JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

ACCELERATING EXPANSION OF TWO FLUID UNIVERSE COUPLED WITH ZERO-MASS SCALAR FIELD IN f(R) GRAVITY

[1]Kalpana Pawar, [1,2*]N. T. Katre, [3]S. H. Shekh
[1]Department of Mathematics, Shri. R. R. Lahoti Science College, Morshi, Dist. Amravati (M.S.), India
[2]Department of Mathematics, Nabira Mahavidyalaya, Katol, Dist. Nagpur (M.S.), India
[3]Department of Mathematics, S. P. M. Science and Gilani Arts Commerce College, Ghatanji, Dist. Yavatmal (M.S.), India
*Corresponding author

Abstract : In this paper, we have considered spatially homogeneous and isotropic flat FRW cosmological model of the universe in the presence of zero-mass scalar fields associated with the barotropic fluid and dark energy distribution on it. We have also discussed various physical and geometrical features of the model.

IndexTerms **- FRW universe, $f(R)$ gravity, zero-mass scalar field.**

I. INTRODUCTION

The most outstanding detection of current cosmology specifies that the present universe is not only expanding but also accelerating. Now a days, this action of the Universe is established by numerous self-governing observational data, comprising the Type Ia Supernovae (SNe Ia) Perlmutter *et al.* [1-4], fluctuation of cosmic microwave background radiation (CMBR) [5] and large scale structure (LSS) [6,7]. In addition, measurements of the cosmic microwave background (CMB) and the large scale structure (LSS) powerfully designate that our universe mainly include a constituent called a dark energy, which is due to the negative pressure. The dark energy model has been categorized in a conventional manner by the equation of state (EoS) parameter, which is not necessarily constant. If it would be equal to -1 (standard Λ CDM cosmology), a little bit more than-1 (the quintessence dark energy) or less than -1 (phantom dark energy), while the possibility of EoS $\ll -1$ ruled out by current cosmological data from SN Ia. The simplest candidate for the dark energy is a cosmological constant Λ. The suggestions that have been put onward to clarify this observed phenomenon can essentially be classified into two classes. One is to adopt that in the framework of Einstein's general theory of relativity (GTR), an exotic component with negative pressure called mysterious energy or dark energy (DE) is necessary to explain this observed phenomenon, for a good review of the dynamics of different DE models, see [8]. Additional another to account for the current accelerating cosmic expansion is to modify GTR.

The conclusion that GTR is not the final theory of gravitational interaction is based on a number of concerns and ensuing limits, particularly the limitation connected to the late-time cosmic acceleration of GTR [9, 10]. Measurements of the cosmological parameters based on the Planck data [11] on the CMB radiation suggest that the Universe is made up of about 4–5% baryons, 25% non-baryonic DM, and 70% dark energy. There have been a few particles named as DM candidates [12–15].The weakly interacting massive particles (WIMPs), SM neutrinos, sterile neutrinos, axions, and super symmetric candidates (neutralinos, neutrinos, gravitinos, axinos) are a few of them. WIMPs are non-baryonic, which consists of the lightest SUSY particles, particularly the neutralino, and it is viewed the most in all likelihood candidate of DM. Axions, which are additionally high candidates of DM, are bosons that have been first proposed to clear up the robust CP problem. The DM particle is still not listed in the table of the nature's elementary particles, which means that the DM's true nature is still unknown almost 80 years after the DM notion was first proposed. In order to justify gravitational interactions different than the one given by GTR, theories of modified gravity [16] were proposed. We have gravity models, Brane world models, Gauss-Bonnet dark energy models, etc., in modified gravity models [17–20]. The $\boldsymbol{f(R)}$gravity is the simplest class of modified gravity theories among them [21, 22]. The section detailing the geometry of Einstein's field equation is modified in this case. It is changed by substituting a function $\boldsymbol{f(R)}$for the Ricci scalar R in the Einstein-Hilbert action. Many authors have studied the aspect of different cosmological models in this gravity [23, 24]. Deriving the exact solution from a power-law $\boldsymbol{f(R)}$cosmological model Capozziello et al. [25] achieve dust matter and Dark Energy phase. Using the same theory Azadi et al. [26] studied vacuum solution in cylindrically symmetric space-time. Bianchi type-III cosmological models with bulk viscosity in $\boldsymbol{f(R)}$ theory investigated by Katore and Shaikh [27]. Miranda et al. [28] discussed a viable singularity-free $\boldsymbol{f(R)}$gravity without a cosmological constant. Sharif and Yousaf [29] studied the impact of dark energy and dark matter models on the dynamical evolution of collapsing self-gravitating systems in this gravity.

The basic obstacle to examining the as yet unsolved issue of the unification of the gravitational and quantum theories is the zero-mass scalar field. In the last few years the theory of gravitation that describes zero-mass scalar fields associated with gravitational field has attracted considerable attention. Recently Chirde and Shekh have studied the Isotropic Background for

Interacting Two Fluid Scenario Coupled with Zero Mass Scalar Field in Modified Gravity [30].The zero-mass scalar field has acquired particular importance. For isotropic background in the presence of a gravitational field 'Big Bang' of universe at the initial stage can be avoided by introducing a zero-mass scalar field [31] along with this some authors [32,33] have investigated cosmological model with zero-mass scalar field.

In this research, we investigate interactions between barotropic fluid and DE with zero-mass scalar field for the spatially homogenous and isotropic flat FRW universe, which is motivated by the ideas above. We take into account an interaction scenario as we proceed.

The structure of this essay is as follows: We provide a succinct overview of the $\boldsymbol{f(R)}$ theory in Section II. The metric and the fundamental equations are explained in Section III. The field equations for interacting two-fluids are solved in Section IV of the paper. The last Section V summarizes the conclusions.

II. $\boldsymbol{f(R)}$ GRAVITY: A BRIEF REVIEW

The $\boldsymbol{f(R)}$ gravity theory is the simple extension of the general theory of relativity. In $\boldsymbol{f(R)}$ theory of gravity, there are three main approaches: (i) metric approach; (ii) Palatini formalism; and (iii) affine $\boldsymbol{f(R)}$ gravity. In the metric approach, there is the Levi-Civita connection and variation of action is done with respect to the metric tensor. In Palatini formalism, the connection and metric are independent of each other, and the variation is done for two mentioned parameters, independently. In metric-affine $\boldsymbol{f(R)}$ gravity, both metric tensor and connection are treated independently, and it is also assumed that the matter action depends on the connection.

The action for this theory is given by

$$S = \frac{1}{2k^2}\int d^4x\sqrt{-g}\, f(R) + \int d^4 x\sqrt{-g}\, L_m(\, g_{ij}\, , \phi_m\,) \tag{1}$$

Here, $f(R)$ is a general function of the Ricci scalar R, $k^2 = 8\pi G = 1$, g is the determinant of the metric g_{ij}, and L_m is the matter Lagrangian that depends on g_{ij} and the matter fields ϕ_m.

The action (1) for this theory is obtained on replacing R by $f(R)$ in the standard Einstein–Hilbert action given by

$$S_{EH} = \frac{1}{2k^2}\int d^4x\sqrt{-g}\, R + \int d^4 x\sqrt{-g}\, L_m(\, g_{ij}\, , \phi_m\,). \tag{2}$$

The variation of action (1) with respect to the metric g_{ij} leads to the following field equations of $f(R)$ gravity,

$$f_R(R)\;\; R_{ij} - \frac{1}{2}\, f(R) g_{ij} - \nabla_i\, \nabla_j\, f_R(R) + g_{ij}\, \nabla^i \nabla_i\, f_R(R) = T_{ij}^{(M)} \quad . \tag{3}$$

Here, $f_R(R) \equiv \frac{d}{dR}\, f(R)\,,$ (4)

∇_i is the covariant derivative, and

$T_{ij}^{(M)}$ is the standard matter-energy momentum tensor derived from the Lagrangian L_m.

III. METRIC AND FIELD EQUATIONS

We consider the spatially homogeneous and isotropic flat FRW universe of the form

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2\, (d\theta^2 + sin^2\theta\, d\varphi^2)] \tag{5}$$

where $a(t)$ is the metric potential or the scale factor of the universe.

The Ricci curvature scalar corresponding to the universe (5) is given by

$$R = 6\,[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}], \tag{6}$$

where overhead dot represents derivative with respect to cosmic time t.

The energy-momentum tensor due to the barotropic fluid, dark energy (DE) and zero-mass scalar field is taken as

$$T_{ij}^{(M)} = (\rho + p)u_i u_j\, + p\, g_{ij} + (\psi,_i \psi,_j - \frac{1}{2} g_{ij}\psi,_h\;\; \psi'^h)\,, \tag{7}$$

together with

$$u^i u_i = -1, \tag{8}$$

where u^i is the four velocity vector; p, ρ and ψ are respectively the isotropic pressure, the energy density and the zero-mass scalar field. p and ρ are given by

$$p = p_m + p_{de} \quad \& \quad \rho = \rho_m + \rho_{de},$$

where p_m and ρ_m are the pressure and energy density of barotropic fluid; p_{de} and ρ_{de} are the pressure and energy density of DE respectively. Also, $p_m = \omega_m \rho_m$ and $\rho_{de} = \omega_{de}\, \rho_{de}$.

The Scalar field ψ satisfies the equation

$$\psi^i_{;\,i} = 0\,. \tag{9}$$

In co-moving co-ordinate system, we have from equation (7),

$$T_1^1 = T_2^2 = T_3^3 = p + \frac{1}{2}\dot{\psi}^2,\;\; T_4^4 = -\rho - \frac{1}{2}\dot{\psi}^2,\;\; T_j^i = 0 \text{ for } i \neq j\,. \tag{10}$$

On solving the field equations (3) for metric (5) and using (10), the Friedman equations for two fluid scenarios with zero-mass scalar field reduces to the following set of equations

$$\left(2\frac{\dot{a}^2}{a^2}\right) f_R - \frac{1}{2}\, f(R) - 2\frac{\dot{a}}{a}\dot{f}_R - \ddot{f}_R = p + \frac{1}{2}\dot{\psi}^2, \tag{11}$$

$$\left(3\frac{\ddot{a}}{a}\right) f_R - \frac{1}{2}\, f(R) - 3\frac{\dot{a}}{a}\dot{f}_R = -\rho - \frac{1}{2}\dot{\psi}^2 \;\; , \tag{12}$$

$$\ddot{\psi} + 3\,\frac{\dot{a}}{a}\dot{\psi} = 0\,. \tag{13}$$

We define some parameters for the universe, which are important in cosmological observations:

Average scale factor $a(t)$ and spatial volume V are given by

$$a = \sqrt[3]{V} \quad , \quad V = a^3; \tag{14}$$

The generalized Hubble's parameter(H), the mean anisotropy parameter (A_m), the expansion scalar (θ), shear scalar (σ) and the deceleration parameter (q) are defined as

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{a}}{a}, \tag{15}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \quad , \tag{16}$$

$$\theta = u^i_{;i} = 3H = 3\frac{\dot{a}}{a}, \tag{17}$$

$$\sigma^2 = \frac{3}{2}A_m H^2, \tag{18}$$

$$q = \frac{-a\,\ddot{a}}{\dot{a}^2} = \frac{-\ddot{a}}{aH^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1, \tag{19}$$

where H_1, H_2 and H_3 are the directional Hubble's parameters in the directions of X, Y and Z axes respectively.

The Hubble's parameter and deceleration parameter are the most important observational quantities for any physically relevant cosmological model. The sign of deceleration parameter q indicates the state of expanding universe. For $q < 0$, it represents inflation (accelerating expansion of the universe); for $q > 0$ it represents deflation (decelerating expansion of the universe); while $q = 0$ shows that the universe expands with constant rate.

IV. Solutions of the Field Equations

To solve the field equations completely, we first assume that the barotropic fluid and DE components interact minimally. Therefore, the energy momentum tensors of the two sources may be conserved separately.

For the barotropic fluid, the energy conservation equation ($T;^{ij}_j = 0$) leads to

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(\rho_m + p_m) = 0, \tag{20}$$

whereas for the DE component, the energy conservation equation ($T;^{ij}_j = 0$) yields

$$\dot{\rho}_{de} + 3\frac{\dot{a}}{a}(\rho_{de} + p_{de}) = 0 \tag{21}$$

We assume that the EoS parameter of the perfect fluid to be a constant (which is considered by Akarsu & Kumar) [34]

$$\omega_m = \frac{p_m}{\rho_m} = constant \tag{22}$$

while ω_{de} has been admitted to be a function of time t .

Since the set of field equations (11 – 13) are coupled system of highly nonlinear differential equations containing six unknowns, namely $a,\ \rho,\ p,\ f_R,\ f,\ \psi$, we can introduce conditions to obtain unique solutions of the field equations.

We consider the volumetric expansion law as

$$a(t) = [\exp(-\gamma\eta t) - 1]^{-\frac{1}{\gamma}} \tag{23}$$

The energy densities of matter and DE no longer satisfy independent conservation laws; instead, they obey

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(\rho_m + p_m) = Q, \tag{24}$$

$$\dot{\rho}_{de} + 3\frac{\dot{a}}{a}(\rho_{de} + p_{de}) = -Q. \tag{25}$$

We consider the interaction term Q between the DE and the barotropic matter components, in the form of $Q \propto H\,\rho_m$, which is already well-thought-out by Saha & Amirhashchi [35]

$$Q = 3\,H\sigma\,\rho_m \ , \tag{26}$$

where the coupling coefficient σ can be considered as a constant.

Using (24) and (26), we get

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(\rho_m + p_m) = 3\,H\sigma\,\rho_m$$

which gives

$$\rho_m = C_1[\exp(-\gamma\eta t) - 1]^{\frac{3\,(1+\omega_m - \sigma)}{\gamma}} \tag{27}$$

From (13), we get

$$\dot{\psi} = C_2[\exp(-\gamma\eta t) - 1]^{\frac{3}{\gamma}} \tag{28}$$

where C_1 and C_2 are constants of integration.

We consider

$$f(R) = R + b\,R^m, \tag{29}$$

then

$$f_R(R) = 1 + b\,m\,R^{m-1} \tag{30}$$

Therefore from (6), (12) and (23), we get

$$\begin{aligned}\rho = {} & 3\,b\,(6)^{m-1}[\exp(-\gamma\eta t) - 1]^{-2m}[\gamma + 2\ \exp(-\gamma\eta t)]^{m-2}\exp(-m\gamma\eta t) \\ & \{\eta^{2m}\,(\gamma + 2\ \exp(-\gamma\eta t))\,[(1-m)\gamma + (2-m)\exp(-\gamma\eta t)] \\ & + m\,(m-1)\,\gamma\,\eta^m[4 + \gamma + \gamma\ \exp(\gamma\eta t)]\ \exp(-\gamma\eta t)\,\} \\ & + 3\,\eta^2\exp(-2\gamma\eta t)\,[\exp(-\gamma\eta t) - 1]^{-2} - \frac{1}{2}{C_2}^2[\exp(-\gamma\eta t) - 1]^{\frac{6}{\gamma}}\end{aligned} \tag{31}$$

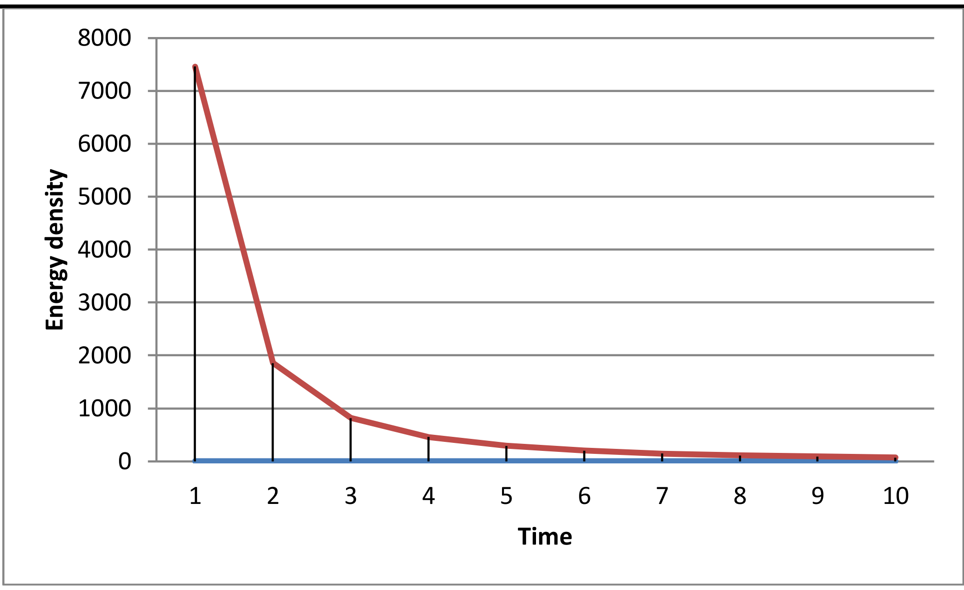

Fig.1 Energy density versus time with appropriate choice of constant.

Now,

$$H = \frac{\dot{a}}{a} = \frac{\eta\ \exp(-\gamma\eta t)}{[\ \exp(-\gamma\eta t)-1]}\ , \tag{32}$$

and $$q = -\frac{\ddot{a}}{a\,H^2} = -[1+\gamma\ \exp(-\gamma\eta t)][\exp(-\gamma\eta t)-1]^{\frac{2}{\gamma}} \tag{33}$$

From (11), we get

$$\begin{aligned} p = {} & (6)^{m-1}\eta^m b\ \exp(-(m+1)\gamma\eta t)\,[\exp(-\gamma\eta t)-1]^{-2m}[\gamma+2\ \exp(-\gamma\eta t)]^{m-3} \\ & \{\ m\,(m-1)\,\gamma\,(\gamma+2\ \exp(-\gamma\eta t))\ \ [-8-2\gamma-2\gamma\exp(\gamma\eta t)) \\ & -\gamma\eta^m\{\gamma\ \exp(2\gamma\eta t)+(4\gamma+8)\exp(\gamma\eta t)+(4+\gamma)\}\,] \\ & +\eta^m[\gamma+2\ \exp(-\gamma\eta t)]^2\,[2m-6-3\gamma\ \exp(\gamma\eta t)\,] \\ & +4\,m(m-1)(m-2)\gamma^2\eta^m[\exp(-\gamma\eta t)-1] \\ & [\,3+\exp(-\gamma\eta t)+\gamma\,(1+\exp(\gamma\eta t))]\} \\ & +\ \eta^2\exp(-\gamma\eta t)\,[\exp(-\gamma\eta t)-1]^{-2}\,[-3\gamma-4\exp(-\gamma\eta t)\,]-\frac{1}{2}{C_2}^2[\exp(-\gamma\eta t)-1]^{\frac{6}{\gamma}} \end{aligned} \tag{34}$$

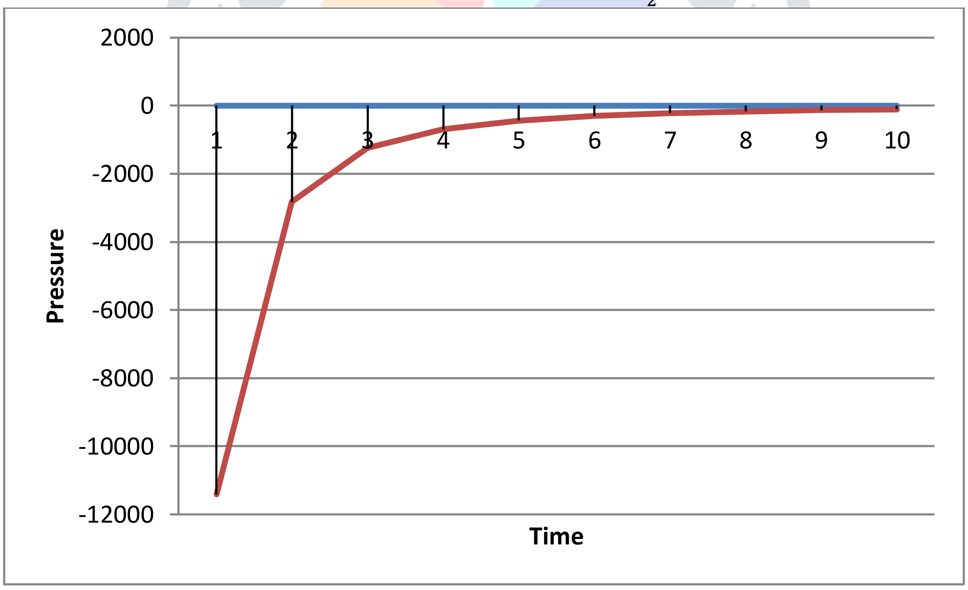

Fig.2. Pressure versus time with appropriate choice of constant.

EoS parameter is

$$\omega=\frac{p}{\rho}=\frac{\begin{array}{c}6^{m-1}\eta^{m}b\,\exp(-(m+1)\gamma\eta t)[\exp(-\gamma\eta t)-1]^{-2m}[\gamma+2\,\exp(-\gamma\eta t)]^{m-3}\\ \{\,m\,(m-1)\,\gamma\,(\gamma+2\,\exp(-\gamma\eta t))\,[\,-8-2\gamma-2\gamma\exp(\gamma\eta t)\\ -\gamma\eta^{m}\{\,\gamma\,\exp(2\gamma\eta t)+(4\gamma+8)\exp(\gamma\eta t)+(4+\gamma)\}\,]\\ +\,\eta^{m}[\gamma+2\,\exp(-\gamma\eta t)]^{2}[2m-6-3\gamma\,\exp(\gamma\eta t)]\\ +\,4\,m(m-1)(m-2)\gamma^{2}\eta^{m}[\exp(-\gamma\eta t)-1]\\ [\,3+\exp(-\gamma\eta t)+\gamma\,(1+\exp(\gamma\eta t))]\}\\ +\,\eta^{2}\exp(-\gamma\eta t)[\exp(-\gamma\eta t)-1]^{-2}[-3\gamma-4\exp(-\gamma\eta t)]\\ -\frac{1}{2}C_{2}{}^{2}[\exp(-\gamma\eta t)-1]^{\frac{6}{\gamma}}\end{array}}{\begin{array}{c}3\,b\,(6)^{m-1}[\exp(-\gamma\eta t)-1]^{-2m}[\gamma+2\,\exp(-\gamma\eta t)]^{m-2}\exp(-m\gamma\eta t)\\ \{\,\eta^{2m}\,(\gamma+2\,\exp(-\gamma\eta t))\,[(1-m)\gamma+(2-m)\exp(-\gamma\eta t)]\\ +\,m\,(m-1)\,\gamma\,\eta^{m}[4+\gamma+\gamma\,\exp(\gamma\eta t)]\,\exp(-\gamma\eta t)\,\}\\ +\,3\,\eta^{2}\exp(-2\gamma\eta t)[\exp(-\gamma\eta t)-1]^{-2}-\frac{1}{2}C_{2}{}^{2}[\exp(-\gamma\eta t)-1]^{\frac{6}{\gamma}}\end{array}} \qquad (35)$$

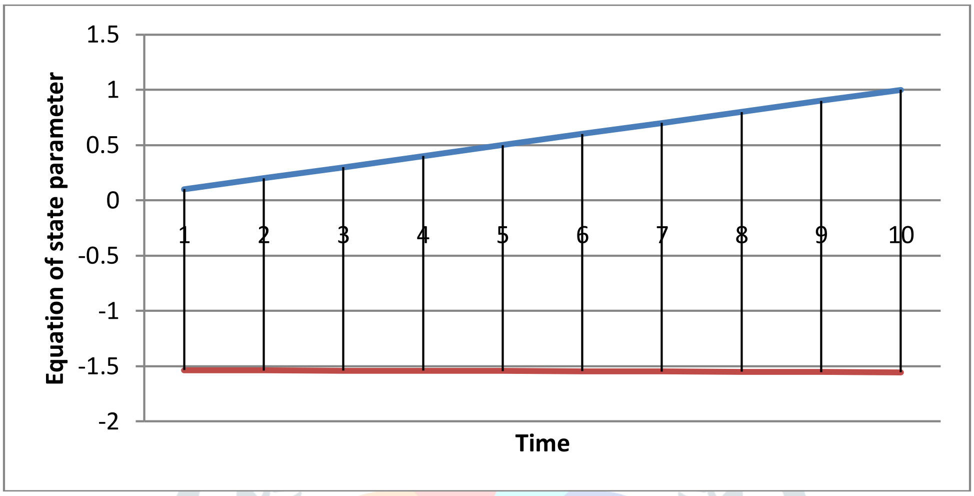

Fig.3. Equation of state parameter versus time with appropriate choice of constant.

Jerk parameter is

$$j=\frac{\dddot{a}}{a\,H^{3}}\;=\;\exp(2\gamma\eta t)\,\{\gamma^{2}[\,1+\,\exp(-\gamma\eta t)]+\,\exp(-\gamma\eta t)\,[\,3\,\gamma+\,\exp(-\gamma\eta t)]\} \qquad (36)$$

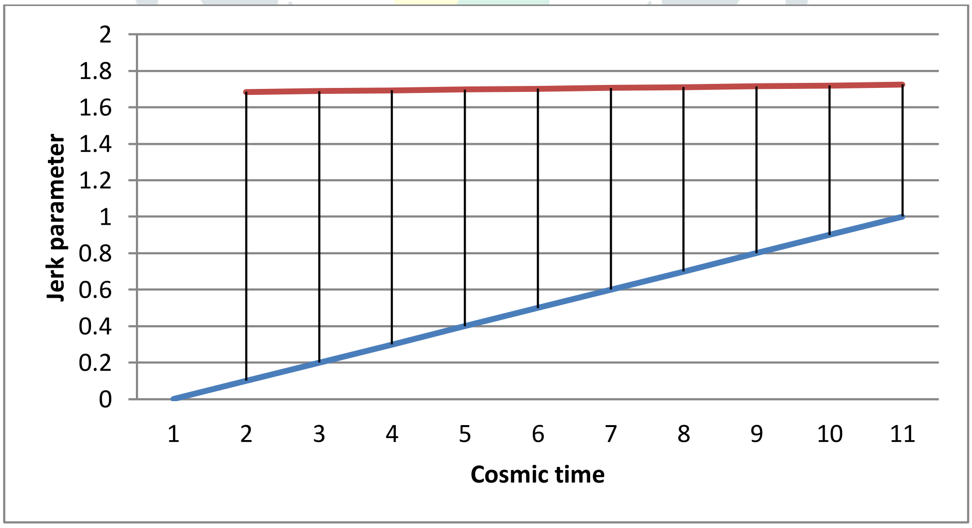

Fig.4. Jerk Parameter verses time with appropriate choice of constant

A deceleration-to-acceleration transition occurs for models with a positive value of j and flat Λ CDM models have a constant jerk $j=1$. This value shows that the jerk parameter in $f(R)$ gravity changes significantly between the deceleration-to-acceleration transition and now, indicating the departure of $f(R)$ gravity models from Λ CDM.

State finder parameter is

$$s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} = \frac{\exp(2\gamma\eta t)\{\gamma^2[1+\exp(-\gamma\eta t)]+\exp(-\gamma\eta t)[3\gamma+\exp(-\gamma\eta t)]\}-1}{3\left\{-[1+\gamma\exp(-\gamma\eta t)][\exp(-\gamma\eta t)-1]^{\frac{2}{\gamma}}-\frac{1}{2}\right\}} \quad (37)$$

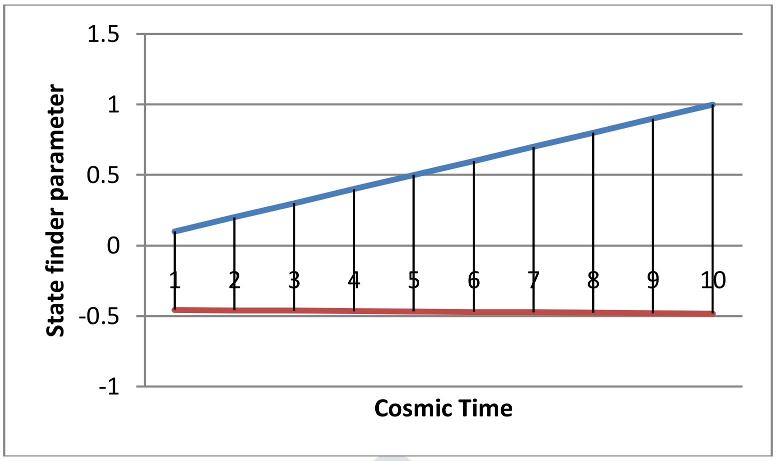

Fig.4 State finder Parameter verses time with appropriate choice of constant

The state finder parameters in two fluids coupled zero- mass scalar field in $f(R)$ Gravity are studied. We determine the values of the state finder parameters that correspond to the unique attractor of the system at hand. Furthermore, we produce plots as shown in fig.5.

Kinematical parameters of the Universe:

The **spatial volume** and the **average scale factor**, $V = a^3 = [\exp(-\gamma\eta t) - 1]^{\frac{-3}{\gamma}}$

The **generalized Hubble parameter**, $H = \frac{\eta\exp(-\gamma\eta t)}{[\exp(-\gamma\eta t)-1]}$

The **expansion scalar**, $\theta = 3H = 3\frac{\eta\exp(-\gamma\eta t)}{[\exp(-\gamma\eta t)-1]}$ and

Deceleration parameter is $q = -[1+\gamma\exp(-\gamma\eta t)][\exp(-\gamma\eta t)-1]^{\frac{2}{\gamma}}$.

We observe that the spatial volume approaches to positive small value at t → 0 and with the increase in time the universe expands exponentially. The results of generalized Hubble's parameter yield an exponential expansion. The value of expansion scalar is exponential and the sign of deceleration parameter is negative which shows the accelerating expansion of the universe as we expect in this exponential law.

V. CONCLUSION

Accelerating Expansion in Two Fluids Coupled Zero- Mass Scalar Field in $f(R)$ Gravity has been investigated in $f(R)$ theory of gravitation. The model has been investigated using the assumption that the perfect fluid's EoS parameter is a constant. Our derived universe displayed an acceleration phase, during which the universe is expanding much more quickly before slowing down in the future. The Hubble parameter and the scalar expansion are time-dependent functions that initially (t→0) have infinitely large values; decrease with expansion, and eventually approach to zero at large expansion. The EoS parameter, on the other hand, is time-dependent, and for short intervals, it behaves as though matter once dominated the universe, but at later times it continues to be present in the quintessence region, as shown by the value of ω > - 1. Therefore, our investigations are supported by the observational fact that the typical matter described by known particle theory is about 4% and the DE cause the accelerating expansion of the universe. This observation is also supported by several high precision observational experiments, particularly the WMAP satellite experiment (DE occupies near about 73% of the energy of the universe and dark matter is about 23% for accelerating expansion). For C_2 > 0, the universe starts out with a constant volume during expansion; for C_2 = 0, the cosmos starts out with zero volume and exponentially expands to an infinite volume. Both the shear scalar and the expansion scalar are fixed. The EoS parameter ω is time dependent, the cosmos grows with a quintessence ω > 1 region, and for a period of time it behaves as though matter formerly predominated while remaining present in later times in phantom ω < −1 region.

REFERENCES

[1] S. Perlmutter et al., Astrophys. J. 517, 5(1999).
[2] S. Perlmutter et al., Nature 391, 51 (1998).
[3] A. G. Riess et al., Astronomical Society of the Pacific 112, 1284 (2000).
[4] J. L. Torny et al., Astrophys. J. 594, 1(2003).
[5] P. De Bernardis et al., Nature 391, 5 (1998).
[6] D. N. Spergel et al., Astrophys. J.Supplement Series 148, 175 (2003).
[7] M. Tegmark et al., Phys. Rev. D 69, 103501(2004).
[8] T. Padmanabhan (2003) Phys. Rep. 380 235.
[9] P.J.E. Peebles, B. Ratra (2003) Rev. Mod. Phys. 75 559.
[10] E.J. Copeland, M. Sami, S. Tsujikawa (2006) Int. J. Mod. Phys. D 15 1753.
[11] M. Li, X.D. Li, S. Wang, Y. Wang (2011) Commun. Theor. Phys. 56 525.

[12] A. G. Riess et al. (1998) Astron. J. 116, 1009-1038
[13] S. Nojiri and S.D. Odintsov (2007) Phys. Lett. B 657, 238-245
[14] P. A. R. Ade, et al. (2016) Astron. Astrophys. 594, A13
[15] R. Massey et al. (2007) Nature 445, 286-290
[16] G. Bertone, D. Hooper and J. Silk (2005) Phys. Rep. 405, 279-390
[17] K. Garrett and G. Duda (2011) Adv. Astron., 968283
[18] J. L. Feng, Ann. Rev. Astron (2010) Astrophys. 48, 495-545
[19] S. Capozziello and M. De Laurentis (2011) Phys. Rep. 509, 167-321
[20] T. Clifton et al. (2012) Phys. Rep. 513, 1-189
[21] S. Nojiri, S.D. Odintsov and V.K. Oikonomou (2017) Phys. Rept. 692, 1-104
[22] S. Nojiri and S.D. Odintsov (2011) Phys. Rept. 505, 59-144
[23] S. Nojiri and S.D. Odintsov (2011) TSPU Bulletin N 8(110), 7
[24] T. P. Sotiriou and V. Faraoni Rev (2010) Mod. Phys. 82, 451
[25] A. De Felice and S. Tsujikawa (2010) Living Rev. Rel. 13, 3
[26] Chirde V, Shekh S: (2015) Astrofiz. 58,1, 121-133
[27] Bhoyar S. R., Chirde V. R., Shekh S.H. (2015) Inter. J. of Adv. Resea. 3 (9) 492
[28] Capozziello S, Martin-Moruno P, Rubano C. (2008) Phys. Lett. B664, 12-15
[29] Azadi A, Momeni D, Nouri-Zonoz M. (2008) Phys. Lett. B670, 210-214
[30] Katore S.D. and Shaikh A. (2014) Afr.Rev.Phys. 9, 0055
[31] Miranda V, Joras S, Waga I, Quartin M. (2009) Phys. Rev. Lett. 102, 221101
[32] Sharif M, Yousaf Z. J. Cosmo. (2014) Astropart. Phys. 06- 019
[33] V.R. Chirde and S.H. Shekh (2016) Bulg. J. Phys. 43, 156–169
[34] Singh, R., Deo, S. (1986) Actaphysicahungarica49, 321
[35] S. Katore, A. Shaikh, M. Sancheti, J. Pawade (2012) Prespacetime J. 3 83-89.
[36] V. Chirde, P. Rahate (2012) Int J. Theor. Phys. 51, 2262-2271.
[37] Akarsu & Kumar (2012) The European Physical Journal Plus, 127,64
[38] Saha, B., Amirhashchi, H. and Pradhan, A. (2012) Astrophysics and Space Science, 342, 257-267.