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SOME APPLICATION OF GRAPH THEORY & NETWORK

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Abstract: In this paper we study of directed graphs arises from making the roads into one-way streets. Use graph theory of various kinds. Also study Euler's and Hamilton paths, the shortest path of problems, network and its applications.

Keywords : 1;Graph, 2;networks, 3;source, 4;sink and 5;path.

1. Introduction:

Graph theory is the branch of Mathematics. A great Swiss Mathematician Leonerd Euler published a paper on what is now called Graph Theory.

In this paper, Eulers developed a theory to solve popular problem known as Konigsberg Bridge problem. A Graph theory is simply a set of curves called edges.

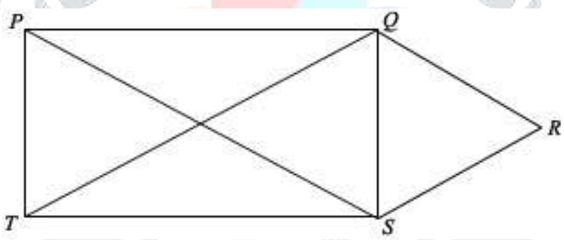

Fig. I

The points *P*, *Q*, *R*, *S* and *T* are called vertices, the lines are called edges and whole diagram is called a graph.

2. Eulerian and Hamiltonian Graphs

One of the oldest problem involving graph is the Konigsberg bridge problem. There were two island linked to each other and to the banks of the Pregel river (earlier known as Konigsberg) by seven bridges.

Shown in Fig. II. The problem was to begin at any of the four land areas to walk across each bridge. One and to return the starting point. In 1736, the great mathematician Lenhered **Euler** concluded that such a walk impossible. He used multi graph to study and solve this problem. Shown in Fig. III for the problem in which *a* and *c* represent the two river banks; *b* and *d* the two islands. The arcs joining them represent the seven bridges.

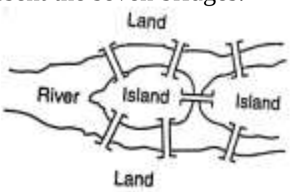

Fig. II

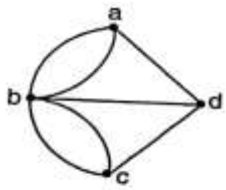

Fig. III

Konigsberg Graph

It is clear that the problem of walking each of the seven bridges exactly once and returning to the starting point is equivalent to finding a circuit in graph-III. The traverses each of the edge exactly once. **Euler** discover a very simple criterion for determining whether such a circuit exists in a graph. Euler today is considered as the father of the graph theory.

The four lands and seven bridges are represented by vertices and edges respectively in *G*. This problem is called as Konigsberg bridge problem.

(i) Eulerian Graph

A circuit in a connected graph is an **Euler circuit** if it contains every edge of the graph exactly once. A connected graph with an **Euler circuit** is called an Euler graph or Eulerian Graph.

(ii) Eulerian Path

A Eulerian path is a path which includes every edge of a graph exactly once where start and end vertices are different.

Example

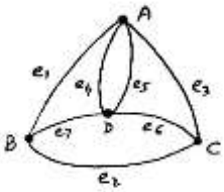

Fig. IV

$B \rightarrow A \rightarrow D \rightarrow A \rightarrow C \rightarrow B \rightarrow D \rightarrow C$ is an Eulerian path.

(iii) Eulerian circuit

It is a path in which edges are traversed and every edge included once and starting end vertices are same.

Example

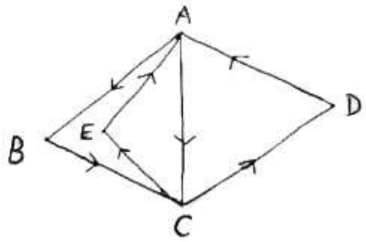

Fig. V

$A \rightarrow B \rightarrow C \rightarrow E \rightarrow A \rightarrow C \rightarrow D \rightarrow A$ is an Eulerian circuit.

(iv) Hamiltonian Graph

Hamiltonian graphs are named after Sir William Hamilton, an Irish mathematician who introduced the problems finding a circuit in which all vertices of a graph appear exactly once.

A circuit in a graph *G* that contains each vertex in *G* exactly once, except for the starting and ending vertex that appears twice is known as Hamiltonian cycle.

A Graph *G* is called a Hamiltonian cycle if it contains a Hamiltonian cycle.

A Hamiltonian path is a simple path that contains all vertices of *G* where the end points may be distinct.

Example:

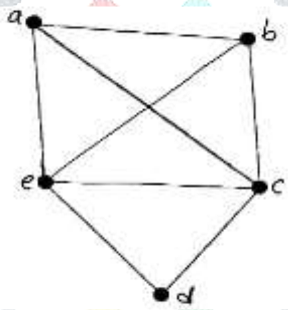

Fig. VI

there is a Hamilton circuit with vertex sequence *a*, *b*, *c*, *d*, *e*, *a*.

(v) The shortest path problem

Suppose that we have a 'map' of the form shown in Fig. VII in which the letters *A*-*L* refer to towns that are connected by roads. If the lengths of these roads are marked, what is the length of the shortest path from *A* to *L*?

Note that the numbers in the diagram need not refer to the lengths of the roads, but could refer to the times taken to travel along them, or to the costs of doing so. Thus, if we have an algorithm for solving this problem in its original formulation, then this algorithm can also be used to give the quickest or cheapest route.

Note that an upper bound for the answer can easily be obtained by taking any path from *A* to *L* and calculating its length. For example, the path $A \rightarrow B \rightarrow D \rightarrow G \rightarrow J \rightarrow L$ has total length 18 and so the length of the shortest path cannot exceed 18.

In such problems our 'map' can be regarded as a connected graph in a non-negative number is assigned to each edge. Such a graph is called a weighted graph, and the number assigned to each edge *e* is the weight of *e*, denoted by *w* €. The problem is to find a path from *A* to *L* with minimum total weight. Note that, if we have a weighted graph in which each edge has weight 1, then the problem reduces to that of finding the number of edges in the shortest path from *A* to *L*.

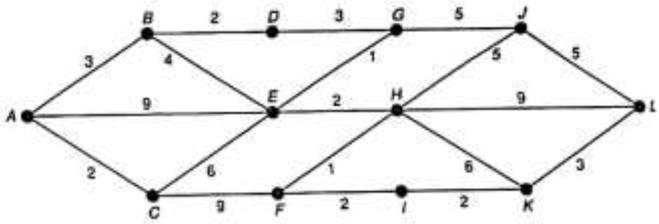

Fig. VII

3. Theorem 3. (1) (Euler 1736), A connected graph *G* is Eulerian if and only if the degree of each vertex of *G* is even.

Proof ⇔ Suppose that *P* is an Eulerian trail of *G*. Whenever *P* passes through a vertex, there is a contribution of 2 towards the degree of that vertex. Since each edge occurs exactly once in *P*, each vertex must have even degree.

⇐ The proof is by induction on the number of edges of *G*. Suppose that the degree of each vertex is even. Since *G* is connected, each vertex has degree at least 2 and so, *G* contains a cycle *C*. If *C* contains every edge of *G*, the proof is complete. If not, we remove from *G* and in which each vertex still has even degree. By the induction hypothesis, each component of *H* has an

Eulerian trail. Since each component of *H* has at least one vertex in common with *C*, by connectedness, we obtain the required Eulerian train of *G* by following the edges of *C* until a non-isolated vertex of *H* is reached, tracing the Eulerian train of the component of *H* that contains that vertex, and then continuing along the edges of *C* until we reach a vertex belonging to another component of *H*, and so on. The whole process terminates when we return to the initial vertex (see Fig. VIII).

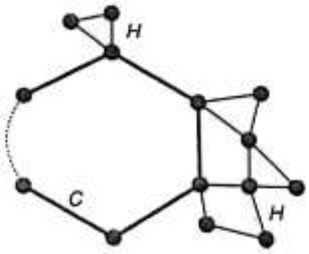

Fig. VIII

This proof can easily be modified to prove the following two results. We omit the details.

Theorem 3.2 (Ore, 1960). If *G* is a simple graph with $n(\geq 3)$ vertices, and if deg (y)+ deg $(w) \geq n$. for each pair of non-adjacent vertices *v* and *w*, then *G* is Hamiltonian.

Proof. We assume the theorem false, and derive a contradiction. So let *G* be a non-Hamiltonian graph with *n* vertices, satisfying the given condition on the vertex degrees. By adding extra edges if necessary, we may assume that *G* is 'only just' non Hamiltonian, in the sense that the addition of any further edge gives a Hamiltonian graph. (Note that adding an extra edge does not violate the condition on the vertex degrees.) It follows that *G* contains a path $v_1 \to v_2 \to \ldots \to v_n$ passing through every vertex. But since *G* is non-Hamiltonian, the vertices v_1 and v_n are not adjacent, and so deg (v_1)+ deg $(v_n) \geq n$. It follows that there must be some vertex v_i adjacent to v_1 with the property that v_{i-1} is adjacent to v_n (see Fig IX). But this gives us the required contradiction since. $v_1 \to v_2 \to \ldots \to v_{i-1} \to v_n \to v_{n-1} \to \ldots \to v_{i+1} \to v_i \to v_1$ is then a Hamiltonian cycle.

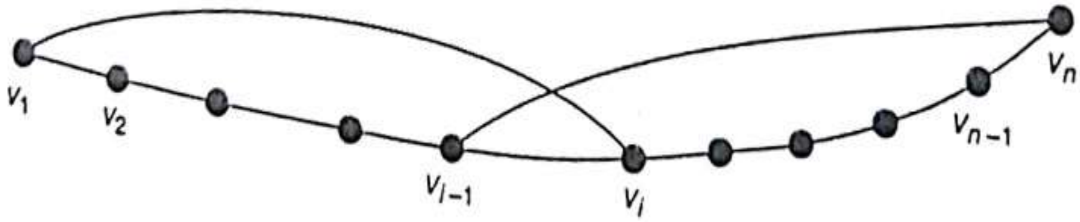

Fig. IX

4. Network:

Our society today is largely governed by Networks transportation, communication, etc. and Mathematical analysis of such networks has becomes of fundamental importance. A network or transport network is a simple, connected, weighted directed graph *N*(*V*, *E*) if

(i) There is a unique vertex $S \in V$ if it has in-degree '*O*'. This vertex is called the source.

(ii) There exists a unique vertex $t \in V$ if it has out-degree '*O*'. This vertex is called sink.

(iii) Every directed edge $e = (V,W) \in E$ has been assigned a non-negative number called the capacity of *e*, denoted by *c*€ = *c*(*V*,*W*). We can think of *c*(*e*) as representing the maximum rate at which commodity can be transported along the edge *e*.

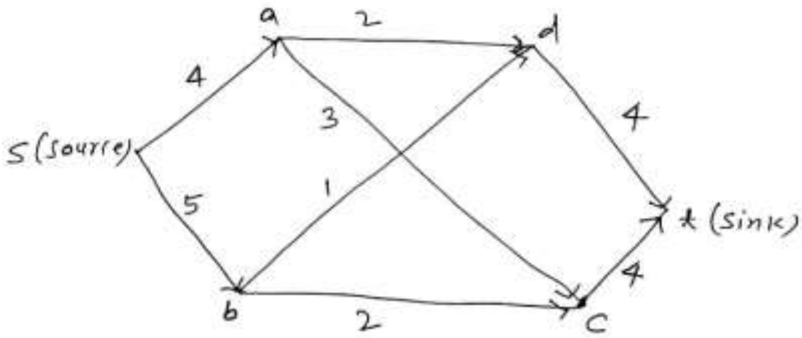

Fig. X

A network showing capacity on each arc.

Example: Find the flow for the given networks.

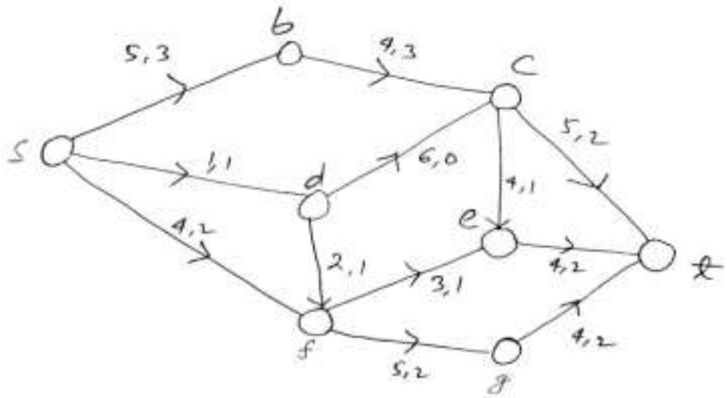

Fig. XI

Solution: The flow out of source *S* is $F_{eb}+F_{sd}+F_{sf}$ = 3+1+2 = 6

The flow into the sink *t* is $F_{gt}+F_{et}+F_{cd}$ = 2+2+2 = 6 ..

$\therefore$ Flow of the network is 6.

Conclusion:

The Graph theory has established itself as an important mathematical tool in a wide variety of subjects, ranging from operational research and chemistry to genetics and linguistics, and from electrical engineering.

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