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# An Inventory Model For Deteriorating Items Beginning With Shortages Under Exponentially Declining Demand

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#### **ABSTRACT**

In the present paper an attempt has been made to formulate an inventory model for deteriorating items where the shortages occur at the beginning of the cycle. Generally, the demand of certain products may decline due to the introduction of more attractive products influencing customers behaviour. Deriving motivation from it, the above model is developed for the demand, which is exponentially declining. The significant features of the model and the results are studied with the help of a numerical example. Sensitivity analysis is carried out to demonstrate the effect of changing parameter values on the optimum solution of the system.

Keywords: Inventory, Deterioration, Shortages, Exponentially Declining Demand.

#### 1. INTRODUCTION

Classical inventory models where the shortages occur at the end the cycle have been widely studied in recent years. A little literature is available on developing the models starting with shortages and ending with no shortages. Such type of models could be used for describing the optimal inventory policies of those new products which are going to be launched in the market or for the products like fruits, vegetables, juices and other food stuffs as well as fashion items which are seasonal in nature. Thus a business could start with shortages like advance booking of such items the demand of which could be fulfilled after a time duration. Moreover, there are many items in the market, which are of high demand for the people for example sugar, wheat, oil, fruits vegetables etc. whose shortages break the customer?s faith and arrival pattern. This motivates the retailers to order the excess of units of items for inventory inspite of the fact that there may be sufficient deterioration while holding them in the inventory. However, due to the use of modern storage technologies deterioration can be managed to a certain extent.

In formulating inventory models, two factors of the problem have been of growing interest to the researchers, one being the deterioration of items and the other being the variation in the demand rate. Controlling and regulating deteriorating item is a major problem in any inventory system. Certain products like vegetables, fruits, food stuffs, fashionable items, chemicals etc. deteriorate during their normal storage period. Hence, when developing an optimal inventory policy for such products, the loss of inventory due to deterioration cannot be ignored. The researches have continuously modified the deteriorating inventory models so as to make them more practicable and realistic.

Demand is the major factor in inventory management. Therefore, decisions of inventory are to be made on the present and future demands. As demand plays a key role in modelling of deteriorating inventory, researchers have recognized and studied the variations (or their combinations) of demand from the viewpoint of real life situations. Demand may be constant, time-varying, stock-dependent and pricedependent, etc. Constant demand is valid only when the phase of the product life cycle is matured and also for finite periods of time. In 1915, the classical EOQ (Economic Order Quantity) was developed where the demand rate of an item was taken as constant. Wagner and Whithin (1958) discussed the discrete case of the dynamic version of EOQ. Covert and Philip (1973), Misra (1975), Dave (1979) and Sarma (1987), etc., established inventory models with constant demand rate. Many inventory items (for example, electronic goods, fashionable clothes, etc.) experience fluctuations in the demand rate. Many products experience a period of rising demand during the growth phase of their product life cycle. On the other hand, the demand of some products may decline due to the introduction of more attractive products influencing customers preference. Moreover, the age of the inventory has a negative impact on demand due to loss of consumer confidence on the quality of such products and physical loss of materials. This phenomenon prompted many researchers to develop deteriorating inventory models with time varying demand pattern. In developing inventory models, two kinds of time varying demands have been considered so far: (a) continuous-time and (b) discrete-time. Most of the continuous-time inventory models have been developed considering either linearly increasing/decreasing demand or exponentially increasing/decreasing demand patterns. Donaldson (1977) promulgated the classical no-shortage inventory policy for linear trend in demand. Ritchie (1980, 1984) discussed the solution of the EOQ formula for a linear increasing time-dependent demand, which was obtained by Donaldson (1984). EOQ models for deteriorating items with trended demand were considered by Bahari-Kasani (1989), Goswami and Chaudhuri (1991, 1992), Xu and Wang (1990), Chung and Ting (1993, 1994), Kim (1995), Jalan et al. (1996), Jalan and Chaudhuri (1999) and Lin et al. (2000), etc. Silver and Meal (1969) developed an approximate solution procedure for the general case of time-varying demand. Generally, this type of demand exists for some particular goods. Many research articles by Silver (1979), Henery (1979), McDonald (1979), Dave and Patel (1981), Sachan (1984), Deb and Chaudhuri (1986), Murdeshwar (1988) and Hargia (1993), etc., analysed linear time varying demand. The consideration of exponentially decreasing demand for an inventory model was first proposed by Hollier and Mak (1983), who obtained optimal replenishment policies under both constant and variable replenishment intervals. Hariga and Benkherouf (1994) generalized Hollier and Mak®s model (1983) by taking into account both exponentially growing and declining markets. Wee (1955(a), 1955(b)) developed a deterministic lot size model for deteriorating items where demand declines exponentially over a fixed time horizon. In this extent, Begum et al. (2009) developed an inventory model with exponential demand rate, finite production rate and shortages.

Later, Ghosh and Chaudhuri (2004, 2006), Khanra and Chaudhuri (2003) and Begum *et al.* (2010), etc., established their models with quadratic time-varying demand. Teng and Chang (2005) established an economic production quantity models for deteriorating items with demand depend on price and stock. Recently, Khedlekar et.al.(2013) and Khedlekar and Shukla (2013) have developed inventory models beginning with shortages with logarithmic demand. Hill (1995) was the first to introduce the ramp type demand rate in inventory model. The ramp type demand is commonly seen when some fresh fruits are brought to the market. In such type of demand, Hill considered increases linearly at the beginning, and then after maturation the demand becomes a constant, a stable stage till the end of the inventory cycle. You (2005) discussed a dynamic inventory policy for product with price and time-dependent demand Skouri et.al. (2009) and Teng et.al. (2011) have developed inventory models beginning with shortages and ending with shortages with ramp type demand rate.

In the present paper, a business process is modelled which starts with shortages of deteriorating items and after certain period of time the retailers can order excess units of item for inventory. The demand rate considered is exponentially decreasing in nature, as among various time varying demand it is more realistic. Generally, the demand of certain products may decline due to the introduction of more attractive products influencing customers behaviour. For such type of business situation, optimal shortages duration is obtained along with some other valuable results. Section 2 consists of assumptions and notations of the model while section 3 deals with the mathematical formulation of the model. In section 4, a numerical example is given to illustrate the model. Section 5 is devoted to sensitivity analysis to check the effectiveness of the model.

#### 2. NOTATIONS AND ASSUMPTIONS

The development of the inventory model is based on the following notations and assumptions.

## 2.1 Notations

T: Cycle time

 $t_1$ : Time for the accumulated shortages

 $I_1$ (t): On hand shortages at any instant of time t

 $I_2$  (t): On hand inventory at any instant of time t

 $C_1$ : Holding cost per unit per unit time

 $C_2$ : Shortage cost per unit per unit time

 $C_{\scriptscriptstyle 3}$  : Deterioration cost per unit per unit time

 $\theta$ :Rate of deterioration  $0 < \theta \le 1$ 

a: Demand at any time

b: Rate by which the demand decreases

 $D_{\scriptscriptstyle T}$ : Deteriorated units

 $\mathbf{S}_{\scriptscriptstyle T}$  : Shortage units

S : Setup cost

 $H_{\scriptscriptstyle C}$ : Holding cost

 $S_c$ : Shortage cost

 $D_{\scriptscriptstyle C}$  Deterioration cost

TC: Total system cost

# 2.2 Assumptions

1. The inventory deals with a single item.

- 2. Shortages are allowed.
- 3. The planning horizon is finite.
- 4. Deteriorated units can neither be repaired nor replaced during the cycle time.
- 5. The functional form of the demand is given as D(t) = a  $e^{-bt}$ ; t > 0; a > 0, 0 < b < 1.

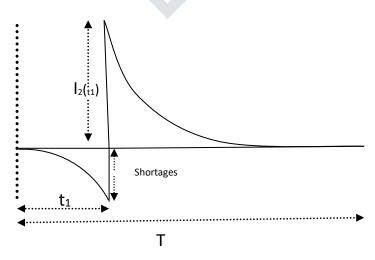
# 3. MATHEMATICAL FORMULATION AND SOLUTION OF THE MODEL

Depletion of the inventory for one cycle is represented in Figure 1.

Suppose the on hand shortages denoted by  $I_1$ (t) are accumulated till time  $t_1$ . Management places the order at time  $t_1$ , which is immediately fulfilled and inventory reaches up to a level  $I_2$ (t).

This inventory level is sufficient to fulfil the demand till time T. After time  $t_1$  the inventory is depleted due to demand and deterioration and reduces to zero at time T. (See Figure 1)

Figure 1



**Inventory-Time Diagram** 

The differential equations for the instantaneous state over (0,T) is given by

$$\frac{d}{dt}I_{1}(t) = -a e^{-bt} \qquad 0 \le t \le t_{1}; \quad a>0; b>0; a>>b$$
 (1)

$$\frac{d}{dt}I_2(t) + \theta I_2(t) = -a e^{-bt} \qquad t_1 \le t \le T$$
 (2)

With the boundary conditions  $I_{\scriptscriptstyle 1}$  (0) =0 and  $I_{\scriptscriptstyle 2}$  (T) =0

On solving equation (1) we get

$$I_1(t) = A - \int a e^{-bt} dt$$

Where A is the constant of integration which is determined by the initial condition  $I_{\perp}$  (0) =0.

Thus, A = 0

Therefore, 
$$I_1(t) = \frac{a}{b} \left( e^{-bt} - 1 \right)$$
 ;  $0 \le t \le t_1$  (3)

On solving equation (2) we get

$$e^{\theta t} I_2(t) = B - \int a e^{t(\theta - b)} dt$$

$$= B - a \frac{e^{t(\theta-b)}}{(\theta-b)}$$

Where B is the constant of integration which will be determined by the boundary condition

$$I_{2}(T) = 0$$

Thus, B = 
$$\frac{a}{(\theta - b)}e^{T(\theta - b)}$$

Therefore,

$$e^{\theta} I_2(t) = \frac{a}{(\theta - b)} e^{T(\theta - b)} - a \frac{e^{t(\theta - b)}}{(\theta - b)}$$

$$=\frac{a}{(\theta-b)}(e^{T(\theta-b)}-e^{t(\theta-b)})$$

$$I_2(t) = \frac{a}{(\theta - b)} (e^{\theta(T - t) - Tb} - e^{-bt})$$
 (4)

Generally, the values  $\theta$  and b are quite small. Therefore using series expansion and neglecting the third and higher order terms in both the exponential expansions we get

$$I_{2}(t) = \frac{a}{(\theta - b)} \left\{ \theta(T - t)(1 - Tb) - b(T - t) + \frac{\theta^{2}}{2}(T - t)^{2} + \frac{b^{2}}{2}(T^{2} - t^{2}) \right\} \qquad t_{1} \le t \le T$$
 (5)

Deteriorating units in time ( $t_1$ , T) are given by

$$D_T = I_2(t) - \int_{t_1}^{T} a e^{-bt} dt$$

$$= \frac{a}{(\theta - b)} \left\{ \theta(T - t_1)(1 - Tb) - b(T - t_1) + \frac{\theta^2}{2}(T - t_1)^2 + \frac{b^2}{2}(T^2 - t_1^2) \right\} + a \left\{ \frac{b}{2}(T^2 - t_1^2) - (T - t_1) \right\}$$
(6)

Deteriorating cost in time (  $t_1$ ,T )

$$= C_3 \frac{a}{(\theta - b)} \left\{ \theta(T - t_1)(1 - Tb) - b(T - t_1) + \frac{\theta^2}{2} (T - t_1)^2 + \frac{b^2}{2} (T^2 - t_1^2) \right\} + C_3 a \left\{ \frac{b}{2} (T^2 - t_1^2) - (T - t_1) \right\}$$
(7)

Holding cost in time  $(t_1, T)$ 

$$= C_1 \int_{t_1}^{T} \frac{a}{(\theta - b)} \left\{ \theta(T - t)(1 - Tb) - b(T - t) + \frac{\theta^2}{2} (T - t)^2 + \frac{b^2}{2} (T^2 - t^2) \right\} dt$$

$$= \frac{a}{(\theta - b)} C_{1} \left[ \theta \left\{ \frac{T^{2}}{2} - \frac{T^{3}b}{2} + (1 - bT)(\frac{t_{1}^{2}}{2} - Tt_{1}) \right\} - b \left\{ \frac{T_{1}^{2}}{2} + \frac{t_{1}^{2}}{2} - Tt_{1} \right\} + \frac{\theta^{2}}{2} \left\{ \frac{T^{3}}{3} - \frac{t_{1}^{3}}{3} - Tt_{1}(T - t_{1}) \right\} \right] + \frac{a}{(\theta - b)} C_{1} \left[ \frac{b^{2}}{2} \left\{ T^{2}(T - t_{1}) - (\frac{T^{3}}{3} - \frac{t_{1}^{3}}{3}) \right\} \right]$$

$$(8)$$

Shortages = 
$$-I_1(t_1)$$

$$=\frac{a}{b}(1-e^{-bt_1})$$

Shortage cost = 
$$C_2 \int_0^{t_1} \frac{a}{b} (1 - e^{-bt_1}) dt$$
  
=  $C_2 a \left\{ \frac{t_1^2}{2} - \frac{bt_1^3}{6} \right\}$  (9)

Number of units including shortages in the business schedule will be

Q= 
$$I_1(t_1) + I_2(t_1)$$

Total inventory cost per cycle will be

TC 
$$(t_1) = \frac{S + H_c + S_c + D_c}{T}$$

$$\frac{1}{T}\left[S + \frac{a}{(\theta - b)}C_{1}\left[\theta\left\{\frac{T^{2}}{2} - \frac{T^{3}b}{2} + (1 - bT)\left(\frac{t_{1}^{2}}{2} - T_{1}\right)\right\} - b\left\{\frac{T_{1}^{2}}{2} + \frac{t_{1}^{2}}{2} - T_{1}\right\} + \frac{\theta^{2}}{2}\left\{\frac{T^{3}}{3} - \frac{t_{1}^{3}}{3} - T_{1}(T - t_{1})\right\}\right] + \frac{a}{(\theta - b)}C_{1}\left[\frac{b^{2}}{2}\left\{T^{2}(T - t_{1}) - \left(\frac{T^{3}}{3} - \frac{t_{1}^{3}}{3}\right)\right\}\right] + C_{2}a\left\{\frac{t_{1}^{2}}{2} - \frac{bt_{1}^{3}}{6}\right\} + C_{3}a\left\{\frac{\theta}{(\theta - b)}\left\{\theta(T - t_{1})(1 - Tb) - b(T - t_{1}) + \frac{\theta^{2}}{2}(T - t_{1})^{2} + \frac{b^{2}}{2}(T^{2} - t_{1}^{2})\right\} + C_{3}a\left\{\frac{b}{2}(T^{2} - t_{1}^{2}) - (T - t_{1})\right\}\right]$$
(10)

Our objective is to minimise total system cost TC ( $t_1$ ) and determine optimal value of  $t_1$  i.e.  $t_1$ \*. The necessary condition for minimising the cost is,

$$\frac{dTC(\mathbf{t}_1)}{d\mathbf{t}_1} = 0$$

provided it satisfy the condition  $\frac{d^2}{dt_1^2}TC(t_1) \ge 0$  at  $t_1 = t_1^*$ .

$$\frac{dTC(t_1)}{dt_1} = \frac{1}{T} \left[ \frac{aC_1}{(\theta - b)} \left\{ \theta(bT - 1)(t_1 - T) - b(t_1 - T) + \frac{\theta^2}{2} (-t_1^2 - (T^2 - 2t_1)) + \frac{b^2}{2} (-T^2 + t_1^2) \right\} + aC_2(t_1 - \frac{bt_1^2}{2}) + \frac{a}{(\theta - b)}C_3 \left\{ \theta Tb - \theta + b - \theta^2 (T - t_1) - \frac{b^2}{2} t_1 \right\} + aC_3(1 - bt_1) \right] = 0$$

(11)

$$\frac{d^{2}TC(\mathbf{f}_{1})}{d\mathbf{f}_{1}^{2}} = \frac{1}{T} \left[ \frac{a\mathbf{C}_{1}}{(\theta - b)} \left\{ \theta(1 - bT) - b + \theta^{2}(1 - \mathbf{f}_{1}) + \mathbf{b}^{2}\mathbf{f}_{1} \right\} + a\mathbf{C}_{2}(1 - b\mathbf{f}_{1}) + \frac{a}{(\theta - b)}\mathbf{C}_{3}(\theta^{2} - b^{2}) - \mathbf{C}_{3}ab \right] > 0$$

at 
$$t_1 = t_1^*$$
. (12)

From equation (11) optimal value of  $t_1$  i.e.  $t_1^{\phantom{1}*}$  is determined.

## 4. NUMERICAL ILLUSTRATION

Let us assume the model parameters as

a = 12, b= .03, 
$$\theta$$
=.08,  $C_1$  = 0.5 ,  $C_2$  = 2.5 ,  $C_3$  = 1.5, T= 10, S= 250

The policies are

$$t_1^* = 1.8154$$

TC (
$$t_1^*$$
) = 54.7765

$$Q^{^*}$$
 = 134 units

$$H_{C}^{*} = 20.0988$$

$$S_{C}^{*} = 4.85370$$

## **5. SENSITIVITY ANALYSIS**

To study the effect of changes in the system parameters a, b and T on the optimal cost and  $t_1^*$  sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 25% and 50% and taking one parameter at a time, while keeping remaining parameters at a original levels.

Changing	% Change in the	$t_1^*$	TC (t <sub>1</sub> *)
Parameters	system	_	
а	50%	1.82	69.66
	25%	1.82	62.22
	-25%	1.83	47.33
	-50%	1.82	39.88
b	50%	2.82	52.54

	25%	2.75	56.92
	-25%	2.66	57.01
	-50%	2.64	57.85
Т	50%	4.88	64.17
	25%	3.7	58.31
	-25%	1.87	55.61
	-50%	1.39	67.05

On the basis of the above table, the following observations can be made:

- 1. The parameter **a** has no effect on  $t_1^*$  but as we increase **a**, the total system cost increases. This indicates that when the initial demand is low, the total cost is reduced.
- 2. The parameter b is highly sensitive to both, time till which the shortages are accumulated and total system cost. As the parameter b increases  $t_1^*$  increases where as the total cost decreases. This implies that when the rate by which the demand decreases is high the total system cost is minimised and the accumulated shortage time increases, which is true in real market situation.
- 3. As the total cycle time is increased, the time by which shortages will accumulate also increases. The total cost increases if we increase. Tor decrease T.

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