JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

A NEW CLASS OF CLOSED SETS IN NANO TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce a new class of closed sets, namely $\boldsymbol{N\delta g^{\wedge}}$ **- closed sets** and $N\delta g^{\wedge}$ - closed sets in Nano topological spaces. Further we investigate fundamental properties are discussed. Additionally we relate with some other Nano topological spaces.

Keywords : Nano topological spaces, generalized closed sets, δg - closed sets, $\delta - closure,\ \ g^{\wedge}$ - open sets.

1 INTRODUCTION

The concept of generalized closed sets as a generalization of closed sets in Topological Spaces was introduced by Levine[10] in 1970.This concept was found to be useful and many results in general topology were improved S. M. Lellis Thivagar [1] introduced Nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. He has also defined Nano closed sets, Nano-interior and Nano-closure of a set. He also introduced the weak forms of Nano open sets. In 2014, K.Bhuvaneswari et al., A. Ezhilarasi introduced the concept of Nano semi-generalized and Nano generalized-semi closed sets in Nano topological spaces. K. Bhuvaneswari and K.Mythili Gnanapriya [7] introduced Nano g-closed sets and obtained some of the basic results. In this paper, we define a study on new class of closed sets is called $\boldsymbol{N\delta g^{\wedge}}$ **- closed sets** in Nano topological space and study the relationships with other Nano sets.

2 PRELIMINARIES

Throughout this chapter$(U,\tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R. Here we recall the following known definitions and properties.

Definition 2.1[8] Let U be a non empty finite set of objects called the ***universe*** and R be a equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be discernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$

1. The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = U_{x\in U}\{R(x) / R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by X.

2. The upper approximation of X with respect to R is the set of all objects which can be possibly defined as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = U_{x\in U}\{R(x) / R(x) \cap X \neq \phi\}$

3. The boundary region of X with respect to R is the set of all objects which can neither as X nor as not X with respect to R and is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2[2] If (U, R) is an approximation space and $X, Y \subseteq U$, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\emptyset) = U_R(\emptyset) = \emptyset$ and $L_R(U) = U_R(U) = U$
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
6. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition 2.3[1] Let U be the universe, R be an ***equivalence relation*** on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the proposition 2.2, $R(X)$ satisfies the following axioms:

1. U and $\emptyset \in \tau_R(X)$
2. The union of the elements of any subcollection of $(U, \tau_R(X))$ is in $(U, \tau_R(X))$.
3. The intersection of the elements of any finite subcollection of $(U, \tau_R(X))$ is in $(U, \tau_R(X))$.

That is $(U, \tau_R(X))$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $(U, \tau_R(X))$ are called as Nano open sets.

Remark 2.4[1] If $(U, \tau_R(X))$ is the Nano topology on U with respect to X, then the set $B = \{U, \emptyset, L_R(X), B_R(X)\}$ is the ***basis*** for $\tau_R(X)$.

Definition 2.5[1] If $(U, \tau_R(X))$ is a Nano topological space with respect to X and if $A \subseteq U$, then the ***Nano interior*** of A is defined as the union of all Nano-open subsets of A and is denoted by $Nint(A)$.That is, $Nint(A)$ is the largest Nano-open subset of A.

The ***Nano closure*** of A is defined as the intersection of all Nano -closed sets containing A and it is denoted by $Ncl(A)$. That is, $Ncl(A)$ is the smallest Nano-closed set containing A.

Definition 2.6[1,5] Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then A is said to be

(i) ***Nano pre-open*** if $A \subseteq Nint(Ncl(A))$

(ii) ***Nano semi-open*** if $A \subseteq Ncl(Nint(A))$

(iii) ***Nano α-open*** if $A \subseteq Nint(Ncl(Nint(A)))$

The complements of the above mentioned sets are called their respective ***Nano-closed*** sets.

Definition 2.7[7] Let $(U, \tau_R(X))$ be a Nano topological space. A subset A of $(U, \tau_R(X))$ is called ***Nano generalized-closed set*** (briefly $Ng -$ closed) if $Ncl(A) \subseteq V$ where $A \subseteq V$ and V is Nano-open.
The complement of Nano generalized -closed set is called as ***Nano generalized-open set***.

Definition 2.8[9] For every set $A \subseteq U$, the ***Nano generalized closure of A*** is defined as the intersection of all Ng- closed sets containing A and is denoted by $Ng - cl(A)$.

Definition 2.9[9] For every set $A \subseteq U$, the ***Nano generalized interior of A*** is defined as the union of all Ng-open sets contained in A and is denoted by $Ng - int(A)$.

Proposition 2.10[9] For any $A \subseteq U$ **,**
(i) $NgCl(A)$ is the smallest Ng closed set containing A.
(ii) A is Ng- closed if and only if $NgCl(A) = A$.
(iii) $A \subseteq NgCl(A) \subseteq Cl(A)$

Proposition 2.11[9] For any two subsets A and B of U,
(i) If $A \subseteq B$, then $NgCl(A) \subseteq NgCl(B)$
(ii) $NgCl(A \cap B) \subseteq NgCl(A) \cap NgCl(B)$

Definition 2.12 The nano δ – interior [13] of a subset A of X is the union of all regular open set of X contained in A and is denoted by $NInt_{\delta}(A)$. The subset A is called $N\delta$-open [13] if $A = NInt_{\delta}(A)$, i.e. a set is $N\delta$-open if it is the union of regular open sets. The complement of a $N\delta$-open is called $N\delta$-closed. Alternatively, a set $A \subseteq (U, \tau_R(X))$ is called $N\delta$-closed [13] if $A = Ncl_{\delta}(A)$, where $Ncl_{\delta}(A) = \{x \in X{:}\, int(cl(U) \cap A \neq \emptyset,\ U \in \tau_R(X)\ and\ x \in U\}$.

Definition 2.13 A subset A of $(U, \tau_R(X)$ is called

- Nano generalized closed (briefly Ng-closed) set[5] if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano open set in $(U, \tau_R(X)$.
- Nano semi-generalized closed (briefly Nsg-closed) set [5] if $Nscl(A) \subseteq U$ whenever $A \subseteq U$ and U is a nano semi-open set in $(U, \tau_R(X))$.
- Nano generalized semi-closed (briefly Ngs-closed) set [4] if $Nscl(A) \subseteq U$ wheneve $A \subseteq U$ and U is nano open set in $(U, \tau_R(X))$.
- Nano α- generalized closed (briefly $N\alpha g$-closed) set [15] if $N\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano open set in $(U, \tau_R(X))$.
- Nano generalized α- closed (briefly $Ng\alpha$-closed) set [6] if $N\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano α-open set in $(U, \tau_R(X))$.
- Nano δ-generalized closed (briefly $N\delta g$-closed) set [3] if $Ncl_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and U is nano open set in $(U, \tau_R(X))$.
- $Ng^{\wedge}$ -closed set [12] if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is a nano semi-open set $(U, \tau_R(X))$.
- $N\alpha g^{\wedge}$-closed set [9] if $N\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\alpha g^{\wedge}$- open set in $(U, \tau_R(X))$.

The complement of $N\alpha g$-closed (resp. Nsg-closed, Ngs-closed, $N\alpha g$-closed, $Ng\alpha$-closed $N\delta g$-closed and $Ng^{\wedge}$-closed and$N\alpha g^{\wedge}$-closed)) set is called Ng-open (resp. Nsg -open,

Ngs -open, $N\alpha g$ -open, $Ng\alpha$ -open, $N\delta g$ -open, $Ng^{\wedge}$-open and $N\alpha g^{\wedge}$-open).

Definition 2.13 A subset A of $(U,\tau_R(X)$ is called

(i) $N\alpha g^*$-closed set [6] if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano α-open set in $(U,\tau_R(X))$.

(ii) $Nr^{\wedge}g$-closed set [6] if $Ngcl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano regular open set in $(U,\tau_R(X))$.

(iii)Nano Strongly generalized closed (briefly Ng^*-closed)[8] if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano g-open set in $(U,\tau_R(X))$.

(iv) A nano regular generalized closed (briefly Nrg-closed)[13] if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano regular-open set in $(U,\tau_R(X))$.

(v) A nano weakly closed (briefly w-closed)[] if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano semi-open set in $(U,\tau_R(X))$.

3 $N\delta g^{\wedge}$-closed sets

We introduce the following definition.

Definition 3.1 A subset A of a space $(U,\tau_R(X))$ is called $\boldsymbol{N\delta g^{\wedge}}$**-closed** if $Ncl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is a $Ng^{\wedge}$-open set in $(U,\tau R(X))$.

Proposition 3.2 Every $N\delta$ – closed set is $N\delta g^{\wedge}$ - closed set.

proof : Let A be an $N\delta$ -closed set and U be any $Ng^{\wedge}$-open set containing A. Since A is $N\delta$-closed , $Ncl_\delta(A) = A$ for every subset A of U. Therefore $Ncl_\delta(A) \subseteq U$ and hence A is $N\delta g^{\wedge}$- closed set.

Remark 3.3 The converse of the above theorem is not true as shown in the following example.

Example 3.4 Let $U = \{a,b,c,d\}$ with $\frac{U}{R} = \{\{a\},\{c\},\{b,d\}\}$ $X = \{a,b\}$. Then $\tau R(X) = \{U,\emptyset,\{a\},\{b,d\},\{a,b,d\}\}$ $N\delta g^{\wedge}C(U,\tau_R(X) = \{U,\emptyset,\{c\},\{b,c\},\{c,d\},\{a,c\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$ Clearly the sets $N\delta C = \{U,\emptyset,\{c\},\{a,c\},\{b,c,d\}\}$ Here $\{b,c\}$ is $N\delta g^{\wedge}C$ but not $N\delta C$ in $(U,\tau R(X))$.

Proposition 3.5 Every $N\delta g^{\wedge}$ – closed set is Ng - closed set.

proof : Let A be an $N\delta g^{\wedge}$ -closed set and U be an any open set containing A in $(U,\tau R(X))$. Since every open set is $Ng^{\wedge}$-open and A is $N\delta g^{\wedge}$-closed, $Ncl_\delta(A) \subseteq U$ for every subset A of X. Since $Ncl(A) \subseteq Ncl_\delta(A) \subseteq U, Ncl(A) \subseteq U$ and hence A is Ng-closed.

Remark 3.6 The converse of the above theorem is not true as shown in the following example.

Example 3.7 Let $U = \{a,b,c,d\}$ with $\frac{U}{R} = \{\{a\},\{c\},\{b,d\}\}$ $X = \{a,b\}$. Then $\tau R(X) = \{U,\emptyset,\{a\},\{b,d\},\{a,b,d\}\}$ $N\delta g^{\wedge}C(U,\tau_R(X) = \{ U,\emptyset,\{c\},\{b,c\},\{c,d\},\{a,c\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$ and $NgC(U,\tau_R(X) = \{ U,\emptyset,\{c\},\{b,c\},\{c,d\},\{a,c\},\{a,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$ Clearly the set $\{a,d\}$is Ng - closed set but not $N\delta g^{\wedge}$ – closed set in $(U,\tau R(X))$.

Proposition 3.8 Every $N\delta g^{\wedge}$ – closed set is Ngs - closed set.

proof: Let A be an $N\delta g^{\wedge}$-closed and U be any open set containing A in $(U, \tau R(X))$. Since every open set is $Ng^{\wedge}$-open, $Ncl_{\delta}(A) \subseteq U$ for every subset A of U. Since $Nscl(A) \subseteq Ncl_{\delta}(A) \subseteq U, Nscl(A) \subseteq U$ and hence A is Ngs -closed.

Remark 3.9 A Ngs -closed set need not be $N\delta g^{\wedge}$-closed as shown in the following example.

Example 3.10 Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{d\}, \{b, c\}\}$ $X = \{a, c\}$. Then $\tau R(X) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ $N\delta g^{\wedge}C(U, \tau_R(X) = \{ U, \emptyset, \{d\}, \{b, d\}, \{c, d\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $NgsC(U, \tau_R(X) = \{ U, \emptyset, \{a\}\{b\}, \{c\}, \{d\}, \{b, d\}, \{c, d\}, \{a, d\}, \{b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Here the set$\{a\}$ is Ngs – closed set but not $N\delta g^{\wedge}$ – closed set in $(U, \tau R(X))$.

Proposition 3.11 Every $N\delta g^{\wedge}$ – closed set is $N\alpha g$ - closed set.

proof: It is true that $N\alpha cl(A) \subseteq Ncl_{\delta}(A)$ for every subset A of U.

Remark 3.12 A $N\alpha g$ -closed set need not be $N\delta g^{\wedge}$-closed as shown in the following example.

Example 3.13 Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{c\}, \{b, d\}\}$ $X = \{a, b\}$. Then $\tau R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ $N\delta g^{\wedge}C(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ and $N\alpha g\mathrm{C}(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$. Here the set $\{a, d\}$ is $N\alpha g$ - closed set but not $N\delta g^{\wedge}$ – closed set in $(U, \tau R(X))$.

Proposition 3.14 Every $N\delta g^{\wedge}$ – closed set is $N\delta g$ - closed set.

proof: Let A be an $N\delta g^{\wedge}$-closed set and U be any open set containing A. Since every open set is $Ng^{\wedge}$-open, $Ncl_{\delta}(A) \subseteq U$, whenever $A \subseteq U$ and U is $Ng^{\wedge}$- open. Therefore $Ncl_{\delta}(A) \subseteq U$ and U is open. Hence A is $N\delta g$ -closed.

Remark 3.15 A $N\delta g$ -closed set need not be $N\delta g^{\wedge}$-closed as shown in the following example.

Example 3.16 Let $U = \{a, b, c\}$ with $\frac{U}{R} = \{\{a\}, \{b, c\}\}$ $X = \{a, b\}$. Then $\tau R(X) = \{U, \emptyset, \{a\}, \{b, c\}\}$ $N\delta g^{\wedge}C(U, \tau_R(X) = \{\{U, \emptyset, \{a\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and $N\delta g\mathrm{C}(U, \tau_R(X) = \{ U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Here the sets $\{b\}$ and $\{c\}$is A $N\delta g$ -closed sets but not $N\delta g^{\wedge}$ – closed set in $(U, \tau R(X))$.

Remark 3.17 The class of $N\delta g^{\wedge}$-closed sets is properly placed in the class of $N\alpha g^{\wedge}$-closed set.

Example 3.18 Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{c\}, \{b, d\}\}$ $X = \{a, b\}$. Then $\tau R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ $N\delta g^{\wedge}C(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ and $N\alpha g^{\wedge}\mathrm{C}(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$.

Proposition 3.19 Every $N\delta g^{\wedge}$ – closed set is Nsg - closed set.

proof: It is true that $Nscl(A) \subseteq Ncl_{\delta}(A)$ for every subset A of $(U, \tau_R(X)$.

Remark 3.20 A Nsg -closed set need not be $N\delta g^{\wedge}$-closed as shown in the following example.

Example 3.21 Let $U = \{a, b, c\}$ with $\frac{U}{R} = \{\{a\}, \{b, c\}\}$ $X = \{a, b\}$. Then $\tau R(X) = \{U, \emptyset, \{a\}, \{b, c\}\}$ $N\delta g^{\wedge}C(U, \tau_R(X) = \{\{U, \emptyset, \{a\}, \{a, b\}, \{b, c\}, \{a, c\}\}$and $NsgC(U, \tau_R(X) = \{ U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Here the sets $\{b\}$ and $\{c\}$are Nsg -closed sets but not $N\delta g^{\wedge}$ – closed set in $(U, \tau R(X))$.

Proposition 3.22 Every $N\delta g^{\wedge}$ – closed set is $Ng\alpha$ - closed set.

proof: It is true that $N\alpha cl(A) \subseteq Ncl_{\delta}(A)$ for every subset A of $(U, \tau_R(X)$.

Remark 3.23 A $Ng\alpha$ -closed set need not be $N\delta g^{\wedge}$-closed as shown in the following example.

Example 3.24 Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{d\}, \{b, c\}\}$ $X = \{a, c\}$. Then $\tau R(X) N\delta g^{\wedge}C(U, \tau_R(X) = \{ U, \emptyset, \{d\}, \{b, d\}, \{c, d\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $Ng\alpha C(U, \tau_R(X) = \{ U, \emptyset, \{d\}, \{b, d\}, \{c, d\}, \{a, d\}, \{a, c\}, \{a, b\}\{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Here the set $\{a, c\}$ and $\{a, d\}$ are $Ng\alpha$ – closed sets but not $N\delta g^{\wedge}$ – closed set in $(U, \tau R(X))$.

Proposition 3.25 Every $N\delta g^{\wedge}$ – closed set is $Ng^{\wedge}$ - closed set.

proof: It is true that $Ncl(A) \subseteq Ncl_{\delta}(A)$ for every subset A of $(U, \tau_R(X)$.

Remark 3.26 A $Ng^{\wedge}$ -closed set need not be $N\delta g^{\wedge}$-closed as shown in the following example.

Example 3.27 Let $U = \{a, b, c\}$ with $\frac{U}{R} = \{\{a\}, \{b, c\}\}$ $X = \{a, b\}$. Then $\tau R(X) = \{U, \emptyset, \{a\}, \{b, c\}\}$ $N\delta g^{\wedge}C(U, \tau_R(X) = \{\{U, \emptyset, \{a\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and $Ng^{\wedge}C(U, \tau_R(X) = \{ U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Here the sets $\{b\}$ and $\{c\}$are $Ng^{\wedge}$ -closed sets but not $N\delta g^{\wedge}$ – closed set in $(U, \tau R(X))$.

Proposition 3.28 Every $N\alpha$ – closed set is $N\delta g^{\wedge}$ - closed set.

proof : Let A be an $N\alpha$ -closed set and U be any $Ng^{\wedge}$-open set containing A. Since A is $N\alpha$-closed , $Ncl_{\delta}(A) = A$ for every subset A of U. Therefore $Ncl_{\delta}(A) \subseteq U$ and hence A is $N\delta g^{\wedge}$- closed set.

Remark 3.29 The converse of the above theorem is not true as shown in the following example.

Example 3.30 Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{d\}, \{b, c\}\}$ $X = \{a, c\}$. Then $\tau R(X) = N\delta g^{\wedge}C(U, \tau_R(X) = \{ U, \emptyset, \{d\}, \{b, d\}, \{c, d\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $N\alpha C(U, \tau_R(X) = \{ U, \emptyset, \{d\}, \{a, d\}, \{b, c, d\}\}$. Here the set$\{b, d\}$ and$\{a, b, d\}$are $N\delta g^{\wedge}$ – closed sets but not $N\alpha$ – closed set in $(U, \tau R(X))$.

Proposition 3.31 Every $N -$ closed set is $N\delta g^{\wedge}$ - closed set.

Example 3.32 Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{c\}, \{b, d\}\}$ $X = \{a, b\}$. Then $\tau R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ $N\delta g^{\wedge}C(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ and $NC(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{a, c\}, \{b, c, d\}\}$.

Proposition 3.33 Every $N\delta g^{\wedge}$ – closed set is $N\alpha g^{*}$- closed set.

proof : It is true that $Ncl(A) \subseteq Ncl_{\delta}(A)$ for every subset A of $(U, \tau_R(X)$.

Remark 3.34 The converse of the above theorem is not true as shown in the following example.

Example 3.35 Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{d\}, \{b, c\}\}$ $X = \{a, c\}$. Then $\tau R(X) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ $N\delta g^{\wedge}C(U, \tau_R(X) = \{ U, \emptyset, \{d\}, \{b, d\}, \{c, d\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $N\alpha g^{*}(U, \tau_R(X) = \{ U, \emptyset, \{d\}, \{a, b\}, \{c, d\}, \{a, c\}, \{b, d\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\{b, c, d\}\}$. Here the sets $\{a, c\}$ and $\{a, b, c\}$ are $N\alpha g^{*}$ – closed sets but not $N\delta g^{\wedge}$ – closed set in $(U, \tau R(X))$.

Proposition 3.36 Every $N\delta g^{\wedge}$ – closed set is $Nr^{\wedge}g$- closed set.

proof: Let A be an $N\delta g^{\wedge}$-closed set and U be any open set containing A. Since every open set is $Ng^{\wedge}$-open, $Ncl_{\delta}(A) \subseteq U$ for every subset A of U. Since $Ngcl(A) \subseteq Ncl_{\delta}(A) \subseteq U, Ngcl(A) \subseteq U$ and hence A is $Nr^{\wedge}g$ -closed.

Remark 3.37 The converse of the above theorem is not true as shown in the following example.

Example 3.38 Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{c\}, \{b, d\}\}$ $X = \{a, b\}$. Then $\tau R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ $N\delta g^{\wedge}C(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ and $Nr^{\wedge}g\mathrm{C}(U, \tau_R(X) = \{ U, \emptyset, \{a\}, \{c\}, \{d\}, \{b, c\}, \{a, b\}, \{c, d\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$. Here the sets $\{a, b\}$ and $\{a, b, d\}$are $Nr^{\wedge}g$ – closed sets but not $N\delta g^{\wedge}$ – closed set in $(U, \tau R(X))$.

Proposition 3.39 Every $Ng^{*} -$ closed set is $N\delta g^{\wedge}$ - closed set.

Example 3.40 Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{b\}, \{c, d\}\}$ $X = \{b\}$. Then $\tau R(X) = \{U, \emptyset, \{b\}\}$. $N\delta g^{\wedge}C(U, \tau_R(X) = \{ U, \emptyset, \{c, d\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, d\}\}$ and $Ng^{*}C(U, \tau_R(X) = \{ U, \emptyset, \{a, b, c\}, \{a, c, d\}\}$

Proposition 3.41 Every $N\delta g^{\wedge} -$ closed set is Nrg - closed set.

proof: Let A be an $N\delta g^{\wedge}$-closed set and U be any open set containing A. Since every open set is $Ng^{\wedge}$-open, $Ncl_{\delta}(A) \subseteq U$ for every subset A of U. Since $Ncl(A) \subseteq Ncl_{\delta}(A) \subseteq U, Ncl(A) \subseteq U$ and hence A is Nrg-closed.

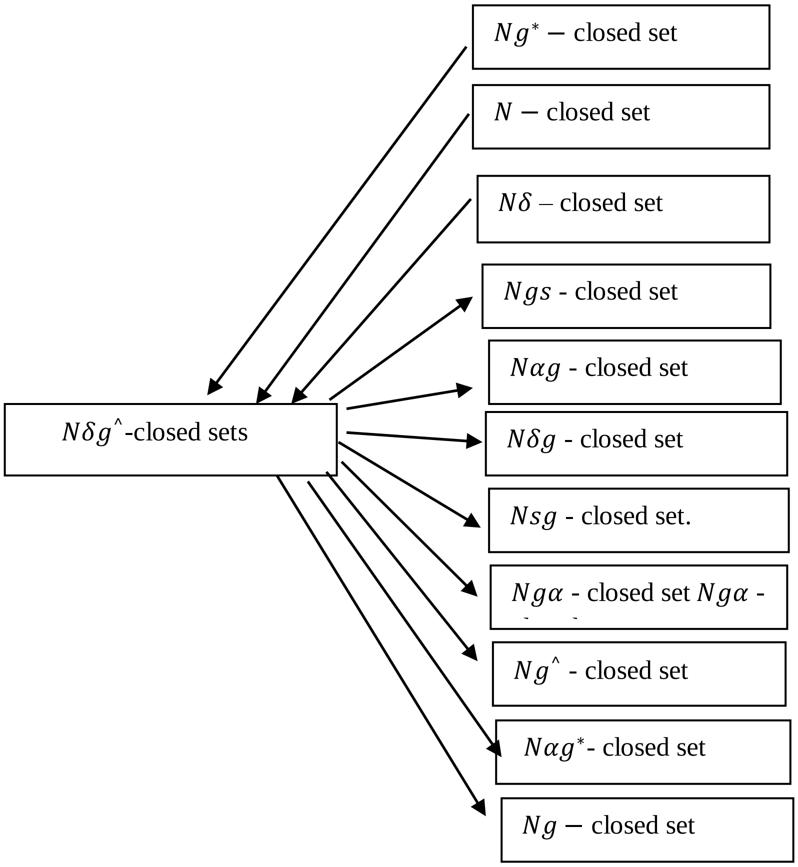
Remark 3.42 The converse of the above theorem is not true as shown in the following example.

Example 3.43 Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{c\}, \{b, d\}\}$ $X = \{a, b\}$. Then $\tau R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ $N\delta g^{\wedge}C(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ and $Nrg\mathrm{C}(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\{b, c, d\}\}$.Here the sets $\{a, b\}$, $\{a, d\}$ and $\{a, b, d\}$ are Nrg - closed sets but not $N\delta g^{\wedge}$ – closed set in $(U, \tau R(X))$.

Remark 3.43 The class of $N\delta g^{\wedge}$-closed sets is properly placed in the class of w-closed set.

Example 3.44 Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{c\}, \{b, d\}\}$ $X = \{a, b\}$. then $\tau R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ $N\delta g$ $C(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}$ and $w\mathrm{C}(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$.

Remark 3.45 The following diagram shows the relationships of $N\delta g^{\wedge}$-closed sets with other known existing sets. $A \rightarrow B$ represents A implies B but not conversely.



4 Characterisation

Theorem 4.1 The finite union of $N\delta g^{\wedge}-$ closed sets is $N\delta g^{\wedge}-$ closed.

proof: Let $\{A_i/i=1,2,\dots,n\}$ be a finite class of $N\delta g^{\wedge}-$ closed subsets of a space $(U,\tau R(X))$. Then for each $Ng^{\wedge}$-open set U_i containing A_i , $Ncl_{\delta}(A_i)\subseteq U_i, i\in\{1,2,\dots,n\}$. Hence $U_iA_i\subseteq U_i, U_i=V$. Since arbitrary union of $Ng^{\wedge}$-open sets in $(U,\tau R(X))$ is also $Ng^{\wedge}$-open set in$(U,\tau R(X))$, V is $Ng^{\wedge}$-open in $(U,\tau R(X))$. Also $U_iNcl_{\delta}(A_i)=Ncl_{\delta}(U_iA_i)\subseteq V$. Therefore U_iA_i is $N\delta g^{\wedge}$-Closed in $(U,\tau R(X))$.

Example 4.2 Let $U=\{a,b,c\}$ with $\frac{U}{R}=\{\{a\},\{b,c\}\}$ $X=\{a,b\}$. Then $\tau R(X)=\{U,\emptyset,\{a\},\{b,c\}\}$ $N\delta g^{\wedge}C(U,\tau_R(X)=\{\{U,\emptyset,\{a\},\{a,b\},\{b,c\},\{a,c\}\}$ Here the sets $\{a\}$ and $\{a,b\}$ both are $N\delta g^{\wedge}$ -closed sets $\{a\}\cup\{a,b\}=\{a,b\}$ is also are $N\delta g^{\wedge}$ -closed set.

Remark 4.3 The intersection of any two $N\delta g^{\wedge}$-Closed sets in $(U,\tau R(X))$ need not be $N\delta g^{\wedge}$-Closed.

Example 4.4 Let $U=\{a,b,c\}$ with $\frac{U}{R}=\{\{a\},\{b,c\}\}$ $X=\{a,b\}$. Then $\tau R(X)=\{U,\emptyset,\{a\},\{b,c\}\}$ $N\delta g^{\wedge}C(U,\tau_R(X)=\{\{U,\emptyset,\{a\},\{a,b\},\{b,c\},\{a,c\}\}$ Here the sets $\{a,b\}$ and $\{b,c\}$ both are $N\delta g^{\wedge}$ -closed sets but not their intersection $\{a,b\}\cap\{b,c\}=\{b\}$ is not in $N\delta g^{\wedge}$– closed set in $(U,\tau R(X))$.

Proposition 4.5 If A is $Ng^{\wedge}$-open and $N\delta g^{\wedge}$-Closed subset of $(U, \tau R(X))$ then A is an $N\delta$ -closed subset of $(U, \tau R(X))$.

proof: Since A is $Ng^{\wedge}$-open and $N\delta g^{\wedge}$-Closed, $Ncl_{\delta}(A) \subseteq A$. Hence A is $N\delta$ -closed.

Example 4.6 Let $U = \{a, b, c\}$ with $\frac{U}{R} = \{\{a\}, \{b, c\}\}$ $X = \{a, b\}$. Then $\tau R(X) = \{U, \emptyset, \{a\}, \{b, c\}\}$ $N\delta g^{\wedge}C(U, \tau_R(X) = \{\{U, \emptyset, \{a\}, \{a, b\}, \{b, c\}, \{a, c\}\}$.Here$Ng^{\wedge}O(U, \tau_R(X)\{\{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and $N\delta C(U, \tau_R(X) = \{\{U, \emptyset, \{a\}, \{b, c\}\}$.

Theorem 4.7 The intersection of a $N\delta g^{\wedge}$-Closed set and a $N\delta$ -closed set is always $N\delta g^{\wedge}$-Closed.

proof: Let A be $N\delta g^{\wedge}$-Closed and let F be $N\delta$ -closed. If U is an $Ng^{\wedge}$-open set with $A \cap F \subseteq U$, then $A \subseteq U \cup F^c$ and so $Ncl_{\delta}(A) \subseteq U \cup F^c$. Now $Ncl_{\delta}(A \cap F) \subseteq Ncl_{\delta}(A) \cap F \subseteq U$. Hence $A \cap F$ is $N\delta g^{\wedge}$-Closed.

Example 4.8 Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{c\}, \{b, d\}\}$ $X = \{a, b\}$. Then $tR(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ $N\delta g^{\wedge}C(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ and $N\delta C(U, \tau_R(X) = \{ U, \emptyset, \{c\}, \{a, c\}, \{b, c, d\}\}$. Here the sets $\{c, d\} \in N\delta g^{\wedge}C(U, \tau_R(X) \cap \{c\} \in N\delta C(U, \tau_R(X) = \{c\}$ is $N\delta g^{\wedge}$-Closed.

Proposition 4.9 If A is a $N\delta g^{\wedge}$-Closed set in a space $(U, \tau R(X))$ and $A \subseteq B \subseteq Ncl_{\delta}(A)$, then B is also a $N\delta g^{\wedge}$-Closed set.

proof: Let U be a $Ng^{\wedge}$-open set of $(U, \tau R(X))$ such that $B \subseteq U$. Then $A \subseteq U$. Since A is $N\delta g^{\wedge}$-Closed set, $Ncl_{\delta}(A) \subseteq U$. Also since $B \subseteq Ncl_{\delta}(A)$, $Ncl_{\delta}(B) \subseteq Ncl_{\delta}(Ncl_{\delta}(A))$ =$(Ncl_{\delta}(A))$. Hence $Ncl_{\delta}(B) \subseteq U$. Therefore B is also a $N\delta g^{\wedge}$-Closed set.

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