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SOLUTIONS OF EINSTEIN MAXWELL FLUIDS FOR CHARGED FLUID SPHERE

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Abstract :

The present paper provides solutions of Einstein-Maxwell fields for charged fluid sphere in general Relativity by taking $e^{\lambda} = Br^{n}$ for general value of n and a suitable form of total charge Q. Pressure density have been calculated.

Key Word : Charged fluid, sphere, charge, pressure, density.

1. INTRODUCTION

Growing interest has been found on finding interior solution of the Einstein-Maxwell equations corresponding to static charged fluid spheres. Bonnor [5], Efinger [8], Kyle and Martin [12] have obtained internal solutions for static spherically symmetric charged fluid spheres under different conditions. There are also some solutions for charged fluid distribution with spherical symmetry which are regular (Wilson [20], Kramer and Neugebauer [9], Krori and Barua [9 (a)]. Bonnor and Vaidya [6] have given the solutions of the Einstein Maxwell equations describing the emission of charged null fluid from a spherically symmetric body. The cases of the interior solutions for charged fluid spheres have been also presented by Bekenstein [4], Bailyn and Eimeral [2] and Bailyn [3].

Nduka [13] has obtained the solution for the interior metric of a uniformly charged static fluid sphere. He has also presented some solutions of static spherical distribution which are not free from singularity at the origin (Nduka [14]). Singh and Yadav [18] have given a quadrature method to solve the problem of charged fluid spheres. Whitman and Burch [19] have also presented some interior solutions for charged fluid spheres. Some other workers in this line are Yadav et. al. [22, 23] Copper stock and Cruz [7] and Zhu Shi Chang [24] Murad [10], Maurya and Gupta [11], Purushottam and Yadav and Pandya et. al. [15].

In this paper we have given some solutions to Einstein-maxwell field equations for a static spherically symmetric charged fluid distribution by taking $e^{\lambda} = Br^{n}$ for general value of n and a suitable form of total charge Q. The solutions have been discussed in different cases in different case in particular if we set n = 0, we get the results due to Nduka [14].

2. THE FIELD EQUATIONS AND THEIR SOLUTIONS

We use here the spherically symmetric metric

(2.1) $ds^2 = e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^{\upsilon}dt^2$

where v and λ are functions of r only.

The Einstein – Maxwell equations for the charged perfect fluid distribution in general relativity are [1]

(2.2) $G_{ij} = -8\pi T_{ij},$

(2.3) $\left[(-g)^{1/2} F^{ij}\right]_{,j} = 4\pi J^{i}(-g)^{1/2},$

(2.4) $F_{(ij,k]} = 0$

Here T_{ij} is the energy momentum tensor, J^{μ} is current four vector and G_{ij} is Einstein tensor defined by the equation.

(2.5) $G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$

where R_{ij} is Ricci tensor and R is scalar of curvature tensor.

For the system under study, the energy momentum tension T^i_j has two parts vis. t^i_j and E^i_j for matter and charge respectively.

(2.6) $T^i_j = t^i_j + E^i_j$

where

(2.7) $t^i_j = \left[(\rho + p) u^i u_j - p g^i_j\right]$

with $u_i\, u^i = 1$. The non-vanishing components of t^i_j are $t^1_1 = t^2_2 = t^3_3 = -p$ and $t^4_4 =$

The electromagnetic energy tension E^i_j in terms of field tensor F_{ij} is given by

(2.8) $E^i_j = -F_{jk} F^{ik} + \frac{1}{2} g^i_j F_{\mu v} F^{\mu v}$

Due to spherical symmetry, the only non-vanishing electric components of field tensor F^{ij} is $F_{41} = - F_{14}$. We take $F_{23} = 0$. It then follows that the non-zero components of E^i_j are

(2.9) $E^4_4 = E^1_1 = -E^2_2 = -E^3_3 = -\frac{1}{8\pi} g_{44} g_{11} (F^{41})^2$

Now we get from equation (2.3)

(3.2.10) $F^{41} = \frac{Q(r) e^{-(\lambda + v)/2}}{r^2}$

were Q(r) represents the total charge contained within a sphere of radius r viz.

(2.11) $Q(r) = 4\pi\int_0^r J^4 r^2 e^{-\frac{\lambda+v}{2}} dr.$

From equation (2.11) we see that outside the fluid sphere Q(r) is constant Q_0. It follows from (2.10) that the asymptotic from of the electric field is $\frac{Q_0}{r^2}$.

That the field equations (2.2) lead to

(2.12) $8\pi\rho + 8\pi E_4^4 = e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$

(2.13) $8\pi\rho - 8\pi E_1^1 = e^{-\lambda}\left(\frac{\lambda'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}$

(2.14) $8\pi\rho - 8\pi E_2^2 = e^{-\lambda}\left(\frac{\lambda''}{2} + \frac{\lambda'^2}{4} - \frac{\lambda' v'}{4} + \frac{\lambda' - \lambda'}{2r}\right)$

Eliminating p from (2.13) and (2.14) we get

(2.15) $v'' + \frac{v'^2}{2} - \frac{1}{r}(v' + \lambda') - \frac{\lambda' v'}{2} + \frac{1}{r^2}(2 - 32\pi E_1^1 r^2 e^{\lambda}) - \frac{2}{r^2} = 0$

We make the substitution

(2.16) v = 2 log y.

Use of equation (2.10) and (2.16) in (2.15) gives the second order differential equation

(2.17) $y^{11} - \left(\frac{1}{r} + \frac{v'}{2}\right)y' + \left(\frac{e^{\lambda}}{r^2} - \frac{\lambda'}{2r} - \frac{1}{r^2} - \frac{2Q^2(r)e^{\lambda}}{r^4}\right)y = 0$

Which is generalization of Wyman's equation (Wyman [21]).

Here we see that in (2.17) there are three unknows y, λ and Q (r). We impose two conditions and assume

(2.18) $Q(r) = Ar^{\zeta}$

(2.19) $e^{\lambda(r)} = Br^n$

Where A, B, ζ and n are constants. Substitution of these in equation (2.17) provides us

(2.20) $y'' - \left(\frac{1}{r} + \frac{n}{2r}\right) y' + (Br^{n-2} - \frac{n}{2r^2} - \frac{1}{r^2} - 2A^2 Br^{2\zeta+n-4})y = 0$

3. SPECTFIC SOLUTION

For different values of A, B, n and ζ equation (3.2.20) gives a number of solutions some of which are already known. The whole class of solutions can be divided into the following cases :

Case (i) Solution corresponding to unoharged cases i.e.

When A = 0

Case (ii) Solutions corresponding to charged cases i.e.

When $A \neq 0$

Here we shall confine ourselves to case (ii) in which charge is present. We take $A^2 = \frac{1}{2}, \zeta = 1$ then equations (2.20) reduces to

(3.1) $y'' - \left(\frac{1}{r} + \frac{n}{2r}\right) y' - \left(\frac{n}{2r^2} + \frac{1}{r^2}\right) y = 0$

which can be written as

(3.2) $r^2 y'' - \left(\frac{n}{2} + 1\right)(ry' + y) = 0$

Equation (3.3.2) may be transfored into well known Euler's equation

(.3.3) $r^2 y'' + h\, r\, y' + q\, y = 0$

where h and q are constants. The solution of equation (3.2) may now be written down and the metric function v(r) is obtained. Equation (2.19) gives other metric coefficient λ (r) while density and pressure can be evaluated from equation (2.12) and (2.13)

To solve equation (3.2) there are three possible cases (Ritger and Rose [17]

Case I. $\Delta > 0 \text{ i.e. } n^2 + 16n + 32 > 0$

The solution in this case is

(3.4) $y = a_1 r^1 + b_1 n^m, e^\lambda = B_1 r^n$

where a_1, b_1 and B_1 are constants to be fixed by the boundary conditions and

(3.5) $1 = \alpha_1 + \alpha_2,\ m = \alpha_1 - \alpha_2$

$$\alpha_1 = 1 + \frac{n}{4}, \alpha_2 = \frac{1}{4}\sqrt{n^2 + 16n + 32}$$

Now metric potential v and λ are known.

The electromagnetic energy tensor is given by

(3.6) $8\lambda r^2 E_1^1 = 8\lambda r^2 E_4^4 = -8\lambda r^2 E_2^2 = -8\lambda r^2 E_3^3 = \frac{1}{2}$

Hence from equation (2.12) and (2.13) density δ and pressure p are given by

(3.7) $8\lambda\rho = \frac{1}{B_1 r^2}\left[(n-1)r^n + \frac{B_1}{2}\right]$

(3.8) $8\lambda p = \frac{1}{B_1 r^{n+2}}\left[\frac{a_1 1 r^1 + b_1 m r^m}{a_1 r^1 + b_1 r^m} + \frac{1}{2} - \frac{B_1 r^n}{4}\right]$

Now to evaluate the constants appearing in the solution

we impose the following boundary conditions.

1. The metric potential g_{44} and g_{11} are continuous across the boundary ($r = r_0$) of the fluid sphere.

2. Derivative of g_{44} i.e. $\frac{d}{dr}(e^{\nu})$ is continuous across the boundary ($r = r_0$) of the fluid sphere.

The line element for $r > r_0$ is given by Reissner Nordstrom metric

(3.3.9) $ds^2 = y\,dt^2 - y^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$

where $y = \left(1 - \frac{2M}{r} + \frac{Q_0^2}{r^2}\right), Q_0 = Q(r_0)$ and M is the total mass of the sphere given by

(3.10) $M = 4\pi\int_0^{r_0} \delta(r) r^2\, dr$

Using the above boundary conditions, the constants a_1, B_1, b_1 are given by

(3.11) $a_1 = \left(1 - y_0 - 2My_0 - \frac{Q_0^2}{r^2}\right) \times \frac{\beta^{-1} r_0^{-1}}{2\sqrt{y_0}}$

(3.12) $b_1 = -\left(1 - y_0 - 2ly_0 - \frac{Q_0^2}{r^2}\right) \times \frac{\beta^{-1} r_0^{-m}}{2\sqrt{y_0}}$

(3.13) $B_1 = r_0 - ny_0^{-1}$

where $\beta = 1 - m, y_0 = \left[1 + \frac{2m}{r_0} + \frac{Q_0^2}{r_0^2}\right]$

Case 2 $\Delta = 0$ i.e. $n^2 + 16n + 32 = 0$

This implies either $\left(n + 8 - 4\sqrt{2}\right) = 0$ or $\left(n + 8 + 4\sqrt{2}\right) = 0$

Case 2 (a) $n = -\left(8 - 4\sqrt{2}\right)$

In this case solution is

(3.14) $y = (a^2 \log r + b^2) r^{1+\sqrt{2}(1-\sqrt{2})}$

$$e^{\lambda} = B_2 r^{4\sqrt{2}(1-\sqrt{2})}$$

Density and pressure are

(3.15) $8\pi\rho = \frac{1}{B_2 r^2}\left[\left(4\sqrt{2}-9\right) r^{4\sqrt{2}(1-\sqrt{2})} + \frac{B_2}{2}\right]$

(3.16) $8\pi p = \frac{2}{B_2 r^{4\sqrt{2}-6}}\left[\frac{\left(\sqrt{2}-1\right)(a_2 \log r + b_2) + a_2}{(a_2 \log r + b_2)} + \frac{1}{2} - \frac{B_2 r^{4\sqrt{2}(1-\sqrt{2})}}{4}\right]$

The constants a_2, b_2 and B_2 can be found as in case I

Case 2(b) When $n = -\left(8 + 4\sqrt{2}\right)$

(3.17) $y = \left(a_2^1 \log r + b_1\sqrt{2}\right) r^{1-\sqrt{2}(1+\sqrt{2})}$

$$e^{\lambda} = B_2^1 r^{-4\sqrt{2}(1+\sqrt{2})}$$

Density and pressure are

(3.18) $8\pi\rho = \frac{1}{B_2^1 r^2}\left[-\left(4\sqrt{2}+9\right) r^{4\sqrt{2}(1+\sqrt{2})} + \frac{B_2^1}{2}\right]$

(3.19) $8\pi p = \frac{2}{B_2^1 r^{2(2\sqrt{2}-3)}}\left[\frac{-\left(\sqrt{2}-1\right)(a_2^1 \log r + b_2^1) + a_2^1}{\left(a_2^1 - \log r + b_2^1\right)} + \frac{1}{2} - \frac{B_2^1 r^{4\sqrt{2}(1+\sqrt{2})}}{4}\right]$

The constants a_2^1, b_2^1 and B_2^1 can be calculated as in case I.

Case 3. When $n^2 + 16n + 32 < 0$. The solution of equation (3.3.2) in this case is

(3.20) $y = r^{\sigma}(\psi_1 r^{i\delta} + \psi_2 r^{i\delta})$

where σ and δ will depend on the value of n and ψ_1, ψ_2 are constants. This solution can be expressed in terms of real functions by noting that

(3.21) $r^{i\delta} = e^{i\delta \log r}$

$= \cos(\delta \log r) + I \sin(\delta \log r)$

Hence the solution (3.20) can be written as

(3.22) $y = r^{\sigma}(a_3 \cos x + b_3 \sin g\ x)$

and

$e^{\lambda} = B_3 r^n$

where $x = \delta \log r$

Density and pressure in this case are given by

(3.23) $8\pi\rho = \frac{1}{B_3 r^2}\left[(n-1)r^n + \frac{B_3}{2}\right],$

(3.24) $8\pi p = \frac{2}{B_3 r^{n+2}}\left[(a_3 \cos x + b_3 \sin x)^{-1} \times\right.$

$\left.\{(\sigma a_3 + b_3\delta)\mathrm{cis}x + (\sigma b_3 - \delta a_3)\sin x\} + \frac{1}{2} - \frac{B_3 r^n}{2}\right]$

The constants a_3, b_3 and B_3 are fixed by the boundary conditions as before

4. REMARKS

The conditions p > 0 and ρ > 0 will impose further restrictions on our solutions. We therefore restrict our solutions to only those values of constants for which the pressure and density are positive.

In these solutions if we set n = 0 then by suitable adjustment of constants, the results coincide with those already reported by Nduka [14]. Thus our solutions may be considered as the generalization of those obtained by Nduka [14].

5. REFERENCES

1. Adler, R.J. Bazin, M. and Sochiffer, M.M. (1965): Introduction to General Relativity, Mc Graw-New York, Chap. 4.
2. Bailyn, M. and Eimeral, D. (1972) : Phys. Rev., D5, 1897.
3. Bailyn, M. (1973) : Phys. Rev., D8, 1036.
4. Bekenstein, J.d. (1971) : Phys. Rev., D4, 2185.
5. Bonnor, W.B. (1960) : Z. Phys., 160, 59.
6. Bonnor, W.B. and Vaidya, P.G. (1970) : General Relativity and Gravitation, 1(2), 127.
7. Cooperstock, F.I. and dela Cruz, V. (1978), G.R.g., 9, 835.
8. Efinger, H.J. (1965) : Z. Phys., 188, 31.
9. Kramer, D. and Neugebauer, g. (1973) : Ann. Phys. (Leipsig), 27, 129.

9.(a) Krori, K.D. and Barua, J. (1975) : J. Phys. A. Math, 8, 508.

10. Murad, M.H. (2016), Astrophys Space Sci., 361, 20.
11. Maurya, S.K., and Gupta, K. (2013), Astrophys. Space Sci., 344, 407.
12. Kyle, C.F. and martin, A.W. (1967) : Nuovo. Cimento., 50, 583.
13. Nduka, A. (1976) : Gen. Relativ. Gravitation, 7, 501.
14. Nduka, A. (1977) : Acta Phys. Pol. B8, 75.
15. Purushottam and Yadav, R.B.S. (2019), Res. Rev. J, 4 810.12.
16. Pandya, D. M. et. al. (2015), Astrophys Space Sc. 356, 285.
17. Ritger, P.D. and Rose, N.J. (1968): Differential Equations with Applications, Mc Graw-Hill, New York, 144.
18. Singh, T. and Yadav, R.B.S. (1978) : Acta Phys. Poln., B9, 475.
19. Whitman, P.G. and Burch, R.C. (1981) : Phys. Rev., D24, 2049.
20. Wilson, S.J. (1967) : Can. J. Phys. 47, 2401.
21. Wyman, M. (1949) : Phys. Rev., 75, 1930.
22. Yadav, R.B.S. and Purshottam (2005), Proc. Math. Scoc., 21, 107.
23. Yadav, R.B.S. and Kumar, J. (2020), Res. Rev. J., 5,124-129.
24. Zhu Shi Chang. (1982); GRG, 15, 4.