JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

gso-Connectedness and gso-Compactness in topological spaces

[1]M. I. Irshad, [2]K. Vithyasangaran

[1]Department of Mathematics, Faculty of Natural Sciences, The Open University of Sri Lanka, [2]Department of Mathematics, Faculty of Science, Eastern University, Sri Lanka

Abstract : In this paper we have introduced new concepts $\boldsymbol{gso}$-connectedness and $\boldsymbol{gso}$-compactness in a topological space $(\boldsymbol{X},\boldsymbol{\tau})$ and obtain some of their properties using $\boldsymbol{gso}$-closed sets.

***Keywords* - gso-closed sets, gso-continuous maps, gso-connectedness and gso-compactness.**

1 INTRODUCTION

In the literature, different type of connectedness and compactness were defined and studied by different authors [1-9]. Connectedness is one of the principal topological space properties that are used to distinguish topological spaces. A subset of a topological space is called a connected set if it is a connected space when viewed as a subspace of that topological space. The notations of compactness resulted in motivating mathematicians to generalize these notations further.

The concept of gso-closed set was introduced in 2019 by Irshad M. I. and Elango P. [10] in topological space and obtained various properties. The aim of this paper is to study gso-connectedness and gso-compactness using gso-closed set and also discuss some of their properties.

2 PRELIMINARIES

Throughout this paper (X,τ), (Y,σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X,τ), $cl(A)$ and $Int(A)$ denote the closure of A and interior of A respectively.

Definition 2.1. Let (X,τ) be topological space. Then, a subset A of $(X,\tau)$is called

a) gso-closed set [10] if A is both a g-closed set and a semi-open set in X.
b) gso-open set [10] if A is a g-open set or a semi-closed set in X.

The collection of all gso-closed sets of X is denoted by $C_{gso}(X)$.

Definition 2.2. A function $f:(X,\tau)\to(Y,\sigma)$ is called

a) gso-continuous [10] if the inverse image of every closed set in (Y,σ) is gso-closed in (X,τ).
b) gso- irresolute [10] if the inverse image of every gso-closed set in (Y,σ) is gso-closed in (X,τ).

3 GSO-CONNECTEDNESS

Definition 3.1. Let A and B be subsets of a topological space X. Then, A and B are called, gso-separated if $A\cap cl_{gso}(B)=\emptyset=cl_{gso}(A)\cap B$.

Definition 3.2. A topological space X is said to be generalized semi-open connected (briefly gso-connected) if X cannot be written as the union of two non-empty disjoint gso-open sets.

Example 3.1. Let $X = \{a, b\}$ and $\tau = \{X, \emptyset, \{a\}\}$. Then, the topological space (X, τ) is gso-connected.

Remark 3.1. Every gso-connected space is connected. But, the converse need not be true in general as seen in the following example.

Example 3.2. Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}\}$. Now, clearly (X, τ) is connected. Then, the gso-open sets of X are $\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Therefore, (X, τ) is not gso-connected space, because $X = \{a\} \cup \{b, c\}$, where $\{a\}$ and $\{b, c\}$ are non-empty gso-open sets.

Theorem 3.3. *If $f: X \to Y$ is a gso-continuous surjective map and X is gso-connected, then Y is connected.*

Proof. Suppose that X is gso-connected and assume that Y is not connected. Then, $Y = A \cup B$, where A and B are non-empty disjoint open sets in Y. Since f is a gso-continuous surjective map, $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty disjoint gso-open sets. This is a contradiction to that X is gso-connected. Hence Y is connected.

Theorem 3.4. *If $f: X \to Y$ is a gso-irresolute surjective map and X is gso-connected, then Y is gso-connected.*

Proof. Suppose that X is gso-connected and assume that Y is not gso-connected. Then, $Y = A \cup B$, where A and B are non-empty disjoint gso-open sets in Y. Since f is a gso-irresolute surjective map, $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty disjoint gso-open sets. This is a contradiction to that X is gso-connected. Hence Y is gso-connected.

Definition 3.3. A subset Y of a topological space X is called the gso-subspace of X if $Y \cap U$ is gso-open, when U is gso-open in X.

Definition 3.4. A gso-subspace Y of a topological space X is gso-disconnected if there exist gso-open subsets U and V of X such that $Y \cap U$ and $Y \cap V$ are disjoint non-empty gso-open sets whose union is Y. The gso-subspace is gso-connected if it is not gso-disconnected.

Lemma 3.5. *If Y is a gso-connected subspace of X and if the sets U and V form a gso-separation of X, then $Y \subset U$ or $Y \subset V$.*

Proof. Since U and V are both gso-open in X, the sets $Y \cap U$ and $Y \cap V$ are gso-open in Y. We have, $(Y \cap U) \cup (Y \cap V) = Y$ and $(Y \cap U) \cap (Y \cap V) = \emptyset$. If $Y \cap U$ and $Y \cap V$ are non-empty, then Y is gso-separated, but Y is gso-connected. Then $Y \cap U = \emptyset$ or $Y \cap V = \emptyset$. Therefore, $Y \subset U$ or $Y \subset V$.

Theorem 3.6. *Let A and B be subspaces of a topological space X. If A and B are gso-connected and not gso-separated, then $A \cup B$ is gso-connected.*

Proof. Assume that $A \cup B$ is not gso-connected. Then, $A \cup B = U \cup V$, where U and V are disjoint non-empty gso-open sets in X. Since A and B are gso-connected, then by Lemma (3.5), either $A \subset U$ or $A \subset V$ and $B \subset U$ or $B \subset V$. If $A \subset U$ and $B \subset U$, then $A \cup B \subset U$ and $V = \emptyset$. This is a contradiction to that V is non-empty. Therefore, $A \cup B$ is gso-connected.

Theorem 3.7. *If $\{A_\alpha : \alpha \in I\}$ is non-empty collection of gso-connected subspaces of a topological space X such that $\bigcap_{\alpha \in I} A_\alpha \neq \emptyset$, then $\bigcup_{\alpha \in I} A_\alpha$ is gso-connected.*

Proof. Assume that $Y = \bigcup_{\alpha \in I} A_\alpha$ is not gso-connected. Then $Y = U \cup V$, where U and V are non-empty disjoint gso-open sets in X. Since $\bigcap_{\alpha \in I} A_\alpha \neq \emptyset$, there is a point p of T $\bigcap_{\alpha \in I} A_\alpha$. Since $p \in Y$, either $p \in U$ or $p \in V$. Suppose that $p \in U$. Since A_α is gso-connected, $A_\alpha \subset U$ or $A_\alpha \subset V$. Since $p \in A_\alpha, A_\alpha \not\subset V$. Hence, $A_\alpha \subset U$ for every α. Then $Y = \bigcup_{\alpha \in I} A_\alpha \subset U$. This is a contradiction to that V is non-empty. Therefore, $\bigcup_{\alpha \in I} A_\alpha$ is gso-connected.

Theorem 3.8. *Let A be a gso-connected subspace of X. If $A \subset B \subset cl_{gso}(A)$, then B is also gso-connected.*

Proof. Assume that B is not gso-connected. Then, $B = U \cup V$, where U and V are disjoint non-empty gso-open sets in B. Since A is gso-connected then by Lemma (3.5), either $A \subset U$ or $A \subset V$. Suppose that $A \subset U$. Then $cl_{gso}(A) \subset cl_{gso}(U)$. Since $cl_{gso}(U)$ and V are disjoint, B cannot intersect V. This contradicts the fact that V is a non-empty subset of B. Therefore, B is gso-connected.

4 GSO-Compactness

Definition 4.1. A collection $\{A_i : i \in I\}$ of gso-open sets in a topological space X is called a gso-open cover of a subset B of X if $B \subset \cup \{A_i : i \in I\}$ holds.

Definition 4.2. A topological space X is gso-compact if every gso-open cover of X has a finite subcover.

Definition 4.3. A subset B of a topological space X is said to be gso-compact relative to X if, for every collection $\{A_i : i \in I\}$ of gso-open subsets of X such that $B \subset \cup \{A_i : i \in I\}$ there exists a finite subset I_0 of I such that $B \subseteq \cup \{A_i : i \in I_0\}$.

Definition 4.5. A subset B of a topological space X is said to be gso-compact if B is gso-compact as a subspace of X.

Theorem 4.1. *Every gso-closed subset of a gso-compact space X is gso-compact relative to X.*

Proof. Let A be a gso-closed subset of gso-compact space X. Then, A^c is gso-open in X. Let $M = \{G_\alpha : \alpha \in I\}$ be a cover of A by gso-open sets in X. Then, $M^* = M \cup A^c$ is a gso-open cover of X. Since X is gso-compact, M^* is reducible to a finite subcover of X, say $X = G_{\alpha_1} \cup G_{\alpha_2} \cup \dots \cup G_{\alpha_m} \cup A^c, G_{\alpha_k} \in M$. But, A and A^c are disjoint hence $A \subset G_{\alpha_1} \cup G_{\alpha_2} \cup \dots \cup G_{\alpha_m} \cup A^c, G_{\alpha_k} \in M$, which implies that any gso-open cover M of A contains a finite subcover. Therefore, A is gso-compact relative to X. Thus, every gso-closed subset of gso-compact space X is gso-compact.

Theorem 4.2. *Every gso-compact space is compact.*

Proof. Let X be a gso-compact space. Let $\{A_i : i \in I\}$ be an open cover of X. Then $\{A_i : i \in I\}$ is a gso-open cover of X as every open set is gso-open set. Since X is gso-compact, the gso-open cover $\{A_i : i \in I\}$ of X has a finite subcover, say $\{A_i : i = 1, \dots, n\}$ for X. Hence X is compact.

Theorem 4.3. *Let $f : X \to Y$ be surjective, gso-continuous function. If X is gso-compact, then Y is compact.*

Proof. Let $\{A_i : i \in I\}$ be an open cover of Y. Since f is gso-continuous function, then $\{f^{-1}(A_i) : i \in I\}$ is gso-open cover of X has a finite subcover, say $\{f^{-1}(A_i) : i = 1, \dots, n\}$. Therefore, $X = \cup_{i=1}^{n} f^{-1}(A_i)$ which implies $f(X) = \cup_{i=1}^{n} A_i$. Since f is surjective, $Y = \cup_{i=1}^{n} A_i$. Thus, $\{A_1, A_2, \dots, A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for Y. Hence Y is compact.

Theorem 4.4. *If a map $f : X \to Y$ is gso-irresolute and a subset B of X is gso-compact relative to X, then the image $f(B)$ is gso-compact relative to Y.*

Proof. Let $\{A_\alpha : \alpha \in I\}$ be any collection of gso-open subsets of Y such that $f(B) \subset \cup \{A_\alpha : \alpha \in I\}$. Then, $B \subset \{f^{-1}(A_\alpha) : \alpha \in I\}$ holds. From the hypothesis, B is gso-compact relative to X. Then, there exists a finite subset I_0 of I such that $B \subset \{f^{-1}(A_\alpha) : \alpha \in I_0\}$. Therefore, we have $f(B) \subset \{A_\alpha : \alpha \in I_0\}$., which shows that $f(B)$ is gso-compact relative to Y.

5 CONCLUSION

In this paper, we defined new kind of connectedness and compactness called gso-connectedness and gso-compactness. A topological space X is said to be generalized semi-open connected (briefly gso-connected) if X cannot be written as the union of two non-empty disjoint gso-open sets. A topological space X is gso-compact if every gso-open cover of X has a finite subcover. The gso-connectedness and gso-compactness fulfilled most of the connectedness and compactness properties in topological spaces.

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