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BIANCHI TYPE – I STRING COSMOLOGICAL MODELS WITH AXIAL SYMMETRY

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ABSTRACT :

Investigations on string cosmology in axially symmetric Bianchi type-I space time with and without magnetic field using different assumptions. Various physical and geometrical parameters of the model have been also discussed.

Key Words : String cosmology, magnetic field, Bianchi type-I, model, axial symmetry.

1. INTRODUCTION

Much interest has been given towards Bianchi type-I string cosmological models with axial symmetry by various authors [2,6,8,17] [Bali and Anjali [4] have found [Bianch I magnetized string dust cosmological model]. Krorietal [11] has studied the exact solutions of string cosmology for Bianchi types II, VI_0, VIII and IX space times. It has been noted [Kibble 10] that the existence of a large scalar network of strings in the early universe does not contradict the present-day observations of the universe and further the vacuum string (Zeldovich 25) can generate density fluctuations sufficient to explain the galaxy formation. These strings have stress energy and they couple to the gravitational field so that it may be interesting to study the gravitational effects which arise from strings. This has been already done by several authors, (Vilenkin [20], Gott [9], Garfinkle [7], although the general relativistic treatment of strings was pioneered by Letelier [12] and Stachel [18]. In geometrical string (massless) models, infinite number of degrees of freedom are possessed by each string for which the end points move at the speed of light. This problem is resolved by considering the realistic (massive) string model of Takabayashi [19]. The energy-momentum tensor for the massive strings has been first formulated by Letelier [12], who considered the massive string being formed by geometric string with particles attached along its extension. Its application to general relativity first appeared in Letelier [13], while Stachel [18] considered massless strings. So the total energy momentum tensor for a cloud of massive strings can be written as

(1.1) $T_i^j = \rho u_i u^j - \tau x_i x^j$

Where ρ is the rest energy density for a cloud of strings with particles attached to them (p-strings). Thus we have

(1.2) $\rho = \rho_p + \tau$

ρ_p being the particle energy density and τ being the string's tension density, u^i is the four velocity for the cloud of particles and x^i is the four vector representing the string's direction which essentially is the direction of anisotropy. Thus

(1.3) $u_1 u^i = -1 = x_i x^i$ and $ux^i = 0$

Banerjee et. al. [3] have found some cosmological solutions in Bianchi I space time following the technique used by Letelier and Stachel with without magnetic field. Melvin [14] in his solution for dust and deelectromagnetic field argued that the presence of magnetic field is not as unrealistic as it appears to be because for a large part of the history of evolution matter was highly ionized, and matter and field were smoothly coupled. Later during cooling as a result of expansion the ions combined to form neutral matter. Some other worker's in this line are Baysal et. al. [1] bali and Pareek [5], Yadav et. al. [22, 23] and Pradhan et. al. [15] and Yilmaz [24].

In this paper we have studied the string cosmology in axially symmetric Bianchi I space time with and without magnetic field using different assumptions. Various physical parameters of the model have been also evaluated.

2. THE FIELD EQUATIONS

Here we take an axially symmetric Bianchi I metric given by

(2.1) $ds^2 = -dt^2 + e^{2A}dx^2 + e^{2B}(dy^2 + dz^2)$

Where A and b are function of t only.

Now, the energy-momentum tensor for the string dust with a magnetic field along the direction of the string, i.e., the x-direction is given by

(2.2) $T_i^j + E_i^j = \rho u_i u^j - \tau x_i x^j + \frac{1}{4\pi}\left(F_i^{\alpha} F_{\alpha}^j - \frac{1}{4} F_{\alpha\beta} \delta_i^j\right)$

Where T_k^j is the stress-energy tensor for a string dust system, E_i^j is that for magnetic field and $F_{\alpha\beta}$ is the electromagnetic field tensor. The other terms have already been explained in the previous section. In the co-moving co-ordinates system $u_i = \delta_4^j$ and

(2.3) $T_4^4 = -\rho, T_1^1 = -\tau, T_2^2 = T_3^3 = 0 = T_i^j (\text{for } i \neq j)$

Again, since the magnetic field is being assumed in the x-direction, F_{23} is the only non-zero component of the electromagnetic field tensor. Maxwell equation $F_{[ij,\alpha]} = 0$ and $\left(F^{ij}(-g)^{1/2}\right) = 0$, now lead to the result

(2.4) $F_{23} = k$

Where k is a constant Therefore the components of stress energy tensor for the electromagnetic field are

(2.5) $E_4^4 = E_1^1 = -E_2^2 = -E_3^3 = -\frac{k^2}{8\pi}\exp(-4B)$

Now, choosing units such that $8\pi G = 1$, the surviving components of Einstein field equations given by

(2.6) $R_i^j - \frac{1}{2}\delta_i^j R = -\left(T_i^j + E_i^j\right)$

are

(2.7) $2A_4B_4 + B_4^2 = \rho + \frac{k^2}{8\pi} + e^{-4B}$

(2.8) $2B_{44} + 3B_4^2 = \tau + \frac{k^2}{8\pi} e^{-4B}$

(2.9) $A_{44} + A_4^2 + B_4^2 + A_4B_4 = \frac{-k^2}{8\pi} e^{-4B}$

The proper volume R^3, expansion scalar (θ) and shear scalar σ^2 are respectively given by

(2.10) $R^3 = \exp(A + 2B)$

(2.11) $\theta = U^i_{,i} = A_4 + 2B_4 = \frac{3R}{R}$

(2.12) $\sigma^2 = \sigma_{ij}.\sigma^{ij} = A_4^2 + 2B_4^2 - \frac{1}{3}\theta^2$

Where

(2.13) $\sigma_{ij} = \frac{1}{2}\left[u_{ij} + u_{ji} + u_i u^\alpha u_{j\alpha} + u_j u^\alpha u_{i\alpha}\right] - \frac{1}{3}\theta\left(g_{ij} + u_i u_j\right)$

Now, one can directly obtain the Ray Chaudhuri's equation (Ray Chaudhuri [16] from the above set of field equations (2.7) to (2.9) and using (2. 11) and (2.12) as

(2.14) $\theta = \frac{1}{3}\theta^2 - 2\sigma^2 = \frac{1}{2}\rho_p - \frac{k^2}{8\pi}\exp(-4B)$

Where

(2.15) $R_{ij}.u^i u^j = -\frac{\rho_p}{2} - \frac{k^2}{8\pi}\exp(-4B)$

Now in view of all the three (strong, weak and dominat) energy conditions (Wald [21]) one find $\rho \geq 0$ and $\rho_p \geq 0$, together with the fact that the sign of τ is unrestricted. It may take values positive, negative or zero as well. This implies in view of (2.14) that even the existence of the strings is unable to halt the collapse. From the above energy conditions we find that τ might even take the negative value and therefore Einstein's equation (2.6) with $\tau < 0$, is the equation for an anistropic fluid with pressure different from zero along the direction of x.

3. SOLUTION OF THE FIELDS EQUATIONS

Here we have three equation (2.7) – (2.9) in four unknowns A, B, τ and ρ and thus the system is indeterminate. To remove indeterminacy, we need one more condition for this we assume.

(3.1) $A = \alpha + \mu B$

Where α, μ, is constants. Now we consider the following cases

Case I : Here we choose $\mu = 0$. Then in this case we have

(3.2) $A = \alpha$

Then equation (2.9) using (3.1) goes to the form

(3.3) $$B_{44} + B_4^2 = -\frac{k^2}{8\pi}\exp(-4B)$$

Equation (3.3) can be written as an integral equation

(3.4) $$\int d\left(B_4^2 e^{2B}\right) = -\frac{k^2}{4\pi} e^{-2B} dB + D$$

Where D is constant of integration. So we get

(3.5) $$B_4^2 = De^{2B} + \frac{k^2}{8\pi} e^{-4B}$$

Which can again be written as an integral form as

(3.6) $$\int \frac{e^{2B} dB}{\left[De^{2B} + \frac{k^2}{8\pi}\right]^{1/2}} = \pm(t - t_0)$$

Where t_0 is another integration constant. Integrating (4.3.6) we get

(3.7) $$e^{2B} = D(t - t_0)^2 - \frac{k^2}{8\pi D}$$

ρ and τ can now be found from (2.7) and (2.8) respectively as

(3.8) $$\rho = \frac{D}{\left[D(t - t_0)^2 - \frac{k^2}{8\pi D}\right]}$$

(3.9) $$\tau = \frac{\left[D^2(t - t_0)^2 - 3\frac{k^2}{8\pi}\right]}{D(t - t_0)^2 - \frac{k^2}{8\pi D}}$$

(3.10) $$\rho_p = \rho - \tau = \frac{\left(\frac{k^2}{4\pi}\right)}{\left[D(t - t_0)^2 - \frac{k^2}{8\pi D}\right]^2}$$

Finally, the proper volume R^3, expansion scalar θ and shear scalar σ are found to be

(3.11) $$R^3 = D(t - t_0)^2 - \frac{k^2}{8\pi D_1}$$

(3.12) $$\theta = \frac{2D(t - t_0)}{R^3}$$

(3.13) $$\sigma^2 = \frac{1}{6}\left[\frac{2.D(t - t_0)}{R^3}\right]^2$$

From the above solution we observe that at the initial epoch.

$$(t-t_0)^2 = \frac{k^2}{8\pi D^2}$$

The string model starts with an initial singularity $R^3 \to 0$. While $\rho, \rho_p\ \tau, \theta, \sigma^2$ etc. diverge. This is a line singularity, since exp (2A) $\to 1$ (when λ = 0) and exp (2B) $\to$ 0. At a later instant when $\left(t - {t_0}^2 = \frac{3k^2}{8\pi D}\right)$ we have $\tau_0 = 0$ and $\rho = \rho_p$. So at this epoch string vanish and we are left with a dust filled universe with a magnetic field. At this stage

(3.14) $$\rho = \frac{4\pi D^2}{k^2}$$

$$R^3 = \frac{k^2}{4\pi D}$$

$$\theta = \frac{2}{k}(6\pi)^{\frac{1}{2}C}$$

$$\sigma^2 = \frac{2\pi D^2}{3k^2}$$

i.e. all these parameters are of finite magnitude. In this solution matter is directly related with the magnetic field as is noted in (3.11). When the magnetic field is absent, the matter is also absent and the solution reduces to that of pure geometric string distribution.

Case II : Here we use α = 0 in A = α + μB so that

(3.15) $A = \mu B$

By the use of (3.15) and (2.9) we get

(3.16) $$(\mu+1)B_{44} + (\mu^2+\mu+1)B_4^2 = -\frac{k^2}{8\pi}\exp(-4B)$$

This equation can be written as an integral equation

(3.17) $$\int d\left[B_4^2 \exp\left(2\left(\frac{\mu^2+\mu+1}{\mu+1}\right)B\right)\right]$$

$$= \frac{-k^2}{4\pi(\mu+1)}\int \exp\left(2\left(\frac{\mu^2-\mu-1}{\mu+1}\right)B\right)dB + k_1$$

Where K is constant of integration. So we have

(3.18) $$B_4^2 = K_1 \exp\left[-2\left(\frac{\mu^2+\mu+1}{\mu+1}\right)B\right] - \frac{k^2}{8\pi(\mu^2-\mu-1}\exp(-4B)$$

Which can again be written as an integral form as

$$(3.19)\quad \int \frac{e^{2B}dB}{\left[k_1 \exp\left(-2\left(\frac{\mu^2-\mu-1}{\mu+1}\right)v\right) - 8\pi\left(\frac{k^2}{\mu^2-\mu-1}\right)\right]^{1/2}} = \pm(t-t_0)$$

Where t_0 is another constant of integration. To solve (3.19) we choose μ such that

$$(3.20)\quad \mu^2 - 3\mu - 3 = 0$$

It is a quadratic equation in μ which can be solved to give.

$$(3.21)\quad \mu = \frac{3 \pm \sqrt{21}}{2}$$

Using (3.21) in (3.19), followed by integration, we get

$$(3.22)\quad e^{2B} = \left[\frac{8\pi k_1}{k^2}\left(5 \pm \sqrt{21}\right) - \frac{k^2(t-t_0)^2}{2\pi\left(5 \pm \sqrt{21}\right)}\right]^{1/2}$$

From (3.22) it is clear that arbitrary constant k_1 must be +ve in this case. So we replace k_1 by λ^2 in the following. The other parameters can be found as before

$$(3.23)\quad \rho = \frac{\frac{\left(9+2\sqrt{21}\right)k^2}{\left(5+\sqrt{21}\right)^2 16\pi}(t-t_0)^2 - \left(5+\sqrt{21}\right)\lambda^2}{\left[\frac{8\pi\lambda^2}{k^2}.\left(5+\sqrt{21}\right) - \frac{k^2(t-t_0)^2}{2\pi\left(5+\sqrt{21}\right)}\right]^2}$$

From energy conditions, ρ > 0 which demands positive sign before $\sqrt{21}$ in equation (3.22). With this choice other parameters are found explicitly as follows

$$(3.24)\quad \tau = \frac{\left(9+\sqrt{21}\right)\lambda^2 - \frac{\left(4+\sqrt{21}\right)}{\left(5+\sqrt{21}\right)^2}\frac{k^2}{16\pi^2}(t-t_0)^2}{\left[\frac{8\pi\lambda^2}{k^2}.\left(5+\sqrt{21}\right) - \frac{k^2(t-t_0)^2}{2\pi\left(5+\sqrt{21}\right)}\right]^2}$$

$$(4.3.25)\quad \rho_p = \frac{4\lambda^2 + \frac{k^2(t-t_0)^2}{16\pi^2\left(5+\sqrt{21}\right)}}{\left[\frac{8\pi\lambda^2\left(5+\sqrt{21}\right)}{k^2} - \frac{k^2(t-t_0)^2}{2\pi\left(5+\sqrt{21}\right)}\right]^2}$$

$$(3.26)\quad R^3 = \left[\frac{8\pi\lambda^2\left(5+\sqrt{21}\right)}{k^2} - \frac{k^2\left(t-t_{0^2}\right)}{2\pi\left(5+\sqrt{21}\right)}\right] \times \frac{\left(7+\sqrt{21}\right)}{8}$$

(3.27) $\theta = -\left(\frac{7+\sqrt{21}}{5+\sqrt{21}}\right)\frac{k^2}{8\pi}\left[\frac{(t-t_0)}{\frac{8\pi\lambda^2(5+\sqrt{21})}{k^2} - \frac{k^2(t-t_0)^2}{2\pi(5+\sqrt{21})}}\right]$

(3.28) $\sigma^2 = \frac{7+\sqrt{21}}{(5+\sqrt{21})^2}\left(\frac{k^2}{48\pi^2}\right)\left[\frac{(t-t_0)}{\frac{8\pi D^2}{k^2}(5+\sqrt{21}) - \frac{k^2(t-t_0)^2}{2\pi(5+\sqrt{21})}}\right]$

It is clear from (3.22) that

(3.29) $T^2 < \frac{16\pi^2\lambda^2}{k^4}(5+\sqrt{21})^2$

Where $T = t_0 - t$. When $t < t_0$ we have $T > 0$ and clearly from equation (3.27) we have

$\theta > 0$

Which in dicates that our model is expanding

Case III : when K = 0 (i.e. when magnetic field is absent)

Then using (3.15) equation (2.9) reduces to

(3.30) $B_{44} + \frac{\mu^2+\mu+1}{\mu+1}B_4 = 0$

This differential equation can be integrated to give solution which is not the special case of that give above

(3.31) $e^{2B} = [\gamma(t-t_0)]^{\frac{2(\mu-1)}{(\mu^2+\mu+1)}}$

Where r is a constant.

Therefore

(3.32) $e^{2A} = [\gamma(t-t_0)]^{\frac{2\mu(\mu-1)}{(\mu^2+\mu+1)}}$

The physical parameters are found to be

(3.33) $R^3 = [\gamma(t-t_0)]^{\frac{(2+\mu)(\mu+1)}{(\mu^2+\mu+1)}}$

(3.34) $\rho = \frac{2\mu+1}{(t-t_0)^2}$

(3.35) $\tau = \frac{1}{(t - t_0)^2}$

(3.36) $\rho_p = \frac{2\mu}{(t - t_0)^2}$

(3.37) $\theta = \frac{(\mu + 2)}{(t - t_0)}$

(3.38) $\sigma^2 = \frac{2}{3}\frac{(\mu - 1)^2}{(t - t_0)^2}$

Here we see that at the initial epoch, $t = t_0, R^3 \to 0$, while $\rho, \tau, \rho_p, \theta, \sigma^2$, all diverge, and exp $(2A), \exp(2B) \to 0$, the point singularity. This is the starting point of the string model. Again at a later stage when $t - \to \propto, R \to \propto$ but all other physical parameters become insignificant. Further we see that for a pure geometric string ($\rho_p = 0$), we have to take $\mu = 0$. For this case the universe starts with string and ends up at a stage when the massive strings themselves disappear without any remnant.

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