JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

# Degree And Total Degree Of Vertex In Tensor Product And Normal Product Of Intuitionistic Coloring Fuzzy Graphs

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## Abstract:

An intuitionistic coloring fuzzy graph can be obtained from two given intuitionistic coloring fuzzy graphs using tensor product and normal product. In particular cases, we discuss the degree and total degree of a vertex in intuitionistic coloring fuzzy graphs formed by these operations in terms of the degree and total degree of vertices in the given intuitionistic coloring fuzzy graphs.

## Keywords:

Degree of vertex, total degree of vertex, tensor product and normal product of two ICFG

## 1. Introduction:

In 1965, Zadeh has introduced the fuzzy graph to describe the uncertainty, considering everything. It has been various quickly developing and applications in different field. In spite of fact that Rosenfeld presented one more explained definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as path, cycles, connected ness, blocks, forest and etc. Atanassov was presented intuitionistic fuzzy graph theory. Pathinathan and Jesintha Rosline [6] discussed the vertex degree of cartesian product of IFG. Nagoor Gani and Sheik Mujibur Rahman [3] developed that the cartesian product and composition of some IFGs of total degree of vertex. Radha and Kumaravel [7] introduced the concept of edge degree of cartesian product and composition and also, we study some properties of IFGs by Sheik Dhavudh and Srinivasan [9]. Nirmala and Vijaya [5] defined the operations on fuzzy graphs. In this paper, we find the degree and total degree of vertices of tensor product and normal product of ICFGs under certain condition. First, we go through some basic definitions in ICFGs.

## 2. Preliminaries

**Definition:2.1[6]:**

An IFG is of the form $G = (V, E)$ where (i) $V = \{x_1, x_2, \ldots, x_n\}$ such that $\mu_A: V \to [0,1]$ and $v_A: V \to [0,1]$ denotes the membership degree and non-membership degree of the element $v_n \epsilon V$ respectively and $0 \leq \mu_A(x_n) + v_A(x_n) \leq 1$ for all $x_n \in V$.
(ii) $E \subseteq V \times V$ where $\mu_A: V \times \mathrm{V} \to [0,1]$; $v_A: V \times \mathrm{V} \to [0,1]$ such that $\mu_B(x_m, x_n) \leq \min(\mu_A(x_m), \mu_A(x_n))$ and $v_B(x_m, x_n) \geq \max(x_A(x_m), x_A(x_n))$ and $0 \leq \mu_A(x_m, x_n) + v_A(x_m, x_n) \leq 1$ for all $(x_m, x_n) \in E$

**Definition:2.2[6]:**

Let $G^* = (V, E)$ be an IFG. Then the degree of vertex x is defined by $d(x) = (d_\rho(x), d_\theta(x))$ where $d_\rho(x) = \sum_{x \neq y} \rho_B(x, y)$ and $d_\theta(x) = \sum_{x \neq y} \theta_B(x, y)$

**Definition:2.3[5]:**

The normal product of two fuzzy graphs on $(\alpha_1, \beta_1)$ on $G_i = (V_i, X_i)$,
i=1, 2 is defined as a fuzzy graph $(\alpha_1 \circ \alpha_2,\ \beta_1 \circ \beta_2)$ on $G = (V, X)$ where $V = V_1 X V_2$ and $X = \{(u, u_2)(u, v_2) \backslash u \in V_1, (u_2, v_2) \in X_2\} \cup \{(u_1, w)(v_1, w) \backslash (u_1, v_1) \in X_1, w \in V_2\} \cup \{(u_1, u_2)\ (v_1, v_2) \backslash (u_1, v_1) \in X_1, (u_2, v_2) \in X_2\}$
Fuzzy set $\sigma_1 \circ \sigma_2$ *and* $\mu_1 \circ \mu_2$ are defined as
$(\alpha_1 \circ \alpha_2)(u_1, u_2) = \alpha_1(u_1) \wedge \alpha_2(u_2), if\ (u_1, u_2) \in V_1 X V_2$
$(\beta_1 \circ \beta_2)\ (u, u_2)(u, v_2) = \alpha_1(u) \wedge \beta_2(u_2, v_2)$, if $u \in V_1$, $(u_2, v_2) \in X_2$
$(\beta_1 \circ \beta_2)\ (u_1, w)(v_1, w) = \beta_1(u_1, v_1) \wedge \alpha_2(w)$, if $w \in V_2$, $(u_1, v_1) \in X_1$
$(\beta_1 \circ \beta_2)\ (u_1, u_2)\ (v_1, v_2) = \beta_1(u_1, v_1) \wedge\ \beta_2(u_2, v_2)$, if $(u_1, v_1) \in X_1, (u_2, v_2) \in X_2$

**Definition: 2.4[3]**

Let $G = (V, E)$ be an IFG. Then the total degree of a vertex x$\in V$ is defined by
$td(x) = (td_\mu(x), td_v(x)) = \sum_{x \neq y} \mu_B(x, y) + \mu_A(x), \sum_{x \neq y} v_B(x, y) + v_A(x),$
$= (d_\mu(x) + \mu_A(x), d_v(x) + v_A(x))$

If every vertex of G has same membership total degree $k_1$ and same non-membership total degree $k_2$ then said to be a total regular intuitionistic fuzzy graph.

**Definition:2.5[3]**

Let $= (V, E)$ be an IFG. If $(d_\mu(x), d_v(x)) = (k_1, k_2)$ for all $x \in V$ that is if every vertex has same membership degree $k_1$ and same non-membership degree $k_2$ then said to be a total regular intuitionistic fuzzy graph.

## 3. Degree of vertex in tensor product of intuitionistic fuzzy graphs

**Definition:3.1**

Let $G_A = (v_i, \mu'_A, \vartheta'_A;\ e_{ij}, \mu''_A, \vartheta''_A)$ and $G_B = (v_i, \mu'_B, \vartheta'_B;\ e_{ij}, \mu''_B, \vartheta''_B)$ and be two intuitionistic fuzzy graphs with underlying vertex sets $V_A$ and $V_B$ and the edge sets $E_A$ and $E_B$ respecti1vely. Then the tensor product of $G_A$ and $G_B$ is a pair of functions

$(\mu'_A \boxdot \mu'_B, \vartheta'_A \boxdot\ \vartheta'_B)$ with underlying vertex set $V_A \boxdot V_B = \{(x_1, y_1): x_1 \epsilon V_A\ \&\ y_1 \epsilon V_B\}$ and underlying edge set $E_A \boxdot E_B = \{(x_1, x_2)(y_1, y_2): (x_1, y_1) \epsilon E_A \& (x_2, y_2) \epsilon E_B\}$ with

$(\mu'_A \boxdot \mu'_B)(x_1, y_1) = \mu'_A(x_1) \wedge \mu'_B(\ y_1)$

$(\vartheta'_A \boxdot\ \vartheta'_B)(x_1, y_1) = \vartheta'_A(x_1) \vee \vartheta'_B(\ y_1)$, if $x_1 \epsilon V_A\ \&\ y_1 \epsilon V_B$ and

$(\mu_A'' \boxdot \mu_B'')(x_1,x_2)(y_1,y_2) = \mu_A''(x_1,y_1) \wedge \mu_B''(x_2,y_2)$

$(\vartheta_A'' \boxdot \vartheta_B'')(x_1,x_2)(y_1,y_2) = \vartheta_A''(x_1,y_1) \vee \vartheta_B''(x_2,y_2), (x_1,y_1)\epsilon E_A \& (x_2,y_2)\epsilon E_B$

**Definition:3.2**

By definition, For any $(x_1,x_2)\epsilon V \times V$

$$d_{G_A \boxdot G_B}(x_1,x_2) = \langle d_{(\mu_A' \boxdot \mu_B'')}(x_1,x_2), d_{(\vartheta_A' \boxdot \vartheta_B'')}(x_1,x_2)\rangle$$

$$= \langle \sum_{\substack{(x_1,x_2)\\(y_1,y_2)\epsilon E}} (\mu_A' \boxdot \mu_B'')(x_1,x_2)(y_1,y_2), \sum_{\substack{(x_1,x_2)\\(y_1,y_2)\epsilon E}} (\vartheta_A' \boxdot \vartheta_B'')(x_1,x_2)(y_1,y_2)\rangle$$

$$= \langle \sum_{(x_1,y_1)\in E_A} \mu_A'(x_1,y_1) \wedge \sum_{(x_2,y_2)\in E_B} \mu_B''(x_2,y_2), \sum_{(x_1,y_1)\in E_A} \vartheta_A'(x_1,y_1) \vee \sum_{(x_2,y_2)\in E_B} \vartheta_B''(x_2,y_2)\rangle$$

**Theorem:3.3**

Let $G_A = (v_i, \mu_A', \vartheta_A';\ e_{ij}, \mu_A'', \vartheta_A'')$ and $G_B = (v_i, \mu_B', \vartheta_B';\ e_{ij}, \mu_B'', \vartheta_B'')$ be two IFG, if

(i) $\mu_B'' \geq \mu_A'$ and $\vartheta_B'' \leq \vartheta_A'$ then $d_{G_A \boxdot G_B}(x_1,x_2) = d_{G_A}(x_1)$ and

(ii) $\mu_A' \geq \mu_B''$ and $\vartheta_A' \leq \vartheta_B''$ then $d_{G_A \boxdot G_B}(x_1,x_2) = d_{G_B}(x_2)$.

**Proof:**

Let $G_A = (V_i, \mu_A', \vartheta_A';\ e_{ij}, \mu_A'', \vartheta_A'')$ and $G_B = (V_i, \mu_B', \vartheta_B';\ e_{ij}, \mu_B'', \vartheta_B'')$ be two IFG,

(i) $d_{G_A \boxdot G_B}(x_1,x_2) = \sum_{(x_1,y_1)(x_2,y_2)\in E}(\mu_A'' \boxdot \mu_B'')(x_1,x_2)(y_1,y_2)$

$= \sum_{(x_1,y_1)\in E_A} \mu_A''(x_1,y_1) \wedge \sum_{(x_2,y_2)\in E_B} \mu_B''(x_2,y_2)$

$= \sum_{(x_1,y_1)\in E_A} \mu_A''(x_1,y_1)$

$= d_{\mu G_A}(x_1)$

And $d_{G_A \boxdot G_B}(x_1,x_2) = \sum_{(x_1,y_1)(x_2,y_2)\in E}(\vartheta_1'' \boxdot \vartheta_2'')(x_1,x_2)(y_1,y_2)$

$= \sum_{(x_1,y_1)\in E_1} \vartheta_1''(x_1,y_1) \vee \sum_{(x_2,y_2)\in E_2} \vartheta_2''(x_2,y_2)$

$= \sum_{(x_1,y_1)\in E_1} \vartheta_1''(x_1,y_1)$

$= d_{\vartheta G_A}(x_1)$

Thus, $d_{G_A \boxdot G_B}(x_1,x_2) = \langle d_{\mu G_A}(x_1), d_{\vartheta G_A}(x_1)\rangle$

(ii) $d_{G_A \boxdot G_B}(x_1,x_2) = \sum_{(x_1,y_1)(x_2,y_2)\in E}(\mu_A'' \boxdot \mu_B'')(x_1,x_2)(y_1,y_2)$

$= \sum_{(x_1,y_1)\in E_A} \mu_A''(x_1,y_1) \wedge \sum_{(x_2,y_2)\in E_B} \mu_B''(x_2,y_2)$

$= \sum_{(x_2,y_2)\in E_B} \mu_B''(x_2,y_2) = d_{\mu G_B}(x_2)$

And $d_{G_A \boxdot G_B}(x_1,x_2) = \sum_{(x_1,y_1)(x_2,y_2)\in E}(\vartheta_A'' \boxdot \vartheta_B'')(x_1,x_2)(y_1,y_2)$

$= \sum_{(x_1,y_1)\in E_A} \vartheta_A''(x_1,y_1) \vee \sum_{(x_2,y_2)\in E_B} \vartheta_B''(x_2,y_2)$

$= \sum_{(x_2,y_2)\in E_B} \vartheta_B''(x_2,y_2)$

$= d_{\vartheta G_B}(x_2)$

Thus, $d_{G_A \boxdot G_B}(x_1,x_2) = \langle d_{\mu G_B}(x_2), d_{\vartheta G_B}(x_2)\rangle$

In this intuitionistic coloring fuzzy graph $G_1$ and $G_2$ and $G_1 \times G_2$ multiplication is given in Fig:1. Here the author used to magnetic concept for multiplication. Suppose that same vertices color present in multiplication we can't multiply the two vertices. If it is different color we can multiply and given the name as which color has minimum value. The value of the vertices and edges has minimum and maximum values. In multiplication we can take only minimum value. Because chromatic number concept used here. Minimum number of color required so we used this concept.

**Example 3.4**

Consider $G_A = (v_i, \mu'_A, \vartheta'_A;\ e_{ij}, \mu''_A, \vartheta''_A)$ and $G_B = (v_i, \mu'_B, \vartheta'_B;\ e_{ij}, \mu''_B, \vartheta''_B)$ be two ICFG in fig.

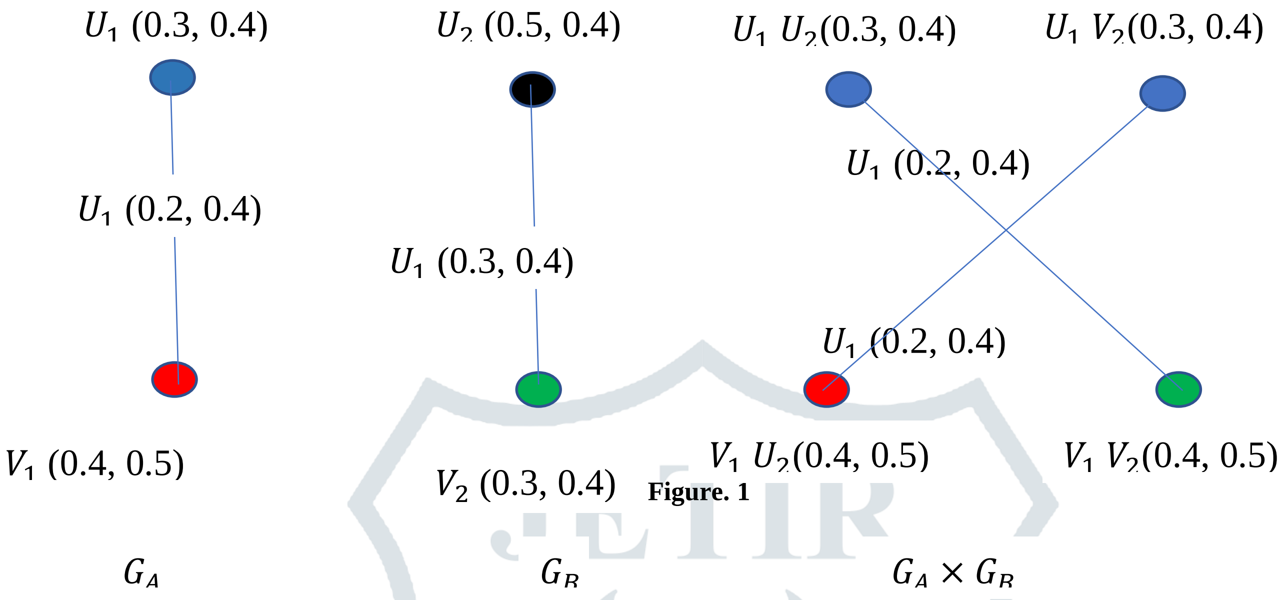


**FIG: 1**

If $\mu''_B \geq \mu''_A$ and $\vartheta''_B \leq \vartheta''_A$, then $d_{G_A \boxdot G_B}(u_1, u_2) = d_{G_A}(u_1)$

$d_{\mu G_A \boxdot G_B}(u_1, u_2) = 0.2$; $d_{\vartheta G_A \boxdot G_B}(u_1, u_2) = 0.4$; $\Rightarrow d_{G_A \boxdot G_B}(u_1, u_2) = (0.2, 0.4)$; and

$d_{\mu G_A}(u_1)$=0.2, $d_{\vartheta G_A}(u_1)$=0.4, $\Rightarrow d_{G_A}(u_1) = (0.2, 0.4)$

Thus, $d_{G_A \boxdot G_B}(u_1, u_2) = d_{G_A}(u_1)$

## 3.5 Total degree of a vertex in tensor product of intuitionistic fuzzy graph

**Definition:3.5.1**

For any $(p_1, p_2) \epsilon V_A \times V_B$,

$$td_{\mu(G_A \boxdot G_B)}(p_1, p_2) = \sum_{(p_1,q_1)(p_2,q_2)\in E} (\mu''_A \boxdot \mu''_B)(p_1, p_2)(q_1, q_2) + (\mu'_A \boxdot \mu'_B)(p_1, p_2)$$

$$= \sum_{(x_1,y_1)\in E_A} \mu''_A(x_1, y_1) \wedge \sum_{(x_2,y_2)\in E_B} \mu''_B(x_2, y_2) + (\mu'_A)(p_1) \wedge \mu'_B(p_2)$$

$$= \sum_{(p_1,q_1)\in E_A} \mu''_A(p_1, q_1) + (\mu'_A)(p_1)$$

$$= td_{\mu(G_A)}(p_1)$$

$$td_{\vartheta(G_A \boxdot G_B)}(p_1, p_2) = \sum_{(p_1,q_1)(p_2,q_2)\in E} (\vartheta''_A \boxdot \vartheta''_B)(p_1, p_2)(q_1, q_2) + (\vartheta'_A \boxdot \vartheta'_B)(p_1, p_2)$$

$$= \sum_{(x_1,y_1)\in E_A} \vartheta''_A(x_1, y_1) \vee \sum_{(x_2,y_2)\in E_B} \vartheta''_B(x_2, y_2) + (\vartheta'_A)(p_1) \vee \vartheta'_B(p_2)$$

$$= \sum_{(p_1,q_1)\in E_A} \vartheta''_A(p_1, q_1) + (\vartheta'_A)(p_1)$$

$$= td_{\vartheta(G_A)}(p_1)$$

Thus, $td_{(G_A \boxdot G_B)}(p_1, p_2) = (td_{\mu(G_A)}(p_1), td_{\vartheta(G_A)}(p_1))$

**Example 3.6**

Consider $G_A = (v_i, \mu'_A, \vartheta'_A;\ e_{ij}, \mu''_A, \vartheta''_A)$ and $G_B = (v_i, \mu'_B, \vartheta'_B;\ e_{ij}, \mu''_B, \vartheta''_B)$ be two ICFG in fig. 2

If $\mu''_B \geq \mu''_A$ and $\mu'_B \geq \mu'_A$; $\vartheta''_B \leq \vartheta''_A$ and $\vartheta'_B \leq \vartheta'_A$ then $td_{G_A \boxdot G_B}(u_1, u_2) = td_{G_A}(u_1)$

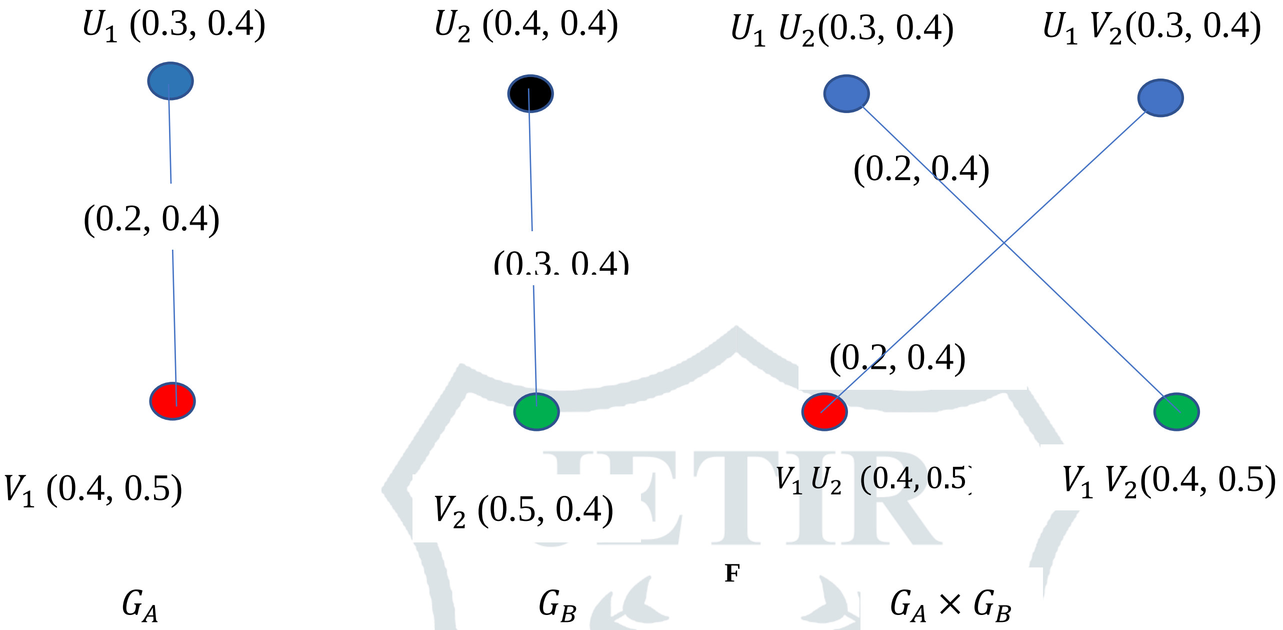


**FIG: 2**

$td_{\mu G_A \boxdot G_B}(u_1, u_2) = 0.2 + 0.3 = 0.5$; $td_{\vartheta G_A \boxdot G_B}(u_1, u_2) = 0.4 + 0.4 = 0.8, then\ td_{G_A \boxdot G_B}(u_1, u_2) = (0.5, 0.8)$ and $td_{\mu G_A}(u_1) = 0.2 + 0.3 = 0.5; td_{\vartheta G_A}(u_1) = 0.4 + 0.4 = 0.8$, then $td_{G_A}(u_1) = (0.5, 0.8)$

Thus, $td_{G_A \boxdot G_B}(u_1, u_2) = td_{G_A}(u_1)$

## 4. Normal products of intuitionistic fuzzy graphs:

**Definition:4.1**

Let $G_A = (V_A, E_A)$ and $G_B = (V_B, E_B)$ be two IFG, Here $V_A = (\mu'_A, \vartheta'_A)$; $E_A = (\mu''_A, \vartheta''_A)$; $V_B = (\mu'_B, \vartheta'_B)$ and $E_B = (\mu''_B, \vartheta''_B)$, then the normal product of $G_A$ and $G_B$ is defined as $(V_A * V_B, E_A * E_B)$, where $V_A * V_B = (\mu'_A * \mu'_B, \vartheta'_A * \vartheta'_B)$ and $E_A * E_B = (\mu''_A * \mu''_B, \vartheta''_A * \vartheta''_B)$

$(\mu'_A * \mu'_B)(p_1, p_2) = \mu'_A(p_1) \wedge \mu'_B(p_2)$;
$(\vartheta'_A * \vartheta'_B)(p_1, p_2) = \vartheta'_A(p_1) \vee \vartheta'_B(p_2)$; $p_1 \epsilon V_A, p_2 \epsilon V_B$ and,

$(\mu''_A * \mu''_B)(p_1, p_2)(q_1, q_2) = \mu'_A(p_1) \wedge \mu''_B(p_2, q_2)$,
$(\vartheta''_A * \vartheta''_B)(p_1, p_2)(q_1, q_2) = \vartheta'_A(p_1) \vee \vartheta''_B(p_2, q_2)$, if $p_1 = q_1$; and $(p_2, q_2) \in E_B$

$(\mu''_A * \mu''_B)(p_1, p_2)(q_1, q_2) = \mu''_A(p_1, q_1) \wedge \mu'_B(p_2)$
$(\vartheta''_A * \vartheta''_B)(p_1, p_2)(q_1, q_2) = \vartheta''_A(p_1, q_1) \vee \vartheta'_B(p_2)$, if $p_2 = q_2$; and $(p_1, q_1) \in E_A$

$(\mu''_A * \mu''_B)(p_1, p_2)(q_1, q_2) = \mu''_A(p_1, q_1) \wedge \mu''_B(p_2, q_2)$
$(\vartheta''_A * \vartheta''_B)(p_1, p_2)(q_1, q_2) = \vartheta''_A(p_1, q_1) \vee \vartheta''_B(p_2, q_2)$, if $(p_1, q_1) \in E_A$ and $(p_2, q_2) \in E_B$

## 4.2 Degree of vertex in normal product of intuitionistic fuzzy graphs:

**Definition: 4.2.1**

Let $G_A = (V_A, E_A)$ and $G_B = (V_B, E_B)$ be two IFG, Here $V_A = (\mu'_A, \vartheta'_A)$; $E_A = (\mu''_A, \vartheta''_A)$; $V_B = (\mu'_B, \vartheta'_B)$ and $E_B = (\mu''_B, \vartheta''_B)$ For any $(u_1, u_2) \in V_A \times V_B$,

$$d_{G_A * G_B}(u_1, u_2) = \langle d_{(\mu''_A * \mu''_B)}(u_1, u_2), d_{(\vartheta''_A * \vartheta''_B)}(u_1, u_2)\rangle$$

$$d_{(\mu''_A * \mu''_B)}(u_1, u_2) = \sum_{(x_1,y_1)(x_2,y_2)\in E} (\mu''_A * \mu''_B)(u_1, v_1)(u_2, v_2)$$

$$= \sum_{(u_1=v_1)(u_2,v_2)\in E_B} \mu'_A(u_1) \wedge \mu''_B(u_2, v_2) + \sum_{(u_2=v_2)(u_1\, v_1)\in E_A} \mu'_B(u_2) \wedge \mu'_A(u_1, v_1) + \sum_{(u_1\, v_1)\in E_A(u_2,v_2)\in E_B} \mu''_A(u_1, v_1) \wedge \mu''_B(u_2, v_2)$$

$$d_{(\vartheta''_A * \vartheta''_B)}(u_1, u_2) = \sum_{(x_1,y_1)(x_2,y_2)\in E} (\vartheta''_A * \vartheta''_B)(u_1, v_1)(u_2, v_2)$$

$$= \sum_{(u_1=v_1)(u_2,v_2)\in E_B} \vartheta'_A(u_1) \vee \vartheta''_B(u_2, v_2) + \sum_{(u_2=v_2)(u_1\, v_1)\in E_A} \vartheta'_B(u_2) \vee \vartheta'_A(u_1, v_1) + \sum_{(u_1\, v_1)\in E_1(u_2,v_2)\in E_B} \vartheta''_A(u_1, v_1) \vee \vartheta''_B(u_2, v_2)$$

**Theorem: 4.3** Let $G_A = (V_A, E_A)$ and $G_B = (V_B, E_B)$ be two IFG, Here $V_A = (\mu'_A, \vartheta'_A)$; $E_A = (\mu''_A, \vartheta''_A)$; $V_B = (\mu'_B, \vartheta'_B)$ and $E_B = (\mu''_B, \vartheta''_B)$.

(i) If $\mu'_A \geq \mu''_B$; $\mu'_B \geq \mu''_A$; $\mu''_A \leq \mu''_B$, then $d_{\mu(G_A * G_B)}(u_1, u_2) = d_{\mu(G_B)}(u_2) + p_2 d_{\mu(G_A)}(u_1)$

(ii) If $\vartheta'_A \leq \vartheta''_B$; $\vartheta'_B \leq \vartheta''_A$; $\vartheta''_A \geq \vartheta''_B$, then $= d_{\vartheta(G_B)}(u_2) + p_2 d_{\vartheta(G_A)}(u_1)$

**Proof:** By definition (4.2.1),

(i) $d_{\mu(G_A * G_B)}(u_1, u_2) = \sum_{(x_1,y_1)(x_2,y_2)\in E}(\mu''_A * \mu''_B)(u_1, v_1)(u_2, v_2)$

$$= \sum_{(u_1=v_1)(u_2,v_2)\in E_B} \mu'_A(u_1) \wedge \mu''_B(u_2, v_2) + \sum_{(u_2=v_2)(u_1\, v_1)\in E_A} \mu'_B(u_2) \wedge \mu'_A(u_1, v_1) + \sum_{(u_1\, v_1)\in E_A(u_2,v_2)\in E_B} \mu''_A(u_1, v_1) \wedge \mu''_B(u_2, v_2)$$

$$= \sum_{(u_1=v_1)(u_2,v_2)\in E_B} \mu''_B(u_2, v_2) + \sum_{(u_2=v_2)(u_1\, v_1)\in E_A} \mu''_A(u_1, v_1) + \sum_{(u_1\, v_1)\in E_A(u_2,v_2)\in E_B} \mu''_A(u_1, v_1)$$

$$= d_{\mu(G_B)}(u_2) + |v_2| d_{\mu(G_A)}(u_1)$$

$$= d_{\mu(G_B)}(u_2) + p_2 d_{\mu(G_A)}(u_1)$$

(ii) $d_{\vartheta(G_A * G_B)}(u_1, u_2) = \sum_{(x_1,y_1)(x_2,y_2)\in E}(\vartheta''_A * \vartheta''_B)(u_1, v_1)(u_2, v_2)$

$$= \sum_{(u_1=v_1)(u_2,v_2)\in E_B} \vartheta'_A(u_1) \vee \vartheta''_B(u_2, v_2) + \sum_{(u_2=v_2)(u_1\, v_1)\in E_A} \vartheta'_B(u_2) \vee \vartheta'_A(u_1, v_1) + \sum_{(u_1\, v_1)\in E_A(u_2,v_2)\in E_2} \vartheta''_A(u_1, v_1) \vee \vartheta''_B(u_2, v_2)$$

$$= \sum_{(u_1=v_1)(u_2,v_2)\in E_B} \vartheta''_B(u_2, v_2) + \sum_{(u_2=v_2)(u_1\, v_1)\in E_A} \vartheta''_A(u_1, v_1) + \sum_{(u_1\, v_1)\in E_A(u_2,v_2)\in E_B} \vartheta''_A(u_1, v_1)$$

$$= d_{\vartheta(G_B)}(u_2) + |v_2| d_{\vartheta(G_A)}(u_1)$$

$$= d_{\vartheta(G_B)}(u_2) + p_2 d_{\vartheta(G_A)}(u_1)$$

**Example 4.4** Consider $G_A = \left(v_i, \mu'_A, \vartheta'_A;\ e_{ij}, \mu''_A, \vartheta''_A\right)$ and $G_B = \left(v_i, \mu'_B, \vartheta'_B;\ e_{ij}, \mu''_B, \vartheta''_B\right)$ be two ICFG in fig. 3

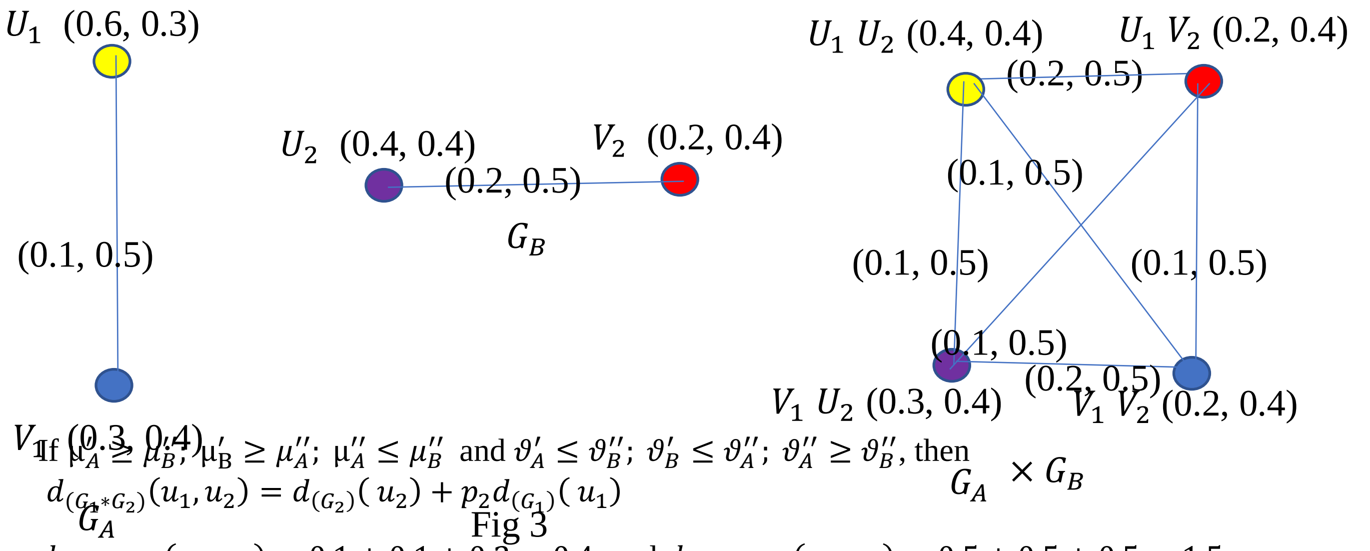


Fig 3

If $\mu'_A \geq \mu''_B$; $\mu'_B \geq \mu''_A$; $\mu''_A \leq \mu''_B$ and $\vartheta'_A \leq \vartheta''_B$; $\vartheta'_B \leq \vartheta''_A$; $\vartheta''_A \geq \vartheta''_B$, then

$d_{(G_A*G_2)}(u_1, u_2) = d_{(G_2)}(u_2) + p_2 d_{(G_1)}(u_1)$

$d_{\mu(G_A*G_B)}(u_1, u_2) = 0.1 + 0.1 + 0.2 = 0.4$; and $d_{\vartheta(G_A*G_B)}(u_1, u_2) = 0.5 + 0.5 + 0.5 = 1.5$

$d_{(G_A*G_B)}(u_1, u_2) = (0.4, 1.5)$ --------- (I)

$d_{\mu(G_B)}(u_2) = 0.2$; $p_2 d_{\mu(G_A)}(u_1) = 2(0.1) = 0.2$; $d_{\mu(G_B)}(u_2)(u_1) + p_2 d_{\mu(G_A)} = 0.2 + 0.2 = 0.4$; and

$d_{\vartheta(G_B)}(u_2) = 0.5$; $p_2 d_{\vartheta(G_A)}(u_1) = 2(0.5) = 1.0$; $d_{\vartheta(G_B)}(u_2) + p_2 d_{\vartheta(G_A)}(u_1) = 0.5 + 1.0 = 1.5$

$d_{(G_B)}(u_2) + p_2 d_{(G_A)}(u_1) = (0.4, 1.5)$--------(II)

Therefore, (I) = (II)

Thus, $d_{(G_A*G_B)}(u_1, u_2) = d_{(G_B)}(u_2) + p_2 d_{(G_A)}(u_1)$

## 4.5 Total degree of a vertex in normal product of intuitionistic fuzzy graph

**Definition: 4.5.1**

By definition 4.1, For any $(u_1, u_2) \in V_A \times V_B$,

$td_{G_A*G_B}(u_1, u_2) = \langle td_\mu(\mu''_A * \mu''_B)(u_1, u_2), td_\vartheta(\vartheta''_A * \vartheta''_B)(u_1, u_2)\rangle$

$$td_\mu(\mu''_A * \mu''_B)(u_1, u_2) = \sum_{(x_1,y_1)(x_2,y_2)\in E} (\mu''_A * \mu''_B)(u_1, v_1)(u_2, v_2) + (\mu'_A * \mu'_B)(u_1, u_2)$$

$$= \sum_{(u_1=v_1)(u_2,v_2)\in E_B} \mu'_A(u_1) \wedge \mu''_B(u_2, v_2) + \sum_{(u_2=v_2)(u_1 v_1)\in E_A} \mu'_B(u_2) \wedge \mu'_A(u_1, v_1) + \sum_{(u_1 v_1)\in E_A (u_2,v_2)\in E_B} \mu''_A(u_1, v_1) \wedge \mu''_B(u_2, v_2) + (\mu'_A)(u_1) \wedge \mu'_B(u_2)$$

$$td_\vartheta(\vartheta''_A * \vartheta''_B)(u_1, u_2) = \sum_{(x_1,y_1)(x_2,y_2)\in E} (\vartheta''_A * \vartheta''_B)(u_1, v_1)(u_2, v_2) + (\vartheta'_A * \vartheta'_B)(u_1, u_2)$$

$$= \sum_{(u_1=v_1)(u_2,v_2)\in E_B} \vartheta'_A(u_1) \vee \vartheta''_B(u_2, v_2) + \sum_{(u_2=v_2)(u_1 v_1)\in E_A} \vartheta'_B(u_2) \vee \vartheta'_A(u_1, v_1) + \sum_{(u_1 v_1)\in E_A (u_2,v_2)\in E_B} \vartheta''_A(u_1, v_1) \vee \vartheta''_B(u_2, v_2) + (\vartheta'_A)(u_1) \vee \vartheta'_B(u_2)$$

**Theorem:4.6** Let $G_A = (V_A, E_A)$ and $G_B = (V_B, E_B)$ be two IFG, Here $V_A = (\mu'_A, \vartheta'_A)$; $E_A = (\mu''_A, \vartheta''_A)$; $V_B = (\mu'_B, \vartheta'_B)$ and $E_B = (\mu''_B, \vartheta''_B)$. For any $(p_1, p_2) \epsilon V_A \times V_B$,

(i) If $\mu'_A \geq \mu''_B$; $\mu'_B \geq \mu''_A$; $\mu''_A \leq \mu''_B$ , then

$$td_{\mu(G_A*G_B)}(r_1, r_2) = td_{\mu(G_B)}(r_2) + P_2 td_{\mu(G_A)}(r) - (P_2 - 1)\mu'_A(r_1) - \mu'_A(r_1) \vee \mu'_B(r_2)$$

(ii)If $\vartheta'_A \leq \vartheta''_B$; $\vartheta'_B \leq \vartheta''_A$; $\vartheta''_A \geq \vartheta''_B$, then

$$td_{\vartheta(G_A*G_B)}(r_1, r_2) = td_{\vartheta(G_B)}(r_2) + P_2 td_{\vartheta(G_A)}(r_1) - (P_2 - 1)\vartheta'_A(r_1) - \vartheta'_A(r_1) \wedge \vartheta'_B(r_2)$$

**Proof:** By definition, 4.5.1,

(i) $td_{\mu(G_A*G_B)}(r_1,r_2) = \sum_{(r_1,s_1)(r_2,s_2)\in E}(\mu''_A * \mu''_B)(r_1,s_1)(r_2,s_2) + (\mu'_A * \mu'_B)(r_1,r_2)$

$$= \sum_{(r_1=s_1)(r_2,s_2)\in E_B} \mu'_A(r_1) \wedge \mu''_B(r_2,s_2) + \sum_{(r_2=s_2)(r_1,s_1)\in E_A} \mu'_B(r_2) \wedge \mu'_A(r_1,s_1) + \sum_{(r_1\, s_1)\in E_A(r_2,s_2)\in E_B} \mu''_A(r_1,s_1) \wedge \mu''_B(r_2,s_2) + \mu'_A(r_1) \wedge \mu'_B(r_2)$$

$$= \sum_{(r_2,s_2)\in E_B} \mu''_B(r_2,s_2) + \sum_{(r_1,s_1)\in E_A} \mu''_A(r_1,s_1) + \sum_{(r_1,s_1)\in E_A} \mu''_A(r_1,s_1) + \mu'_A(r_1) + \mu'_B(r_2) - \mu'_A(r_1) \vee \mu'_B(r_2)$$

$$= \sum_{(r_2,s_2)\in E_B} \mu''_B(r_2,s_2) + \mu'_B(r_2) + \sum_{(r_1,s_1)\in E_A} \mu''_A(r_1,s_1) + \mu'_A(r_1) + \sum_{(r_1,s_1)\in E_A} \mu''_A(r_1,s_1) - \mu'_A(r_1) \vee \mu'_B(r_2)$$

$$= d_{\mu(G_B)}(r_2) + |v_2| \sum_{(r_1,s_1)\in E_A} \mu''_A(r_1,s_1) + \mu'_A(r_1) - \mu'_A(r_1) \vee \mu'_B(r_2)$$

$$= td_{\mu(G_B)}(r_2) + p_2\left|\sum_{(r_1,s_1)\in E_A} \mu''_A(r_1,s_1) + \mu'_A(r_1)\right| - (P_2-1)\mu'_A(r_1) - \mu'_A(r_1) \vee \mu'_B(r_2)$$

$$= td_{\mu(G_B)}(r_2) + p_2 td_{\mu(G_A)}(r_1) - (P_2-1)\mu'_A(r_1) - \mu'_A(r_1) \vee \mu'_B(r_2)$$

(ii) $td_{\vartheta(G_A*G_B)}(r_1,r_2) = \sum_{(x_1,y_1)(r_2,s_2)\in E}(\vartheta''_A * \vartheta''_B)(r_1,s_1)(r_2,s_2) + (\vartheta'_A * \vartheta'_B)(r_1,r_2)$

$$= \sum_{(r_1=s_1)(r_2,s_2)\in E_B} \vartheta'_A(r_1) \wedge \vartheta''_B(r_2,s_2) + \sum_{(r_2=s_2)(r_1,s_1)\in E_A} \vartheta'_B(r_2) \wedge \vartheta'_A(r_1,s_1) + \sum_{(r_1,s_1)\in E_1(u_2,v_2)\in E_B} \vartheta''_A(r_1,s_1) \wedge \vartheta''_B(r_2,s_2) + \vartheta'_A(r_1) \vee \vartheta'_B(r_2)$$

$$= \sum_{(r_2,s_2)\in E_B} \vartheta''_B(r_2,s_2) + \sum_{(r_1,s_1)\in E_A} \vartheta''_A(r_1,s_1) + \sum_{(r_1,s_1)\in E_A} \vartheta''_A(r_1,s_1)\, \vartheta'_A(r_1) + \vartheta'_B(r_2) - \vartheta'_A(r_1) \wedge \vartheta'_B(r_2)$$

$$= \sum_{(r_2,s_2)\in E_B} \vartheta''_B(r_2,s_2) + \vartheta'_B(r_2) + \sum_{(r_1,s_1)\in E_A} \vartheta''_A(r_1,s_1) + \vartheta'_A(r_1) + \sum_{(r_1,s_1)\in E_A} \vartheta''_A(r_1,s_1) - \vartheta'_A(r_1) \wedge \vartheta'_B(r_2)$$

$$= d_{\vartheta(G_B)}(r_2) + |v_2| \sum_{(r_1,s_1)\in E_A} \vartheta''_A(r_1,s_1) + \vartheta'_A(r_1) - \vartheta'_A(r_1) \wedge \vartheta'_B(r_2)$$

$$= td_{\vartheta(G_B)}(r_2) + p_2\left|\sum_{(r_1,s_1)\in E_A} \vartheta''_A(r_1,s_1) + \vartheta'_A(r_1)\right| - (P_2-1)\vartheta'_A(r_1) - \vartheta'_A(r_1) \wedge \vartheta'_B(r_2)$$

$$= td_{\vartheta(G_B)}(r_2) + P_2 td_{\vartheta(G_A)}(r_1) - (P_2-1)\vartheta'_A(r_1) - \vartheta'_A(r_1) \vee \vartheta'_B(r_2)$$

**Example: 4.7**

Consider $G_A = (v_i, \mu'_A, \vartheta'_A;\ e_{ij}, \mu''_A, \vartheta''_A)$ and $G_B = (v_i, \mu'_B, \vartheta'_B;\ e_{ij}, \mu''_B, \vartheta''_B)$ be two IFG in fig. 4

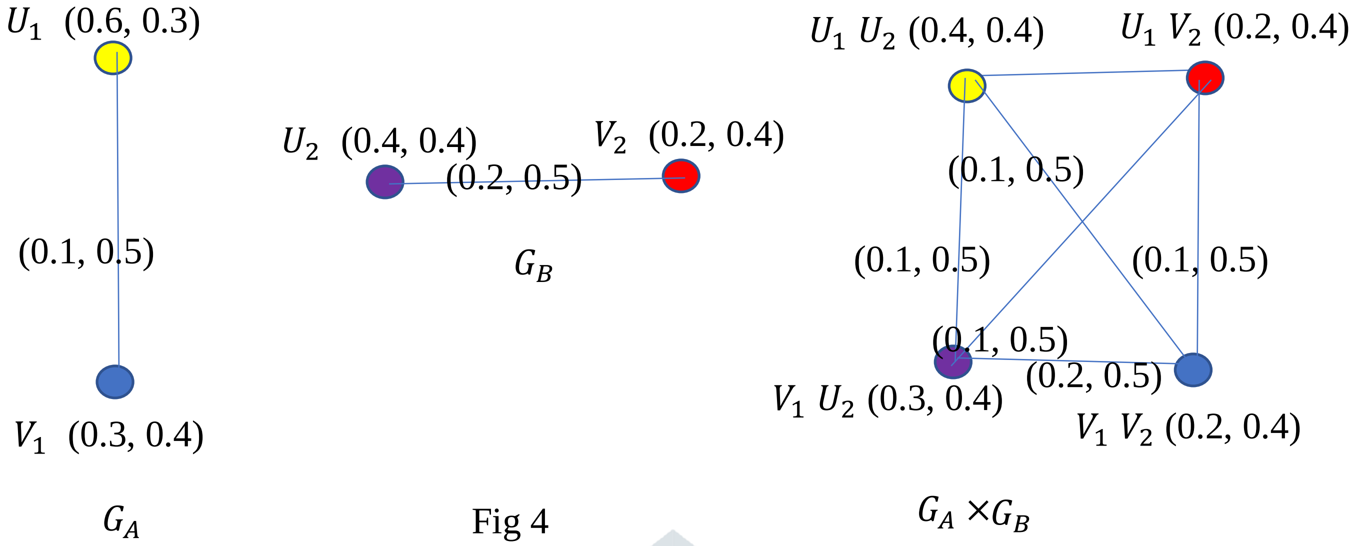


Fig 4

(i) If $\mu'_A \geq \mu''_B;\ \mu'_B \geq \mu''_A;\ \mu''_A \leq \mu''_B$ , then

$$td_{\mu(G_A*G_B)}(u_1,u_2) = td_{\mu(G_B)}(u_2) + P_2 td_{\mu(G_A)}(u_1) - (P_2-1)\mu'_A(u_1) - \mu'_A(u_1) \vee \mu'_B(u_2)$$

$$td_{\mu(G_A*G_B)}(u_1,u_2) = 0.1 + 0.1 + 0.2 + 0.4 = 0.8$$

$$td_{\mu(G_B)}(u_2) + P_2 td_{\mu(G_A)}(u_1) - (P_2-1)\mu'_A(u_1) - \mu'_A(u_1) \vee \mu'_B(u_2)$$
$$= 0.4 + 0.2 + 2(0.5+0.1) - (2-1)(0.5) - 0.5$$
$$= 0.6 + 2(0.6) - 1(0.5) - 0.8 = 0.8$$

(ii) If $\vartheta'_A \leq \vartheta''_B;\ \vartheta'_B \leq \vartheta''_A;\ \vartheta''_A \geq \vartheta''_B$, then

$$td_{\vartheta(G_A*G_B)}(u_1,u_2) = td_{\vartheta(G_B)}(u_2) + P_2 td_{\vartheta(G_A)}(u_1) - (P_2-1)\vartheta'_A(u_1) - \vartheta'_A(u_1) \wedge \vartheta'_B(u_2)$$

$$td_{\vartheta(G_A*G_B)}(u_1,u_2) = 0.5 + 0.5 + 0.5 + 0.4 = 1.9$$

$$td_{\vartheta(G_B)}(u_2) + P_2 td_{\vartheta(G_A)}(u_1) - (P_2-1)\vartheta'_A(u_1) - \vartheta'_A(u_1) \wedge \vartheta'_B(u_2)$$
$$= 0.4 + 0.5 + 2(0.3+0.5) - (2-1)(0.3) - 0.3$$
$$= 0.9 + 1.6 - 0.6 = 1.9$$

Thus, $td_{(G_A*G_B)}(u_1,u_2) = \langle td_{\mu(G_A*G_B)}(u_1,u_2),\ td_{\vartheta(G_A*G_B)}(u_1,u_2)\rangle = (0.8, 1.9)$

## Conclusion:

In this paper, we have found the degree and total degree of vertices in $G_A \boxdot G_B$ and $G_A * G_B$ in terms of the degree and total degree of vertices of $G_A$ and $G_B$ under some condition and illustrated some examples. This will be very useful in studying the various properties of tensor product and normal product of two intuitionistic fuzzy graphs.

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