)\left(1 + \frac{T^2}{T_0^2}\right)^{-\lambda-1}\right]\left[3 - \frac{\eta T_0}{T}\left(\left(1 + \frac{T^2}{T_0^2}\right)^{-\lambda} - 1\right) + 4\eta\lambda\left(\frac{T}{T_0}\right)\left(1 + \frac{T^2}{T_0^2}\right)^{-\lambda-1}\right]$$

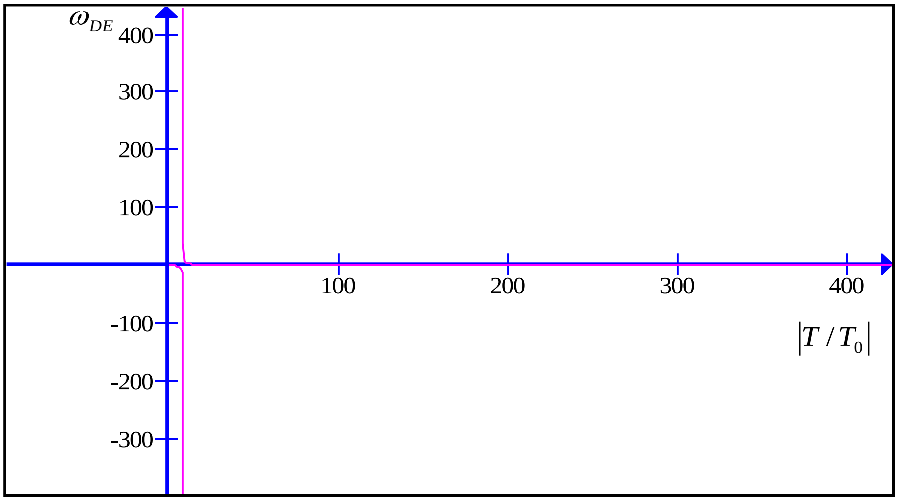

Fig.4 Variation of ω_{DE} vs $|T/T_0|$ for $\eta = 3$ and $\lambda = 2$

The behavior of ω_{DE} in terms of $|T/T_0|$ is found to be similar to that of model-I. Still some singularities appear and the universe remains in the non-phantom phase for a very short interval. For greater values of $|T/T_0|$, matter dominance in the universe can be seen.

On inserting Eq.(26) in Eq.(30), ω_{DE} as a function of *z* becomes as

$$\omega_{DE} = \frac{1}{M}\left[-\eta(+Z)^{-2}\left\{\left(1+(1+Z)^{-4}\right)^{-\lambda} - 1\right\} - 2\eta\lambda(1+Z)^{-2}\left(1+(1+Z)^{-4}\right)^{-\lambda-1}\right] \quad (31)$$

where

$$M = \left[2 + 2\eta\lambda(1+Z)^{-2}\left(1+(1+Z)^{-4}\right)^{-\lambda-1}\right]$$
$$\left[3 - \eta(1+Z)^{2}\left\{\left(1+(1+Z)^{-4}\right)^{-\lambda} - 1\right\} + 4\eta\lambda(1+Z)^{-2}\left(1+(1+Z)^{-4}\right)^{-\lambda-1}\right]$$

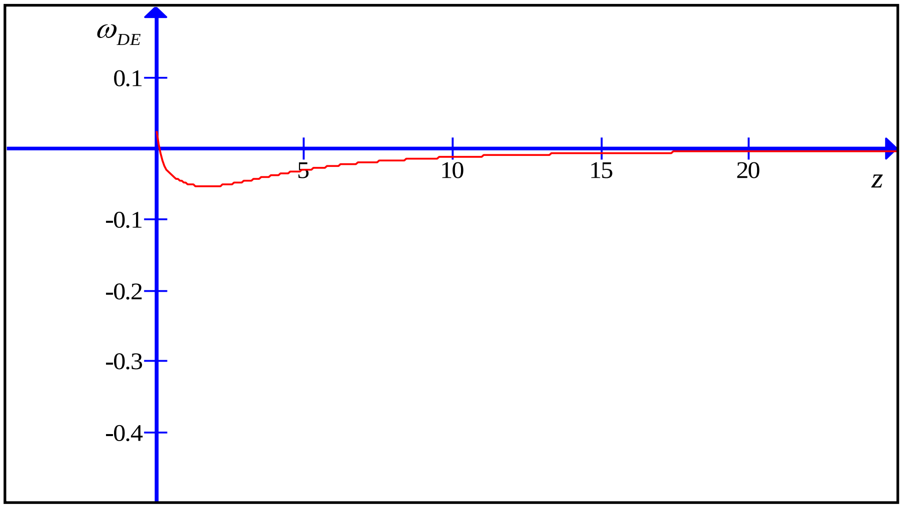

Fig.5 Variation of ω_{DE} vs z for $\eta = 3$ and $\lambda = 2$

Variation of ω_{DE} vs z is shown in Fig.5 It is found that for smaller values of z, ω_{DE} appears to be in non-phantom phase but as the value of z increases ω_{DE} approaches to zero. Universe appears to be matter dominated.

Using Eq.(26) in Eq.(21), we get $\overset{(*)}{\rho}_{DE}$ in the form of z as

$$\overset{(*)}{\rho}_{DE} = \frac{3\dot{Z}^2(1+Z)^{-2}}{8\pi G \overset{(0)}{\rho}_{DE}}\left[3+\eta\left[\left(1+(1+z)^{-4}\right)^{-\lambda}\left((1+z)^2+4\lambda(1+z)^{-2}\left(1+(1+z)^{-4}\right)^{-1}\right)-(1+z)^2\right]\right] \qquad (32)$$

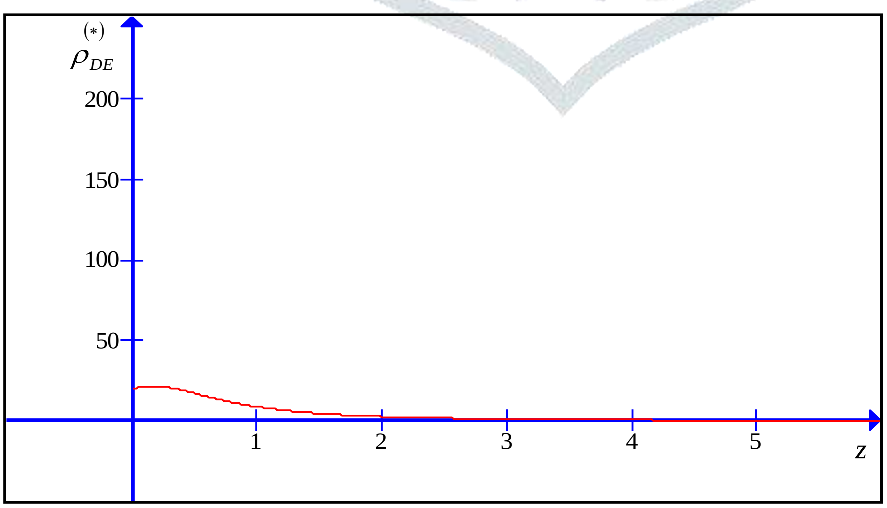

Fig.6 Variation of $\overset{(*)}{\rho}_{DE}$ vs z for $\eta = 3$ and $\lambda = 2$

Fig.6 shows the variation of $\overset{(*)}{\rho}_{DE}$ vs *z*. Dominance of matter over dark energy can be observed.

[5] Conclusion

In this paper, we have investigated the Bianchi type-III cosmological model in the presence of *f(T)* theory of gravity for two models. We have obtained the cosmological model by using hybrid expansion law of average scale factor. The observations are as follows:

- In model I, the variable ω_{DE} $(|T/T_0|)$ transits from phantom phase to non-phantom phase and remains in it whereas the dominance of matter over DE is observed for the variable ω_{DE} *(z)* as *z* increases. The variable $\overset{(*)}{\rho}_{DE}$ (*z*) converges to zero with increases in *z*.

- Universe turns out to be matter dominated in models II.

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