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EXACT SOLUTIONS OF EINSTEIN'S FIELD EQUATIONS FOR HIGHER DIMENSIONS

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Abstract :

In the present paper we have found interior solutions of the higher dimensional Einstein's field equations in three different cases using judicious choice of material density ρ and specific relation between ρ and p which are physically interesting. For example in Case I, we have chosen ρ = H (constant), in case II, $\rho = \rho_c \left(1 - \frac{r^2}{r_0^2}\right)^{\mu}$ where r_0 is radius of sphere, ρ_c is density at r = 0 and μ is +ve integer, in case-III, ρ + 3p = k (constant). The special case of these solutions provide different analogues of earlier works by different researchers in four dimensions.

Key Words :

Dimension, density, pressure, energy momentum tansor.

1. INTRODUCTION

In these days various relativists have paid their attention towards the study of Einstein's field equations in higher dimensions. As a matter of fact, in view of the recent developments in superstring theory (Schwarz [26], Weinberg et. al. [30]) as well as in supergravity theory. It becomes much more important and necessary to discuss theories in more than four-dimensional space-time. Kolb [13] has provided the ideas about the role of extra dimensions is cosmology. Panigrahi, Paul and Chatterjee [22] have studied a higher dimensional cosmological model and explained the recent acceleration with a chaplygin type of gas. Dimensional reduction of extra space is possible in this case. Their solutions are general in nature because all the well known results of 4D chaplygin driven cosmology are recovered when D = 4. Emelyanov et. al. [6] have analyzed the physical phenomena in Riemannian space of arbitrary dimension. Duff et. al. [5] have reviewed the Kaluza-Klein supergravity. Myers [21] and Myera and Perry [20] have obtained the higher dimensional analogues of the metrics of Schwarzachild. Reissnet-Nordstrom and Kerr (Refer Chodos and Detweiler [4], Gibbons and Wiltshire [8], Frolv et. al. [7]. The Reissner-Nordstrom-de Sitter and Rerr – de Sitter metrics for the higher dimensional space-time have been found and present by Xu Dianyan [32]. The Schwarzschild interior solution in higher dimensions when the matter density p is constant have been given by Kroriet at. al. [14]. They have also found morder like solution in higher dimensions [16]. Shen & Tan [27] have found higher dimensional generalization of the interior Wyman solution. Generalization of the interior solution of Gonzalez-Diaz has been obtained by Shen and Tan [28] using equation of state p^4 ρ = 0. Iyer and Vishveshwara [11] have presented Vaidya's radiating star in higher dimensions where as higher dimensional Vaidya metric with an electromagnetic has been obtained by Chetterjee et. al. [3]. Liddle et. al. [17] have discussed the conneguences of the existence of hidden extra dimensions for the structure of meutron stars. Some other workers in this line are Ahmad and Alamri [1] Chatterjee and Panigrahi [2], Panigrahi et. al. [22, 23, 24], Ibrahim and Nutku [10].

Here in this paper, we have obtained interior solutions of the higher dimensional Einstein field equations in different cases using judicius choice of ρ and specific relations between ρ and p which are physically interesting. For example in case I, we have chosen ρ = H (constant) in case II, $\rho = \rho_c \left(1 - \frac{r^2}{r_0^2}\right)^{\mu}$ where r_0 is radius of sphere, p_c in density at r = 0 and μ is +ve integer, in case III, ρ + 3p = K (constant). The special cases of these solutions provide different analogues of earlier works by different workers in four dimensions.

2. FIELD EQUATIONS

The static spherically symmetric metric for higher dimensions (D = n + 3) is given by

(2.1) $$ds^2 = e^B dt^2 - e^A dr^2 - r^2(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1$$

$$....\sin^2\theta_n d\theta_{n+1}^2), \text{ etc.}$$

where B and A are functions of radial coordinate r alone, Here

$$x^0 = t, x^1 = r, x^2 = \theta_1, x^3 = \theta_2$$

when D = 4 it reduces to the well known four dimensional spherically symmetry metric

we use Einstein's field equations for the D-dimensional

Space-time given by Sahdev [25] which are

(2.2) $$R_{ij} = -8\pi\left(T_{ij} - \frac{1}{D-2} g_{ij} T_k^k\right)$$

where we have taken $c = G = 1$ (in natural units). The energy momentum tension T_j^i is found to be

(2.3) $$T_j^i = \text{diag}\ (\underbrace{\rho, -p, -p, -p, ...-p}_{n+2})$$

where p is the pressure and ρ is material density. The field equations (2.2) for the metric (2.1) take the form

$$(2.4)\quad e^{-A}\left(\frac{A'}{r}-\frac{n}{r^2}\right)+\frac{n}{r^2}=\frac{16\pi}{(n+1)}\rho$$

$$(2.5)\quad e^{-A}\left(\frac{B'}{r}+\frac{n}{r^2}\right)-\frac{n}{r^2}=\frac{16\pi}{(n+1)}p$$

$$(2.6)\quad e^{-A}\left(\frac{B''}{2}-\frac{A'B'}{4}+\frac{B'^2}{4}-\frac{(B'+nA')}{2r}-\frac{n}{r^2}\right)+\frac{n}{r^2}=0$$

$$(2.7)\quad p'+(p+\rho)\frac{B'}{2}=0$$

where prime denotes differentiation with respect to r and we have taken G = 1, c = 1 (in natural units)

It is well known that equation (2.7) can be obtained from equations (2.4) – (2.6)

3. SOLUTIONS OF THE FIELD EQUATIONS

Here we have four unknowns A, B, ρ and p in three equations. So the system is inderminate. To make the system determinate we need one more relation or condition. For this we have chosen judicious choice of density ρ or suitable relation between p and ρ which are physically interesting.

Case – I

Here we choose density ρ = H (constant)

The integration of equation (2.4) provides

$$(3.1)\quad e^{A}=\frac{(n+1)(n+2)}{[(n+1)(n+2)-16nHr^2]}$$

From equation (2.7), we find

(3.2) $p = k_1 e^{-B/2} - H$

where k_1 is constant of integration

By use of equation (3.1) and (3.2) in equation (2.5), we get

(3.3) $e^B = \left[\frac{(n+2)k_1}{2K} + k_2\left\{(n+1)(n+2) - 16nHr^2\right\}^{\frac{1}{2}}\right]^2$

Using equation (3.3) in equation (2.9), we find

(3.4) $p = -\frac{nk_1H + 2k_2H^2\left\{(n+1)(n+2) - 16nHr^2\right\}^{1/2}}{\left[(n+2)k_1 + 2k_2H\left\{(n+1)(n+2) - 16nHr^2\right\}^{1/2}\right]}$

This is higher-dimensional analogue of Schwarzachild four dimensional interior solution, Similar solution has been obtained by Krori et. al. [15] also.

Case II

Here we chose $\rho = \rho_c\left(1 - \frac{r^2}{r_0^2}\right)^{\mu}$

where r is radius of the sphere and μ is a positive integer; ρ_c is the matter density at the centre r = 0. Using this relation equation (2.4) can be put as

(3.5) $-e^{-A}A' + \frac{n}{r}e^{-A} = \frac{n}{r} - \frac{16\pi\rho_c r}{(n+1)}\left(1 - \frac{r^2}{r_0^2}\right)^{\mu}$

The solution of equation (3.5) can be written as

(3.6) $e^{-A} = 1 + \frac{16\pi\rho_c}{(n+1)}\sum_{i=1}^{\mu+1}(-1)^i\binom{n}{i-1}\frac{r^{2i}}{r_0^{2(i-1)}(2i+n)}$

Here constant of integration has been taken as zero to avoid singularity at r = 0

Now we try for special solution when μ =1

In this case we find from equation (3.6)

$$(3.7)\quad e^{-A}=1-\frac{16\pi\rho_c}{(n+1)}\left[\frac{r^2}{(n+2)}-\frac{r^4}{r_0^2(n+4)}\right]$$

By taking y = A/2 and x = r^2 and using equation (3.7). We get from equation (2.6) get

$$(3.8)\quad \left[1-\frac{16\pi\rho_c}{(n+1)}\left\{\frac{x}{(n+2}-\frac{x^2}{r_0^2(n+4)}\right\}\right]\frac{d^2y}{dx^2}$$

$$-\frac{8\pi\rho_c}{(n+1)}\left[\frac{1}{(n+2)}-\frac{2x}{r_0^2(n+4)}\right]\frac{dy}{dx}+\left[\frac{4\pi\rho_c}{(n+1)(n+4)r_0^2}\right]y=0$$

To solve equations (3.8) we use the transformation given by

$$(3.9)\quad \theta=\frac{\sqrt{n}}{2}\cosh^{-1}$$

$$\left[\frac{2\pi\rho_c(2(n+2)x-(n+4)r_0^2}{a[(n+4)\pi\ \{4(n+4)\pi\rho_c^2r_0^2-(n+1)(n+2)^2\rho_c\}]^{1/2}}\right]$$

Then equation (3.8) goes to the form

$$(3.10)\quad \frac{d^2y}{d\theta^2}+y=0$$

Solution of equation (3.10) may be written as

$$(3.11)\quad \left[\begin{array}{l} y=\phi\sin\theta+\psi\cos\theta \\ \text{or} \\ e^B=(\phi\sin\theta+\psi\cos\theta)^2 \end{array}\right.$$

where ϕ and ψ are constants of integration.

Substituting the values of A and B from equations (3.7) and (3.11) into equation (2.5) the pressure is found to be

$$(3.12)\quad p=\left[\frac{n(n+1)\rho_c}{4\pi(n+4)A^2}\right]^{\frac{1}{2}}\left[1-\frac{16\pi\rho_c}{(n+1)}\left\{\frac{r^2}{(n+2)}-\frac{r^4}{(n+4)r_0^2}\right\}\right]^{\frac{1}{2}}$$

$$\left[\frac{\phi\cos\theta-\psi\sin\theta}{\theta\sin\theta+\psi\cos\theta}\right]-\frac{n\rho_c}{(n+2)}+\frac{n\rho_c^2r^2}{(n+4)r_0^2}$$

This may be considered as higher dimensional analogue of Mehra's solution [18] in four dimension.

The solutions for other values of μ i.e. density distribution ρ can be considered as done by Knutsen [12] in four dimensions but the equations are more complicated and so we do not try for them and leave them

Case III

Here we choose $\rho + 3p = k$ (constant)

The equation (2.7) in this case reduces to

$$(3.13)\quad \frac{dp}{dr}+(k=2p)\frac{B'}{2}=0$$

Integration of (3.3.13) finally provides the solution

$$(3.14)\quad 2p-k=k_3e^B$$

Where k_3 is a constant of integration.

Now Adding equation (2.4) and (2.5) and using equation (3.14), we get after integration

$$(3.15)\quad e^{A+B}=\frac{(n+1)}{[(n+1)k_4+8\pi k_3r^2]}$$

Using equations (3.14) and (3.15) in (2.5) we find

$$(3.16)\quad e^B=\frac{n\left[(n+1)k_4+8\pi k_3r^2\right]^{1/2}}{A\sqrt{(n+1)r^n}}\left[\int r^{n-1}\zeta dr+\frac{8\pi k}{n(n+1)}\int r^{n+1}\zeta dr\right]$$

and

$$(3.17)\quad e^{A} = \frac{r^{n}}{n}\zeta\left[\int r^{n-1}\zeta dr + \frac{8\pi c}{n(n+1)}\int r^{n+1}\zeta dr\right]^{-1}$$

where

$$\zeta = \left[k_4 + \frac{8\pi k_3}{(n+1)}r^2\right]^{-3/2}$$

Now we can write pressure and density in the form given by

$$(3.18)\quad p = \frac{1}{2}k + \frac{nk_3\{(n+1)k_4 + 8k_3 r\}^{1/2}}{2r^{n}\sqrt{(n+1)}}$$

$$\left[\int r^{n-1}\zeta dr + \frac{8\pi k}{n(n+1)}\int r^{n+1}\zeta dr\right]$$

$$(3.19)\quad \rho = -\frac{1}{k} + \frac{3nk_3\left\{(n+1)k_4 + 8k_3 r^2\right\}^{1/2}}{r^{n}\sqrt{(n+1)}}$$

$$\left[\int r^{n-1}\zeta dr + \frac{8\pi k}{n(n+1)}\int r^{n+1}\zeta dr\right]$$

Which may be considered as higher dimensional analogue of Whittaker's solution [31] in four dimensions for constant effective gravitational mass density (ρ + 3p).

4. REMARKS:

Solutions obtained here in this paper are very significant in the way that they provide higher dimensional analogues of different workers in four dimensions (e.g. in Case I Schwareschild solution and solution by Krori et. al. [5] in case II, Mehra solution [18] and Knutsen [12] in case III, Wittakar's woluton [31] in four dimensions.

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