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ANALYZING INITIAL VALUE PROBLEM SOLUTIONS: METHODS AND TRANSFORMATIONS

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Abstract : An initial value problem is an ordinary differential equation that includes initial conditions. The methods of solving initial value problems include the analytical process, numerical method and using Laplace transforms. First-order and second-order differential equations were used in this study.

***IndexTerms* – Initial value Problem, differential equation, integrating factor, exact equation, approximations.**

I. INTRODUCTION

An initial value problem is an ordinary differential equation that includes initial conditions. Integrate the provided differential equation to obtain the general solution for an initial value problem. Next, ascertain the precise constants of integration using the given initial conditions. Typically, an initial value problem can be described as follows: $f(t, y) = dy/dt$ with $y(t_0) = y_0$, where t_0 is the given point in the domain. The initial condition, which represents the value of the function y at the point t_0, is $y(t_0) = y_0$. The solution is precisely the one that passes through the point (t_0, y_0) on the ty-plane, is the solution y(t) to the initial value problem.

II. OBJECTVES

The methods of solving initial value problems include the analytical method, numerical method and making use of Laplace transforms. Analytical methods include the linear differential equation of the first order with an integrating factor, the method of variable separability, and constructing an exact equation from a differential equation. Numerical methods include Taylor's method and Picard's method.

III. METHODOLOGY

Employed the following techniques to show how to solve the first-order differential equation of first degree $dy/dt + y = 2$ with initial condition $y(0) = 1$ and second order $d^2y/dt^2 + y = t$ with initial conditions $y(0) = 1$ and $y^I(0) = 1$:

Analytical methods

- The linear differential equation of the first order with an integrating factor.
- The method of variable separability.
- Constructing an exact equation from a differential equation.
- The linear differential equation of the second order with constant coefficients.

Numerical methods

- Taylor's method.
- Picard's method

Appling Laplace transform

- To determine $L\{y(t)\}$, apply the Laplace transform on both sides and simplify. The inverse transform is used to obtain the solution.

1. Solution by Analytical Methods

1.1 linear differential equation

An equation $dy/dt + Py = Q$, P and Q are functions of variable t or constants, is called the first order linear differential equation of first degree. In this method, to find the solution, we use the following formula: $y.(I.F.) = \int (Q.(I.F.)dt + c$ …(i), where c is the constant and the integrating factor, $I.F.=\exp(\int Pdt)$.

Take the equation $dy/dt = 2-y$ with the initial condition $y(0)=1$. As a linear differential equation rewrite this as $dy/dt + y = 2$. Here, $P = 1$ and $Q = 2$.The formulas above can be used to get, $I.F.=\exp(\int 1.\,dt) = \exp(t)$

By (i), solution is $y.\exp(t) = \int 2.\exp(t)dt + c = 2.\exp(t) + c$.

$c = -1$,when $t = 0$ & $y = 1$.Hence,$y.\exp(t) = 2.\exp(t)-1$.

On simplifying, $y = 2-\exp(-t) = 1+ t/1!-t^2/2! + t^3/3!- t^3/4! +...$,is the result.

1.2 Separation of variables

Given a first-order differential equation, sometimes we can separate the variables. In this case, obtain the solution by direct integration.

Here, $dy/dt = 2-y$ can be written as $dy/2-y = dt$.

Integrating, $-\log(2-y) = t + c$ or, $\log(2-y) = -(t + c)$.

$y(0) = 1$ shows that $c = 0$.

Simplifying, $y = 2-\exp(-t) = 1+ t/1!-t^2/2! + t^3/3!- t^3/4! +...$, is the result.

1.3 By constructing an exact equation.

A differential equation of the form $M\,dt + N\,dy=0$ …(ii), where M and N are functions of t and y is called an exact equation if the partial derivative of M with respect to y is equal to the partial derivative of N with respect to t. The formula for finding the solution is $\int M\,dt$ (Keep y as costant in M) $+ \int N\,dy$ (terms in N not contatining t) $= c$. If the given equation is not exact, multiply the integrating factor (I.F.) to (ii) convert it to an exact equation and find the solution using formula mentioned above.

$(2-y)\,dt + dy = 0$ …(iii)

Comparing (ii) with (iii), $M = 2-y$, $N = 1$. Also, $\partial M/\partial y = 1$, $\partial N / \partial t = 0$.

This equation is not exact because the partial derivatives $\partial M/\partial y \neq \partial N/\partial t$. To make this as an exact equation, consider the function $1/N(\partial M/\partial y - \partial N/\partial t) = 1$. Therefore, $I.F.=\exp(\int 1.\,dt) = \exp(t)$.

By (iii),$(2-y)\exp(t)\,dt + \exp(t)\,dy =0$…(iv).

Solution of (iv) is $\int (2-y)\exp(t)\,dt = c$.

$c=1$,when $t=0$ & $y=1$.

∴ Solution reduces to $y= 2-\exp(-t)=1+t/1!-t^2/2!+ t^3/3!- t^3/4!+...$

1.4 Homogeneous linear differential equation with constant coefficients

The solution of the second order differential equation is the complementary function(solution of the auxiliary equation).Consider $d^2y/dt^2 + y = 0$ with initial conditions $y(0) = 1$ and $y^I(0) = 1$.

Auxiliary equation is $m^2 + 1 = 0 \Rightarrow m = \pm 1$. Therefore, $y(t) = A\cos x + B\sin x$ … (a) and $y^I(t) = -A\sin x + B\cos x$ …(b)

By (a) and (b), $y(0) = 1$ and $y^I(0) = 1 \Rightarrow A=1$ and $B =1$. Hence, $y(t) = \cos x + \sin x$, is the result.

2. Solution by numerical Method

2.1 Taylor's method.

Taylor's expansion is $y(t)= y_0 + (t-t_0)/1!\,(y_0)^I + (t-t_0)^2/2!\,(y_0)^{II} + (t-t_0)^3/3!\,(y_0)^{III} + (t-t_0)^4/4!\,(y_0)^{IV} + \ldots$ (v) ,where $y_0 = y(t_0)$.

Here, we rewrite $dy/dt =2-y$ and $y(0)=1$ as $y^I(t) = 2-y(t)$ and $y_0 =1$ and $t_0 =0$.

Calculate successively, $(y_0)^I$,$(y_0)^{II}$,$(y_0)^{III}$etc.

$y^I(t) = 2-y(t)$ gives $(y_0)^I = 2-y_0 =2-1=1$

$y^{II}(t) = -y^I(t)$ gives $(y_0)^{II} =-(y_0)^I = -1$

$y^{III}(t) = -y^{II}(t)$ gives $(y_0)^{III} = -(y_0)^{II} = 1$

$y^{IV}(t) = -y^{III}(t)$ gives $(y_0)^{IV} = -(y_0)^{III} = -1$ and so on.

By (v), $y(t) = 1 + (t-0)/1!\,(1) + (t-0)^2/2!\,(-1) + (t-0)^3/3!\,(1) + (t-0)^4/4!\,(-1) + \ldots. = 1 + t/1!-t^2/2!+ t^3/3!- t^3/4!+... +...$, is the result.

2.2 Picard's method.

The formula for finding successive approximation in Picard's method is $y_n = y_0 + \int_{t_0}^{t} f(t,y)dt$…(vi) where $f(t,y_{n-1}) = dy/dt$.

Here, $dy/dt = 2-y$ and $y(0) =1$ implies $f(t,y)=2-y$ and initial approximation $y_0 =1$. By applying the formula (vi), successive approximations can be obtained.

First approximation is,

$y_1 = y_0 + \int_{t_0}^{t} f(t, y_0)dt$

$=1+\int_0^t f(t,1)dt$

$=1+\int_0^t (2-1)dt$

$=1+t = 1+t/1!$

Second approximation,

$y_2 = y_0 + \int_{t_0}^t f(t,y_1)dt$

$= 1+\int_0^t f(t,1+t)dt$

$=1+\int_0^t (2-(1+t))dt$

$=1+ t- t^2/2 = 1+t/1!-t^2/2!.$

Third approximation,

$y_3 = y_0 + \int_{t_0}^t f(t,y_2)dt = 1+\int_0^t f(t,1+t+t^2/2)dt$

$=1+\int_0^t (2-(1+t+t^2/2))dt$

$=1+ t-t^2/2 + t^3/6 = 1 + t/1!-t^2/2! + t^3/3!$ and so on.

n^{th} approximation , $y_n = 1 + t/1!-t^2/2!+ t^3/3!- t^3/4!+... +(-1)^{n-1}t^n /n!+...$

In general, $y(t)= 1 + t/1!-t^2/2!+ t^3/3!- t^3/4!+...$, is the solution.

3.Solution by using Laplace transform

3.1 Apply Laplace Transform on both sides, for the differential equation $dy/dt = 2-y$ with the initial condition $y(0)=1$.

$$L\left\{\frac{dy}{dt}\right\} + L\{y(t)\} = L\{2\}$$

$$\Rightarrow sL\{y(t)\} - y(0) + L\{y(t)\} = \frac{2}{s}$$

$$\Rightarrow sL\{y(t)\} - 1 + L\{y(t)\} = \frac{2}{s}$$

$$\Rightarrow (s+1)L\{y(t)\} - 1 = \frac{2}{s}$$

$$\Rightarrow (s+1)L\{y(t)\} = \frac{2}{s} + 1$$

$$\Rightarrow L\{y(t)\} = \frac{2+s}{s(s+1)} = \frac{2+2s-s}{s(s+1)} = \frac{2(s+1)-s}{s(s+1)} = \frac{2}{s} - \frac{1}{s+1}$$

$$\therefore y(t) = L^{-1}\left\{\frac{2}{s}\right\} - L^{-1}\left\{\frac{1}{s+1}\right\}$$

$=2-\exp(-t) = 1+ t/1!-t^2/2! + t^3/3!- t^3/4! +...$, is the solution.

3.2 Apply Laplace Transform on both sides, for the second order differential equation $d^2y/dt^2 + y = t$ with initial conditions $y(0) = 1$ and $y^I(0) = 1$.

$$L\left\{\frac{d^2y}{dt^2}\right\} + L\{y(t)\} = 0$$

$$\Rightarrow s^2L\{y(t)\} - sy(0) - y^I(0) + L\{y(t)\} = 0$$

$$\Rightarrow s^2L\{y(t)\} - s - 1 + L\{y(t)\} = 0$$

$$\Rightarrow (s^2+1)L\{y(t)\} = s + 1$$

$$\Rightarrow L\{y(t)\} = \frac{s}{s^2+1} + \frac{1}{s^2+1}$$

$$\therefore y(t) = L^{-1}\left\{\frac{s}{s^2+1}\right\} + L^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$= \cos t + \sin t$, is the solution.

IV. CONCLUSION

In this paper, we showed how to apply the analytical method, numerical method and Laplace transforms to find the solution of first order differential equations of first degree and second order differential equations. The paper emphasizes the usage of various methods to verify the result. We encourage everyone to work through a variety of differential equation problems including first and second order solutions.

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