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FUZZY PRE- Z-CONTINUOUS FUNCTIONS IN FUZZY TOPOLOGICAL SPACES

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Abstract: In this paper, we introduced the classical form of Z-continuous functions under the fuzzy setting in the name of fuzzy pre- Z-continuous functions in fuzzy topological spaces. We discussed fuzzy pre- Z-continuous functions and their properties and characterizations.

Index Terms – fuzzy continuous, fuzzy z continuous, fuzzy z- pre- continuous, fuzzy pre- z-continuous.

INTRODUCTION

Fuzzy set theory and fuzzy topology are approached as generalization of ordinary set theory and ordinary topology. Fuzzy topologyis one branch of mathematics, which combines the ordered structure with the topological structure. Topological relation between spatial objects is used in ageographic information system with positional and attributes information. Topological relations can be crisp or fuzzy depending on the certainty or uncertainty of spatial objects and the nature of their relations. Therefore fuzzy sets provide a useful tool to describe the uncertainty of topological properties of figures and surfaces.

After the discovery of fuzzy sets by Zadeh in 1965, much attention has been paid to generalize the basic concepts of the topological theory in thefuzzy setting. In 1968, C.L. Chang [1] initiated a study on fuzzy topological spaces and defined its properties. Fuzzy sets on the universe X will be denoted by greek letters μ, γ, ζ etc.,

The concept of fuzzy pre-open and fuzzy semi open sets were studied by S. Thakur and S. Singh [7]. In 1991, Bin Shahan [8] have defined and studied the concept of fuzzy pre-continuity. Navalagi [4] introduced the pre-zero sets and copre-zero sets of a space with the help of pre-continuous functions. A weaker form of continuity called Z-continuous was introduced by Singal and Nimse [5] in 1997. We already introduced fuzzy Z - pre continuous functions in fuzzy topological spaces, now we introduced fuzzy pre- Z- continuous functions in fuzzy topological spaces.

1. PRELIMINARIES

DEFINITION 1.1

A family $T \subseteq I^X$ of fuzzy sets is called a fuzzy topology for X if it satisfies the following three axioms:
1. $0, 1 \in T$
2. If λ, $\mu \in T$, then $\lambda \wedge \mu \in T$
3. If $\lambda_i \in T$ $\forall i \in I$, then $\vee \lambda_i \in T$

The pair (X, T) is a fuzzy topological space. Every member of T is called fuzzy open sets. The complements of fuzzy open sets are called fuzzy closed sets.

DEFINITION 1.2

A fuzzy set in X is called a fuzzy point if and only if it takes the value 0 $\forall$ y $\in$ X except one, say x $\in$ X. If its value at x is λ $(0 < \lambda \leq 1)$ then the fuzzy point is denoted by p_x where the point x is called its support.

DEFINITON 1.3

Let (X, T) and (Y, S) be two topological spaces. Any function f: $(X, T) \to (Y, S)$ is said to be **fuzzy continuous** if $f^{-1}(\lambda)$ is fuzzy openin (X, T) for each fuzzy open set λ in (Y, S).

DEFINITION 1.4

Let (X, T) and (Y, S) be any two topological spaces. A function f : $(X, T) \to (Y, S)$ is said to be **fuzzy Z-continuous** if the inverse imageof every fuzzy cozero set of (Y, S) is fuzzy open in (X, T).

DEFINITION 1.5

Let (X, T) and (Y, S) be any two topological spaces. A function f : $(X, T) \to (Y, S)$ is said to be **fuzzy Z-pre-continuous** if for every fuzzycozero set γ of (Y, S) containing $f(x_t)$, there exists a fuzzy preopen set μ in (X,T) such that $x_t \in \mu$ and $f(\mu) \le \gamma$.

2. Fuzzy pre Z - continuous

DEFINITION 2.1

Let (X, T) and (Y, S) be any two topological spaces. A function f : $(X, T) \to (Y, S)$ is said to be **fuzzy pre- Z- continuous** if for everyfuzzy copre-zero set γ of (Y, S) containing $f(x_t)$, there exists a fuzzy open setμ in (X, T) such that $x_t \in \mu$ and $f(\mu) \le \gamma$.

REMARK 2.2

Every fuzzy pre- Z-continuous function is fuzzy Z-continuous, (sinceevery fuzzy cozero set is a fuzzy copre-zero set).

REMARK 2.3

The following implications between fuzzy continuous functions are true and the converse need not be true.

THEOREM 2.4

Let (X, T) and (Y, S) be any two topological spaces. For a function f: $(X, T) \to (Y, S)$, the following statements are equivalent:

(a) f is fuzzy pre- Z-continuous.
(b) The inverse image of every fuzzy copre-zero set of (Y, S) is fuzzy open in (X, T).
(c) The inverse image of every fuzzy pre-zero set of (Y, S) is fuzzy closed in (X, T).

Proof :

(a) ⇒ (b)

Let γ be any fuzzy copre-zero set in (Y, S).
For each $x_t \in f^{-1}(\gamma)$ we have by (a),
There exists a fuzzy open set $\mu \in I^X$ such that $x_t \in \mu$ and $f(\mu) \le \gamma$.
$\Rightarrow x_t \in \mu \le f^{-1}(\gamma)$.
Thus $f^{-1}(\gamma)$ is a fuzzy neighbourhood of each of its points.
Therefore, $f^{-1}(\gamma)$ is fuzzy open in (X, T).

(b) ⇒ (c)

Let ζ be any fuzzy pre-zero set in (Y, S).
Then $1 - \zeta$ is a fuzzy copre-zero set in (Y, S).
Consider, $1 - f^{-1}(\zeta) = f^{-1}(1) - f^{-1}(\zeta)$ and by (b),
$f^{-1}(1-\zeta) = 1 - f^{-1}(\zeta)$ is fuzzy open in (X, T).
Therefore, $f^{-1}(\zeta)$ is fuzzy closed in (X, T).

(c) ⇒ (a)

Let $x_t \in$ (X, T).
Let γ be any fuzzy copre-zero set in (Y, S) containing $f(x_t)$.
Then $x_t \notin 1 - \gamma$ and $1 - \gamma$ is a fuzzy pre-zero set.
Since, $f^{-1}(1-\gamma) = f^{-1}(1) - f^{-1}(\gamma)$.
by (c), $f^{-1}(1-\gamma) = f^{-1}(\gamma)$ is a fuzzy closed set and $x_t \notin 1 - f^{-1}(\gamma)$.
Thus $f^{-1}(\gamma)$ is a fuzzy open set containing x_t and $f(f^{-1}(\gamma)) \le \gamma$.
Therefore, f is fuzzy pre- Z-continuous.

THEOREM 2.5

Let (X, T) and (Y, S) be any two topological spaces. For a function f: $(X, T) \to (Y, S)$, the following statements are equivalent:

(a) f is fuzzy pre Z-continuous.

(b) The inverse image of every fuzzy pre-zero set of (Y, S) is fuzzy zero in (X, T).

(c) The inverse image of every fuzzy copre-zero set of (Y, S) is fuzzy cozero in (X, T).

Proof:

(a)⇒ (b)

Let f be fuzzy pre- Z-continuous.

Let μ be any fuzzy pre-zero set in (Y, S).

Then there exists a fuzzy pre-continuous function g: S → I such that, $\mu = \vee_{i\in J} \{\lambda_i \in I^{Y} : g(\lambda_i) = 0, \lambda_i \in S^{'}, \lambda_i \neq 1\}$

We claim that $g \circ f$: (X, T) → (Y, S) is fuzzy continuous.

Let γ be any fuzzy closed set in I with $\gamma \neq 1$.

Since every fuzzy closed set is a fuzzy zero set we have, γ is a fuzzy zero set and hence there exists a fuzzy continuous function h: I → I such that

$\gamma = \vee_{i\in J} \{\lambda_i \in I^{Y} : h(\lambda_i) = 0, \lambda_i \in I^{'}, \lambda_i \neq 1\}$.

As g is fuzzy pre-continuous, we have $h \circ g$: (Y, S) → I is fuzzy pre-continuous and

$(h \circ g)^{-1}\{0\} = g^{-1}(h^{-1}\{0\}) = g^{-1}(\gamma)$,

Where

$g^{-1}(\gamma)$ is a fuzzy pre-zero set in (Y, S).

Now, $(g \circ f)^{-1}(\gamma) = f^{-1}(g^{-1}(\gamma))$ is fuzzy closed in (X, T) as f is fuzzy pre Z-continuous.

Hence $g \circ f$ is fuzzy continuous.

Now $f^{-1}(\mu) = f^{-1}(g^{-1}\{0\}) = (g \circ f)^{-1}\{0\}$ is a fuzzy zero set in (X, T) as $g \circ f$ is continuous.

Hence the inverse image of every fuzzy pre-zero set of (Y, S) is a fuzzy zero set in (X, T).

(b)⇒ (c)

Let μ be any fuzzy copre-zero set in (Y, S).

Thus 1 - μ is a fuzzy pre-zero set in (Y, S).

$\Rightarrow f^{-1}(1-\mu)$ is a fuzzy zero set in (X, T).

Consider, $1 - f^{-1}(\mu) = f^{-1}(1) - f^{-1}(\mu)$.

$\Rightarrow 1 - f^{-1}(\mu) = f^{-1}(1-\mu)$ is a fuzzy zero set in (X, T).

Therefore, $f^{-1}(\mu)$ is a fuzzy cozero set in (X, T).

(c) ⇒ (a)

Let μ be a fuzzy copre-zero set in (Y, S).

$\Rightarrow f^{-1}(\mu)$ is a fuzzy cozero set in (X, T).

$\Rightarrow f^{-1}(\mu)$ is a fuzzy open set in (X, T).

∴ f is fuzzy pre- Z-continuous.

THEOREM 2.6

Let (X, T) and (Y, S) be any two topological spaces, for a function f: $(X, T) \to (Y, S)$, the following statements are equivalent:

(a) f is fuzzy pre- Z-continuous.

(b)The inverse image of every fuzzy pre Z-open set of (Y, S) is fuzzy Z-open in (X, T).

(c)The inverse image of every fuzzy pre Z-closed set of (Y, S) is fuzzy Z-closed in (X, T).

Proof.

(a) ⇒ (b)

Let f be fuzzy pre- Z-continuous.

Let λ be any fuzzy pre Z-open set in (Y, S).

Then there exists a fuzzy copre-zero set μ in (Y, S) such that $\mu \leq \gamma$.

Since the inverse image of every fuzzy copre-zero set of (Y, S) is fuzzy cozero in (X, T), we have

$f^{-1}(\mu)$ is fuzzy cozero in (X, T).

$\Rightarrow f^{-1}(\mu) \leq f^{-1}(\lambda)$ is fuzzy cozero in (X, T).

Thus $f^{-1}(\lambda)$ is fuzzy Z-open in (X, T).

Hence the inverse image of every fuzzy pre Z-open set of (Y, S) is fuzzy Z-open in (X, T).

(b) ⇒ (c)

Let μ be any fuzzy pre Z-closed set in (Y, S).

Thus 1 - μ is a fuzzy pre Z-open set in (Y, S).

$\Rightarrow f^{-1}(1-\mu)$ is a fuzzy Z-open set in (X, T).

Consider, $1 - f^{-1}(\mu) = f^{-1}(1) - f^{-1}(\mu)$.

$\Rightarrow 1 - f^{-1}(\mu) = f^{-1}(1-\mu)$ is fuzzy Z-open in (X, T).

$\therefore f^{-1}(\mu)$ is fuzzy Z-closed in (X, T).

Hence the inverse image of every fuzzy pre Z-closed set of (Y, S) is fuzzy Z-closed in (X, T).

(c) ⇒ (a)

Let μ be a fuzzy pre Z-closed set in (Y, S).

$\Rightarrow f^{-1}(\mu)$ is a fuzzy Z-closed set in (X, T).

$\Rightarrow 1 - f^{-1}(\mu)$ is a fuzzy Z-open set in (X, T).

$\Rightarrow 1 - f^{-1}(\mu)$ is a fuzzy cozero set in (X, T).

$\Rightarrow f^{-1}(\mu)$ is a fuzzy zero set in (X, T).

$\Rightarrow f^{-1}(\mu)$ is a fuzzy open set in (X, T).

Hence f is fuzzy pre- Z-continuous.

THEOREM 2.7

Let (X, T) and (Y, S) be any two topological spaces. A function f: (X, T) → (Y, S) is fuzzy pre- Z-continuous if and only if for each $\lambda \in (X, T)$, $f(\lambda_z) \leq (f(\lambda))_{pz}$.

Proof :

Let f be fuzzy pre- Z-continuous.

Then the inverse image of every fuzzy pre Z-closed set of (Y, S) is fuzzy Z-closed in (X, T).

Let $\lambda \in (X, T)$, then $(f(\lambda))_{pz}$ is fuzzy pre Z-closed in (Y, S).

Thus $f^{-1}((f(\lambda))_{pz})$ is fuzzy Z-closed in (X, T).

Now $f(\lambda) \leq (f(\lambda))_{pz}$

$\Rightarrow \lambda = f^{-1}(f(\lambda)) \leq f^{-1}((f(\lambda))_{pz})$

$\Rightarrow \lambda_z = f^{-1}(f(\lambda))_z \leq (f^{-1}(f(\lambda)_{pz}))_z = f^{-1}(f(\lambda))_{pz}$

$\Rightarrow f(\lambda_z) \leq (f(\lambda))_{pz}$

Conversely,

Let μ be a fuzzy pre Z-closed set in (Y, S).

Then $f(f^{-1}(\mu)_z) \leq (f(f^{-1}(\mu)))_{pz} \leq \mu_{pz} = \mu$

$\Rightarrow f^{-1}(\mu)_z = f^{-1}(\mu)$

$\Rightarrow f^{-1}(\mu)$ is fuzzy Z-closed.

Hence f is fuzzy pre Z-continuous.

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