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Three Stage Sequential Procedure for Minimum Risk Point Estimation

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Abstract: A distributional relationship among the estimators of parameters of interest and those of nuisance parameters involved therein under different continuous probability models was developed by Chaturvedi. A., Pandey S.K, and Gupta M [5This distributional relationship was exploited to develop classes of sequential procedures for the estimation problems considered. In the present paper, the classes for three-stage point estimation procedures are proposed for the estimation problems of minimum risk point estimation.

INTRODUCTION

Hall [4] developed a time-stage procedure to construct a confidence interval of preassigned width and coverage probability for a normal mean, assuming variance to be unknown. He established asymptotic properties of the estimation procedure and showed that it combines the advantages of the two-stage procedure of Stein [6] and purely sequential procedures of Ray [5], Anscombe [1] and Chow and Robbins [3] to deal with the same estimation problem. Similar procedures to deal with other fired width (fixed-size) confidence interval (confidence-region) estimation problems have been developed and studied by various researchers In the present paper, the classes for three-stage point estimation procedures are proposed for the estimation problems of minimum risk point estimation. We study the asymptotic properties of the proposed classes of three-stage schemes and the asymptotic expansion of the risk. The set up [2] of the problem is:

$\mathbf{X}_1, \ldots, \mathbf{X}_n$ be a random sample of size $n(\geqslant t+1)$, from a t variate continuous population, with parameter θ of order $t \times 1$ of interest and Ψ a scalar unknown parameter, let $(\theta', \Psi)' \in R^t \times R^+$. The estimators of θ and Ψ are $\hat{\theta}_n = \hat{\theta}(\mathbf{X}_1, \ldots, \mathbf{X}_n)$ and $\hat{\Psi}_n = \hat{\Psi}(\mathbf{X}_1, \ldots, \mathbf{X}_n)$. The following hypotheticals are made

(A,): A known positive definite matrix Q, of order t by t, a number $\delta \in (0,1]$ and a positive integer $r \geq 1$ exist ,s.t. $n[\psi^{-1}(\theta_n - \theta)'Q(\theta_n - \theta)]^\delta \sim \chi^2_{(r)}$

(A_2) : $\hat{\theta}_n$ and $\hat{\Psi}_n$ are independent for all values of n.

(A_3) : For integers $s(\geqslant 1)$,then for all n greater than or equal to s+1,

$r(n-s)\hat{\Psi}_n/\Psi = \sum_{j=1}^{n-s} Z_j^{(r)}$

where $Z_j^{(r)}$'s are- iid rv's with $Z_j^{(r)} \sim \chi^2_{(r)}$.[2]

(A_4): $\hat{\Psi}_n$ is a consistent estimator of

Minimum risk corresponding to Loss function L = A$[(\theta_n - \theta)'Q(\theta_n - \theta)]^\alpha$ +C n^t

R= $(1+{}^{\delta t}/_{\alpha})C{n^t}_0$

In the absence of any knowledge about ψ,risk can not be minimized by fixed sample size procedure. A class of three stage procedure is is derived for the purpose. For this we propose following three stage procedure:

Let $[y]^+$ denote the positive integral part of y. The class C_A^* of three stage procedure is as follows:

Start with a sample of size m$\geq max.\{s+1, t+1\}$,;m is chosen in a way that m=$o(C^{-\delta}/_{\alpha+\delta t})$ and $\lim\operatorname{Sup}_{C\to 0}(\frac{m}{n_0})<1$

Based on these m observations, compute $\hat{\psi}_{\mathrm{m}}$ and for specified $0<\eta<1$, take a second stage sample of size M,where

$$M = \max\cdot\left\{m.\left[\eta\left\{\frac{k(\alpha,\delta,r)}{ct}\right\}_m^{\delta/(\alpha+\delta t)}\hat{\psi}^{\alpha}_{m/(\alpha+\delta t)}\right]^+ +1\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.1)$$

Then, the final (third) stage sample size being given by

$$N = \max\cdot\left\{M,\left[\left\{\frac{k(\alpha,\delta,r)}{ct}\right\}^{\delta/(\alpha+\delta t)}\cdot\hat{\psi}_M^{\alpha/(\alpha+\delta t)}\right]^+ +1\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.2)$$

After stopping, $\hat{\theta}_{\mathrm{N}}$ is used for estimating $\underline{\theta}$ -

Utilizing (A_2), the risk corresponding to the class C_T^* of three-stage procedures (1.1) - (1.2)is again same as that of pure sequential procedure, given by

$$R_N(C) = (Ct)n_0^t E\left\{\left(\frac{n_0}{N}\right)^{\alpha/\delta}\right\} + CE(N^t)\ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.3)$$

with N determined by the present rule. The 'regret' of the class C_T^* is defined by

$$R_g(C) = R_N(C) - R_{n_0}(C) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.4)$$

Lemma 1.1: For the three-stage procedure (1.1) $-$(1.2), as $C\to 0$

$$E(N) = n_0 - \frac{\alpha s}{\eta(\alpha+\delta t)} + \frac{1}{2} + o(1)\ldots \quad \ldots\ldots.\ldots\ldots. (1.5)$$

and,

$$E(N^2) = n_0^2 + \left\{\frac{\eta(\alpha+\delta t)-2\alpha s}{\eta(\alpha+\delta t)}\right\}n_0 + \frac{\eta^2(\alpha+\delta t)^2+3\alpha s(\alpha+\delta t)}{3\eta^2(\alpha+\delta t)^2} + \frac{3\alpha(\alpha-\delta t)s^2-3\alpha s\eta(\alpha+\delta t)}{3\eta^2(\alpha+\delta t)^2}$$

$$+o\left(c^{\delta/(\alpha+\delta t)}\right)$$

Proof: By the

$$E(N) = E\left[NI\left(\{M\leq m\}U\left\{N\leq\left[\eta\left(\frac{k(\alpha,\delta,r)}{ct}^{\delta}/_{\alpha+\delta t}\right)\cdot\hat{\psi}_m^{\alpha/\alpha+\delta t}\right]^+ +1\right\}\right]+E\left[NI\left(\left[\frac{k(\alpha,\delta,r)}{ct}^{\delta}/_{\alpha+\delta t}\right)\hat{\psi}_M^{\alpha/(\alpha+\delta t)}\right]^+ +1>M\right]$$

$$= \text{I+II} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.7)$$

. as $C\to 0$, $I=O(1)$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (1.8)

$$II = E\left[\left\{\frac{k(\alpha,\delta,r)}{ct}\right\}^{\delta/\alpha+\delta t}\cdot\hat{\psi}_M^{\alpha/(\alpha+\delta t)}+\beta_M\right]\ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.9)$$

Where

$$\beta_M = 1 \quad -\left\{\left(\frac{k(\alpha,\delta,r)^{\delta/(\alpha+\delta t)}}{ct}\right)^{\delta/(\alpha+\delta t)}\cdot\hat{\psi}_M^{\alpha/(\alpha+\delta t)}\right\}-\left[\left(\frac{k(\alpha,\delta,r)}{ct}\right)^{\delta/(\alpha+\delta t)}\cdot\hat{\psi}_M^{\alpha/(\alpha+\delta t)}\right]^+$$

It follows from Hall (1981), that as $M \to \infty$,

$\beta_M \xrightarrow{L} U(0,1)$

Thus ,we obtain from (1.9) that, as $C \to 0$

$$II = \left\{\frac{k(\alpha,\delta,r)}{ct}\right\}^{\delta/(\alpha+\delta t)} \cdot \dot{E}\left\{\hat{\psi}_M^{\alpha/(\alpha+\delta t)}\right\} + \frac{1}{2} \quad \text{……………………… (1.10)}$$

Using (A3) value of $E\{\hat{\psi}_M^K\}$, for K > 0.

$$\bar{E}\left(\hat{\psi}_M^K\right) = \psi^K E\left[\sum_{l=0}^K (l_l^K)\left(\sum_{j=1}^{m=s} z_j(q)\right)^l \{q(M-m)\}^{-l} \cdot \{1 + O(M^{-1})\}\right] + O(M^{-1})$$

$$= \psi^K E\left[\sum_{l=0}^K \binom{k}{l}\left(\sum_{j=1}^{m=s} z(q),\right)_j q^{-l} M^{-l} + o(M^{-1})\right. \quad \text{…………… …….. (1.11)}$$

(Now, expanding M^{-l} around ηn_0 by Taylor's expansion

$$M^{-l} = (\eta n_{0)})^{-l} - l(\eta n_{0)})^{-l-1} + R_{n_0} \quad \text{……………….. (1.12)}$$

where R_{n_0} is the remalnder term.

From (1.11) and (1.12) are gets

$$\bar{E}\{\hat{\psi}_M^K\} = \psi^K \left[\sum_{l=0}^K (l^K)\left(\sum_{j=1}^m z_j^{(q)}\right)^l q^{-l}(\eta n_0)^{-l} - l_{\Sigma=0}^k\, l(l_l^K)\left(\sum_{j=1}^{m-s} z_j^{(q)}\right)^{l+\alpha/(\alpha+\delta t)} \frac{q^{-l}(\eta n_0)^{-l}}{\{q(m-\delta)\}^{\alpha/(\alpha+\delta t)}} - \right.$$

$$\left.\sum_{l=0}^k l(l_l^k)\left(\sum_{j=1}^{m=\infty}\left(\sum_{j=1}^{m-s} z_j^{(q)}\right)^l\right) \text{-}q^{-l}(\eta_0)^{-l-1}\beta_m + \sum_{l=0}^k l(l_l^k) C^l q^{-l}(\eta n_0)^{-l}\right] + R_{n_0} \quad \text{… … … (1.13)}$$

It follows from Cauchy - Schwartz inequality

$$\text{Cov.}^{\,2}\left[\beta_m, \sum_{j=1}^{m-s} z_j^{(q)}\right] \le Var.(\beta_m) Var.\left(\sum_{j=1}^{m-s} z_j^{(q)}\right)$$

$$= \frac{\mathrm{q(m-s)}}{6} = \mathrm{o}(1)[\text{ by the cholce of } m]$$

Implying that β_m and $\left(\sum_{j=1}^{m-s} z_j^{(q)}\right)$are asymptotically uncorrelated. Utilizing this result and (1.10), we obtain from (1.13)

Utilizing (1.14), we obtain on combining (1.8) and (1.10)

$$E(N) = n_0\left\{\left\{1 + \frac{m-s}{\eta n_0}\right\}^{\alpha/(\alpha+\delta t)} - \frac{\alpha(m-s)}{2(\alpha+\delta t)} \frac{1}{(\eta n_0)^2}\left\{1 + \left(\frac{m-s}{\eta n_0}\right\}^{\alpha/(\alpha+\delta t)-1}\right]\right. + 0(1)$$

$$= n_0\left[1 - \frac{\alpha s}{(\alpha+\delta t)(\eta n_0)}\right] + \frac{1}{2} + o(1).$$

and (1.5) follows.

In order to prove (1.6), we have

$$E(N^2) = E\left[\left\{\frac{k(\alpha,\sigma,r)}{ct}\right\}^{\delta/\alpha+\delta t} \cdot \hat{\psi}_M^{\alpha/(\alpha+\delta t)} + \beta_M\right]^2$$

$$= \left\{\frac{K(\alpha,\delta,r)}{ct}\right\}^{2\delta/\alpha+\delta t} E\left(\hat{\psi}_M^{2\alpha/\alpha+\delta t}\right) + E(\beta_M^2) + 2\left\{\frac{K(\alpha,\delta,r)}{ct}\right\}^{2\delta/\alpha+\delta,t} E\left\{\beta_M \hat{\psi}_M^{\alpha/(\alpha+\delta,t)}\right\} \quad \text{........(1.15)}$$

.

Utilizing (1.14), we get.

$$\left\{\frac{K(\alpha,\delta,r)}{ct}\right\}^{2\alpha/(\alpha+\delta t)} E(\psi_M)^{2\alpha/(\alpha+\delta t)}$$
$$= n_0^2\left[\left\{1+\frac{m-s}{\eta n_0}\right\}^{2\alpha/(\alpha+\delta t)} - \frac{\alpha}{(\alpha+\delta t)(\eta n_0)}\frac{m-s}{(\eta n_0)}\right] + o(m^{-1})$$

$$= n_0^2\left[1 - \frac{2\alpha s}{\eta n_0(\alpha+\delta t)} + \frac{\alpha(\alpha-\delta t)s^2}{\eta^2(\alpha+\delta t)^2 n_0^2} + \frac{\alpha s}{(\alpha+\delta t)(\eta n_0)^2}\right] + 0(m^{-1})$$

$$= n_0^2 - \frac{2\alpha s}{\eta(\alpha+\delta t)}n_0 + \frac{\alpha(\alpha-\delta t)s^2}{\eta^2(\alpha+\delta t)^2} + \frac{\alpha s}{\eta^2(\alpha+\delta t)} + 0(m^{-1}) \ldots\ldots\ldots \quad (1.16)$$

similarly, we can prove that

$$2\left\{\frac{k(\alpha,\delta,r)}{Ct}\right\}^{\delta/(\alpha+\delta t)} E\left(\beta_M \hat{\psi}_M^{\alpha/(\alpha+\delta t)}\right)$$
$$= 2\left\{\frac{k(\alpha,\delta,r)}{ct}\right\}^{\delta/(\alpha+6t)} E(\beta_M)E\left(\hat{\psi}_M^{\alpha/(\alpha+\delta t)}\right)$$
$$= 2\cdot\frac{1}{2}n\left[\left\{1+\frac{m-s}{n_0}\right\}^{\alpha/((x+\delta t)} + 0(m^{-1})\right.$$
$$= n_0 - \frac{\alpha s}{\eta(\alpha+\delta t)} + o\left(c^{\frac{\delta}{\alpha+\delta t}}\right) \ldots\ldots\ldots \quad (1.17)$$

Making substitutions from (1.16) and (1.17) in (1.15), gives

$$E(N^2) = n_0^2 - \frac{2asn_0}{\eta(\alpha+\delta t)} + \frac{as}{\eta^2(\alpha+\delta t)^2} + \frac{\alpha(\alpha-\delta t)s^2}{\eta^2(\alpha+\delta t)^2}$$
$$+n_0 - \frac{\alpha s}{\eta(\alpha+\delta t)} + o\left(c^{\frac{\delta}{\alpha+\delta t}}\right) + \frac{1}{3}$$
$$= n_0^2 + \frac{\eta(\alpha+\delta t) - 2\alpha s}{\eta(\alpha+\delta t)}n_0$$
$$+ \frac{\eta^2(\alpha+\delta t)^2 + 3\alpha s(\alpha+\delta t) + 3\alpha(\alpha-\delta t)s^2 - 3\alpha s\,\eta(\alpha+\delta t))}{3\eta^2(\alpha+\delta t)^2} + o\left(c^{\frac{\delta}{\alpha+\delta t}}\right)$$

Which proves (1.6)

The asymptotic expansion for the' regret' of three-stage procedures $(1.1)-(1.2)$ is proven in the following theorem.

Theorem: For the three stage procedure (1.1) –(1.2),Risk Efficiency $R_g(C)$ is given by, as $C \to 0$

$$R_g(C) = o\left(C^{\alpha+2\delta/(\alpha+\sigma t)}\right)$$

Proof : It follows from (1.3) that

$$R_N(C) = Cn_0^t E\left[f\left(\frac{N}{n_0}\right)\right]$$

where $f(x) = \left(\frac{\delta t}{\alpha}\right)x^{-\alpha/\delta} + x^t$. Expanding $f(x)$ around $x = 1$ by Taylor's serles, we obtain for

$$|u-1| < \left|\frac{N}{n_0}-1\right| f\left(\frac{N}{n_0}\right) = f(1) + \left\{\frac{N}{n_0}-1\right\} f'(1) + \frac{1}{2}\left\{\frac{N}{n_0}-1\right\}^2 f''(U)$$

$$R_N(C) = \quad R_{n_0}(C) + \frac{Cn_0^t}{2n_0^2}\left[\{E(N^2) - 2n_0E(N) + n_0^2\} \cdot \left\{\left(\frac{\alpha}{\delta}+1\right) tu^{-\left(\frac{\alpha}{\delta}+2\right)} + t(t-1)u^{t-2}\right\}\right]$$

Using, $U \xrightarrow{a \cdot s} 1$, as $C \to 0$ and applying Lemma 1. ,We get

$$\begin{aligned} R_g(C) = &\quad R_N(C) - R_{n_0}(C) \\ = &\quad \frac{Cn_0^t}{2n_0^2} t\left(\frac{\alpha}{\delta}+t\right)[E(N^2) - 2n_0E(N) + n_0^2] \\ = &\quad \frac{Cn_0^t}{2n_0^2} t\left(\frac{\alpha}{\delta}+t\right)\left[n_0^2 + \frac{\eta(\alpha+\delta t) - 2\alpha s}{\eta(\alpha+\delta t)} n_0 + \frac{\eta^2(\alpha+\delta t)^2 + 3\alpha s(\alpha+\delta t) + 3\alpha(\alpha-\delta t)s^2 - 3\alpha s\eta(\alpha+\delta t)}{3\eta^2(\alpha+\delta t)^2}\right. \end{aligned}$$

$$\begin{aligned} &\left. + o\left(c^{\delta/(\alpha+\delta t)}\right) - 2n_0^2 + \frac{2n_0\alpha s}{\eta(\alpha+\delta t)} - n_0 + o(1) + n_0^2\right] \\ &= \left(\frac{cn_0^t}{2n_0^2}\right)\frac{t(\alpha+\delta t)}{0}\left[\frac{\eta(\alpha+\delta t) - 2\alpha s + 2\alpha s - \eta(\alpha+\delta t)}{\eta(\alpha+\delta t)} n_0 + \frac{3\alpha s(a+\delta t)(1-\eta) + 3a^2(\alpha-\delta t)s^2 + \eta^2(\alpha+\delta t)^2}{3\eta^2(\alpha+\delta t)^2}\right. \\ &\left. + o\left(c^{\delta/(\alpha+\delta t)}\right)\right] \\ &= 0(cn_0^{t-2}) \\ &= 0\left(c^{\alpha+2\delta/(\alpha+\delta t)}\right). \end{aligned}$$

and the theorem follows.

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