



USABILITY OF INTERCEPT VALUES OF REGRESSION ANALYSIS AS THE PAYOFF VALUES OF GAME THEORY MODEL

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Abstract: The advancement in technology has seen a rise in the number of social networking sites and their distinct features. These social networking sites appear to have become rivals of one another and in order to remain in the competition, their features are updated frequently. These features are utilized by the end-users for communication, irrespective of the technical algorithms implemented behind these features. The portability of common features reflects the dependencies and can be measured using regression analysis. Data analytics collected from respective domain experts using a designed survey can provide the intercept values of regression analysis. A two-person game theory model is the best optimization tool to plan their strategies in terms of competitors sharing common features. This research work aims to provide the intercept values calculated from the concept of regression analysis, as supplements for the payoff values of a Game theory model. The bridge between the two domains can be proved by the characteristics of the intercept which can be served as the payoff value..

IndexTerms - Data Analytics, Intercept, Payoff, Game theory

1. INTRODUCTION

Social networks that are also known as “relationship networks,” help people and organizations connect online to share information and ideas. In the last two decades, many organizations are using social media platforms for simple marketing that helps them to create awareness of their available services and products. The resultant of this approach has influenced the public communication, customers interaction, and valuable opinions of the stakeholders Icha O. (2015). In another perspective, social media is becoming a competitive space. Competitive analysis helps in identifying the various opportunities for growth. It also provides a platform to develop new strategies against the current ones that are not performing as well as they should. There are many social media competitor analysis tools available viz.- Buzz sumo, Hoot suite, Streams, Brand watch, Facebook Analytics, Google Analytics, which provide the analytics regarding identifying the close competitors, opportunities and threats, their strengths and weakness in terms of graphics, stakeholder reviews, page likes, etc. Kautish (2017) .

As a step ahead, a problem about the decision-making tool that can provide information gain and building strategies with the nearest competitor is considered. Thus, this research work is an attempt to build an innovative competitor analysis tool based on a game theory model. For this, a reverse engineering approach from competitor analysis to data analysis is used to build up a game theory model Uparkar S. (2019). The walkthrough of this research work is as follows- The following section is the literature survey wherein, the available literature on the three dimensions i.e., social networks, game theory modeling, and regression analysis are discussed. The research methodology section reflects the requirements of game theory and regression analysis as an analytical tool. The replacement of payoff values with the intercept values having proper justification and proves are discussed in the innovative concept section. The conclusion section highlights the findings and future scope of the research work.

2. LITERATURE SURVEY

The available literature for carrying out this research work has been discussed in three dimensions. Initially, the necessity for choosing social networking was surveyed followed by the concept and application for Game theory modeling. Lastly, regression analysis as a mathematical tool has been reviewed.

Kapoor K. (2018) ., provided a systematic survey study based on the past two decades. This paper helped in identifying various Social Networking sites (SNS) and their major highlights. Xie, E. (2019) in his research had taken experimental datasets into consideration to determine the monetary payoff in terms of utility under risk factors. i.e. uncertainty of the respondent values. Each strategy has a numerical value, referred to as attraction, that determines the probability of choosing an action Acemoglu (2009).

Daron Acemoglu and Asu Ozdaglar, talked about the network theory, especially in terms of social networks and its co-existent with the game theory concept. They have put forward the notion of interactions with other agents (s) to remain connected and affect payoff, well-being, and utility. The concept of utility can be extended to the calculation of expected payoff value Xie, E. (2019). I. N. (2015) designed a simulation for a business game based on the various stakeholders and worked on the different phases in developing the ecosystem. The rules for developing various strategies have worked out to provide the output of the game theory model.

Vaidya N. (2013) provided an innovative approach to solve the game theory model. This can help to condense an approach to solve the non-cooperative game to optimize the strategies under consideration. Uparkar S. (2020).. implemented the game theory decision making using the social networking platform players. The approach of regression was used to find a case where the information is not shared among the two players.

Johnson B.T. (2011) proposed the concept of meta-regression models applied to the health care data set. A moving constant approach builds the confidence interval for the lower bound value and the upper bound value of the intercept component. This was the base for using the intercept value for our research work. Khan S. (2005) studied different types of estimators for the intercept value of the linear regression model. The estimator for intercept provided by sample information was more efficient as compared to non-sample prior information. One of the major findings in this paper was that the biasness constraint can influence the estimator in the case of sample information.

3. RESEARCH METHODOLOGY

This section is divided into two parts. The first part is about the concept of the game theory model whereas the second part talks about the concepts of regression analysis.

3.1 Game theory Model:

The mathematical decision-making during a disagreement (in case of risk and uncertainty) is Game theory. The various game theory model(s) can be categorized based on aspects like the total number of players participating, the total amount of gains or losses, and the number of strategies utilized in the game. In a two-person zero-sum game, two players are involved such that the winning of one person implies the loss of another. The strategic form is one of the plainest mathematical narratives of a game. In a two-person zero-sum game, the payoff function of Player II is the negative of the payoff of Player I, hence, focus can be limited to Player I's payoff function only. which is referred to as P in this work. The strategic form, or normal form, of a two-person zero-sum game, is given by a triplet (S_1, S_2, P) where,

- (1) S_1 is a nonempty set, the set of strategies of Player I
- (2) S_2 is a nonempty set, the set of strategies of Player II
- (3) P is a real-valued function defined on $S_1 \times S_2$. (Thus $P(S_1, S_2)$ is a real number for every $s \in S_1$ and every $t \in S_2$.)

The interpretation is as follows. Simultaneously, Player I chooses $s \in S_1$ and Player II chooses $t \in S_2$, each unaware of the choice of the other. Then their choices are made known and I wins the amount $P(S_1, S_2)$ from II. Depending on the monetary unit involved $P(S_1, S_2)$ will be cents, dollars, pesos, beads, etc. If P is negative, I pays the absolute value of this amount to II. Thus, $P(S_1, S_2)$ represents the winnings of I and the losses of II Ferguson. (2018).

A payoff can be defined as the quantifiable measure of satisfaction a person receives after each play. Suppose player I has 'm' activities and player II has 'n' activities, then a payoff matrix can be formed by adopting the following rules

- Row designations for each matrix are the activities available to player I
- Column designations for each matrix are the activities available to player II
- Cell entry a_{ij} is the payment to player I in I's payoff matrix when Player I chooses activity i and II chooses the activity j.
- With a zero-sum, two-person game, the cell entry in player II's payoff matrix will be negative of the corresponding cell entry a_{ij} in player I's payoff matrix so that sum of payoff matrices for a player I and player II is ultimately zero.
- A finite two-person zero-sum game in strategic form (S_1, S_2, P) , is sometimes called a matrix game because the payoff function P can be represented by a matrix. If $S_1 = \{s_1, ..., s_m\}$ and $S_2 = \{t_1, ..., t_n\}$, then by the game matrix or payoff matrix we mean the matrix

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \quad \text{where } a_{ij} = P(s_i, t_j),$$

A payoff is a number, also called a utility, that reflects the desirability of an outcome to a player, for whatever reason Orsini N. (2005).

Here, Player I chooses a row, Player II chooses a column, and II pays I the entry in the chosen row and column. Note that the entries of the matrix are the gains of the row picker and the losses of the column picker.

A mixed strategy for Player I may be represented by an m-tuple, $p = (p_1, p_2, ..., p_m)$ of probabilities that add to 1. If I uses the mixed strategy $p = (p_1, p_2, ..., p_m)$ and II chooses column j, then the (average) payoff to I is $\sum_{i=1}^m p_i a_{ij}$. Similarly, a mixed strategy for Player II is an n-tuple $q = (q_1, q_2, ..., q_n)$. If II uses q and I uses row i the payoff to I is $\sum_{j=1}^n a_{ij} q_j$. More generally, if player I uses the mixed strategy p and II uses the mixed strategy q, the (average) payoff to player I is $p^T A q = \sum_{i=1}^m \sum_{j=1}^n p_i a_{ij} q_j$.

Non-cooperative game theory manages similarly to how rational agents manage with each other to achieve their goals. A strategic game is the most common non-cooperative game wherein, only the available strategies and the outcomes resulting from a combination of alternatives are listed. Negative-sum games and Zero-sum games are two types of non-cooperative games.

3.2 Regression Model:

Regression analysis is often used to model the relationship between a single dependent variable Y and one or more predictors. When one predictor is available, it can be called as "simple" linear regression:

$$E[Y] = \beta + \alpha X + \varepsilon \quad \dots(1)$$

That is, the **expected value** of Y is a linear function of X . The values of α & β are selected by choosing the line which minimizes the squared distance between each Y value and the line of best fit. α & β are chosen, such that they minimize this expression Kumari K. (2018). β is the intercept or a by default value when the independent variable is at zero. α is the slope value and the ε is the small error which can be neglected.

There are four assumptions associated with a linear regression model:

1. **Linearity:** The relationship between X and the mean of Y is linear.
2. **Homoscedasticity:** The variance of residual is the same for any value of X .
3. **Independence:** Observations are independent of each other.
4. **Normality:** For any fixed value of X , Y is normally distributed.

4. INNOVATIVE CONCEPT

The base for considering the parametric value of the regression tool in terms of its intercept for the payoff value with a proper justification is the main motive of this section.

Proposition: The intercept component of a regression model is an unbiased estimator of payoff value in a game theory model.

The payoff in a game theory model is the payout received by a player by arriving at a certain outcome. This payoff in a game can be in terms of price, benefits, satisfaction, etc. There is no uncertainty when the events are not random like weather, stock price, etc. On the other hand, mixed strategies have a random behavior. To determine a player's payoff, a larger number can be assigned to achieve a better outcome for the player. The game theory states that choosing a better outcome maximizes the payoff value. Further, in case of any uncertainty, the payoff values can be specifically assigned either by risk choice or by safe choice. In other words, a player's payoff can be the utility of the outcome. Utility, an economic term can be defined as the total satisfaction received from consuming a good or service. A step ahead in terms of expected utility we have: Davies (2002)

Definitions.

- 1) R is a set of outcomes $\{o_1, o_2, o_3, o_4\}$
- 2) $o_1 > o_2$ means o_1 is preferable to o_2 (which is known as strict preference)
- 3) $o_1 \sim o_2$ means o_1 is indifferent to o_2
- 4) $o_1 \geq o_2$ means o_1 is either preferable or indifferent to o_2 . (which is known as weak preference)
- 5) $(o_1 p o_2)$ means a lottery (an unknown outcome) here probability p will fetch an outcome o_1 where probability $(1-p)$ will fetch an outcome o_2 .

Axioms. For all o_1, o_2, o_3, o_4 in R , and $p, q \in (0,1)$

1. Closure. (o_1, p, o_2) is in R .
2. Weak ordering.
 - (i) $o_1 \geq o_1$. (Reflexive)
 - (ii) $o_1 \geq o_2$ or $o_2 \geq o_1$ (Connectedness)
 - (iii) $o_1 \geq o_2$ and $o_2 \geq o_3 \Rightarrow o_1 \geq o_3$. (Transitive)
3. Reducibility: $((o_1, p, o_2), q, o_2) \sim (o_1, pq, o_2)$
4. Independence: If $(o_1, p, o_3) \sim (o_2, p, o_3)$ then $(o_1, p, o_4) \sim (o_2, p, o_4)$
- 5) Betweenness: If $o_1 > o_2$ then $o_1 > (o_1, p, o_2) > o_2$
- 6) Solvability: If $o_1 > o_2 > o_3$, then there exists p such that $o_2 \sim (o_1, p, o_3)$

Theorem: (J. von Neumann & O. Morgenstern, Theory of Games and Economic Behavior, 1944)

If axioms 1-6 are satisfied for all outcomes in R , then there exists a real-valued utility function u defined on R , such that

- (1) $o_1 > o_2 \Leftrightarrow u(o_1) > u(o_2)$, and $o_1 \sim o_2 \Leftrightarrow u(o_1) = u(o_2)$
- (2) $u(o_1 p o_2) = pu(o_1) + (1-p)u(o_2)$;
- (3) u is an interval scale, that is, if v is any other function satisfying 1 and 2, then there exist real numbers b , and $a > 0$, such that $v(o_1) = au(o_1) + b$ (2)

The above equation resembles the linear regression model where $v(o_1)$ is the expected value of a dependent variable, a is the slope, $u(o_1)$ is the utility function value and b is the intercept.

Thus, a regression model can be utilized to generate the payoff values.

In a non-cooperative game theory for more than one player, each player acts independently with no binding agreements.

Thus, if we put $u(o_1) = 0$ where the independent variable does not share any type of information, the expected value of the dependent variable is the intercept value.

An unbiased estimator of a parameter is an estimator whose expected value is equal to the parameter. Thus, the proposition holds in this case, to conclude that the **"Intercept component of a regression model is an unbiased estimator of payoff value in a game theory model"**.

5. CONCLUSION

This research work focuses on developing a competitive analytical tool for a game theory model. The scope of this research is to provide a zero-sum two-person type of non-cooperative game. Acemoglu (2009) suggest that social networking players may be taken into consideration for their utility values. At this point, the existing data analytical model aids in deciding the best possible values of utility in terms of payoff value. The requirement of the experimental data collection approach is achieved through the regression analysis assumption. The regression analytical model is noticed to be the most suitable choice for finding utility values. The proposition under consideration holds true when the acquired intercept value can be substituted against the payoff value. The same can be extended while the information regarding the available data or the consumer's responses is shared.

There are two types of regression equations Y on X and X on Y . These terms can be extended for the optimizing strategies of player I and player II and vice versa. This could help to develop the game theory payoff matrix for either mixed or pure strategy games. The future study of the research work involves optimizing the strategies of two giant players in the domain of social networking, Facebook and Twitter. The major output with respect to its analytical approaches is the strategies like graph and text analytics. A questionnaire method can be implemented in order to get the responses from various data scientists, as well as the social network scientists who have enough experience and expertise in graph and text analytical approaches. The study indicates that the utility values can be used as satisfaction for interpreting a particular strategy. The concept can be extended to find optimal values for $m \times m$ and $m \times n$ games. Thus, this study is an attempt to involve the predictive model of regression to determine the decisions under conflict situations.

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