



GRAPH THEORY & ITS APPLICATIONS IN COMPUTER SCIENCE & TECHNOLOGY

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Abstract: - The main objective of this paper is to describe the applications of graph theory in the real life problems by representing them in terms of graphs. Many real- world problems can be conveniently represented in terms of graphs by means of points (vertices) and lines (edges). This article gives the Graph theory applications in various fields of connectivity (vertex or edge) problems in an efficient way by minimizing the waiting time of the participants. Mathematics plays an important role in various fields. The use of mathematics is also seen indifferent areas of computer science such as design of algorithms, Artificial intelligence, software architecture, multiprocessing etc. Graph theory is an important field in mathematics which is used in structural models. Graph can be used in research areas of computer science such as data mining, clustering, image capturing, networking, data structure etc. This paper gives an overview of different types of graphs and applications of graph in various fields of computer science & technology.

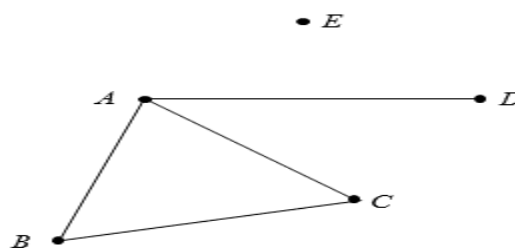
Keywords: Graphs, Edges and Vertices, Directed graph, Undirected graph, multi graph , planer graph,.

INTRODUCTION: -

Graph theory is the sub-field of mathematics and computer science which deals with graphs, diagrams that contain points and lines and which often pictorially represents mathematical truths. A graph is a pictorial and mathematical representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as vertices or nodes and the links that connect the vertices are called edges or arcs or lines. In computer science graph theory is the study of graphs which are mathematical structures used to model pair wise relations between objects from certain collection. Graph can also be represented as tree which is used in designing data structures making use of vertices and edges.

In other words, a graph is an ordered pair $G = (V, E)$ where, G specifies the graph, V is the vertex-set whose elements are called the vertices, or nodes of the graph. This set is often denoted by $V(G)$ or just V , E is the edge-set whose elements are called the edges, or connections between vertices of the graph. This set is often denoted by $E(G)$ or just E .

For Example: In the following graph, $V = \{A, B, C, D, E\}$ and $E = \{AB, BC, CA, AD\}$



A graph can be used to show any data in an organized manner with the help of pictorial representation. We can show the relationship between the variable quantities with the help of a graph. In graph theory, we usually use the graph to show a set of objects, and these objects are connected with each other in some sense. The objects can be described as mathematical concepts, which can be expressed with the help of nodes or vertices, and the relation between pairs of nodes can be expressed with the help of edges.[1]

Leonhard Euler was introduced the concept of graph theory. He was a very famous Swiss mathematician. On the basis of the given set of points, or given data, he was constructed graphs and solved a lot of mathematical problems. He says that different types of data can be shown in various forms, such as line graphs, bar graphs, line plots, circle graphs, frequency tables, etc, with the help of graphical representation.

Concepts used in Graph:-

Vertex (Node) : - A node v is a terminal point or an intersection point of a graph. It is the abstraction of a location such as a city, an administrative division, a road intersection or a transport terminal (stations, terminuses, harbors and airports).

Edge (Link): - An edge e is a link between two nodes. The link (i, j) is of initial extremity i and of terminal extremity j . A link is the abstraction of a transport infrastructure supporting movements between nodes. It has a direction that is commonly represented as an arrow. When an arrow is not used, it is assumed the link is bi-directional.

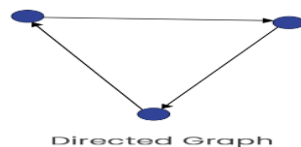
Sub-Graph:- A sub-graph is a subset of a graph G where p is the number of sub-graphs. For instance $G' = (v', e')$ can be a distinct sub-graph of G . Unless the global transport system is considered in its whole, every transport network is in theory a sub-graph of another. For instance, the road transportation network of a city is a sub-graph of a regional transportation network, which is itself a sub-graph of a national transportation network.

History of Graph Theory:

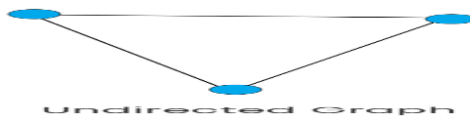
The origin of graph theory started with the problem of Koinsber bridge, in 1735. This problem lead to the concept of Eulerian Graph. Euler studied the problem of Koinsberg bridge and constructed a structure to solve the problem called Eulerian graph. In 1840, A.F Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems. The concept of tree, (a connected graph without cycles[7]) was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. In 1852, Thomas Guthrie found the famous four color problem. Then in 1856, Thomas. P. Kirkman and William R. Hamilton studied cycles on polyhydra and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. In 1913, H.Dudeney mentioned a puzzle problem. Eventhough the four color problem was invented it was solved only after a century by Kenneth Appel and Wolfgang Haken. [2]

TYPES OF GRAPH : - THERE ARE BASICALLY TWO TYPES OF GRAPHS, I.E., UNDIRECTED GRAPH AND DIRECTED GRAPH. THE DIRECTED GRAPH AND UNDIRECTED GRAPH ARE DESCRIBED AS FOLLOWS:

Directed graph: The directed graph can be made with the help of a set of vertices, which are connected with the directed edges. In the directed graph, the edges have a direction which is associated with the vertices.

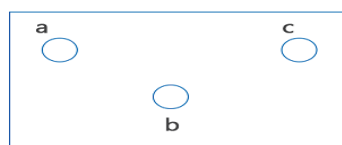


Undirected Graph: The undirected graph can also be made of a set of vertices which are connected together by the undirected edges. All the edges of this graph are bidirectional. We can sometimes call this type of graph an undirected network.



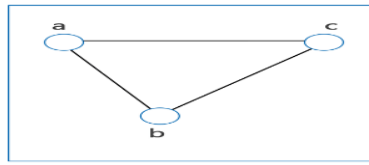
Other types of graph: There are also some other types of graphs, which are described as follows:

Null Graph: A graph will be known as the null graph if it contains no edges. With the help of symbol N_n , we can denote the null graph of n vertices. The diagram of a null graph is described as follows:

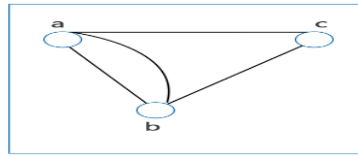


In the above graph, vertices a , b and c are not connected with any edge, and there is no edge. So this graph is a null graph.

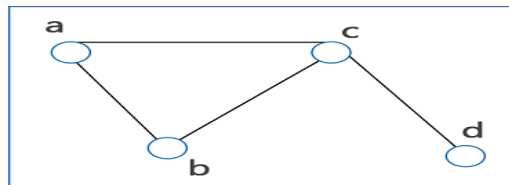
Simple Graph: A graph will be known as a simple graph if it does not contain any types of loops and multiple edges. The simple graph must be an undirected graph. The diagram of a simple graph is described as follows:



Multi-Graph: A graph will be known as a multi-graph if the same sets of vertices contain multiple edges. In this type of graph, we can form a minimum of one loop or more than one edge. The diagram of multi-graph is described as follows:

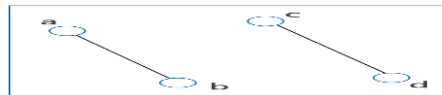


Connected Graph: A graph will be known as a connected graph if it contains two vertices that are connected with the help of a path. The diagram of a connected graph is described as follows:



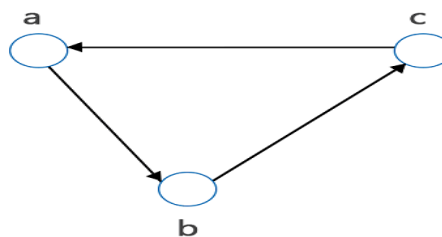
In the above graph, the two vertices, a and b, are connected by a single path. Similarly, other vertices such as (a and c), (c and b), (c and d), (a and d) are all connected by a single path. So this graph is a connected graph.

Disconnected Graph: A graph will be known as the disconnected graph if it contains two vertices which are disconnected with the help of a path. If there is a graph G, which is disconnected, in this case, every maximal connected sub-graph of G will be known as the connected component of the graph G. The diagram of a disconnected graph is described as follows:

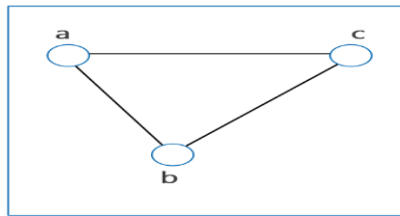


In the above graph, there are vertices a, c, and b, d which are disconnected by a path. So this graph is a disconnected graph.

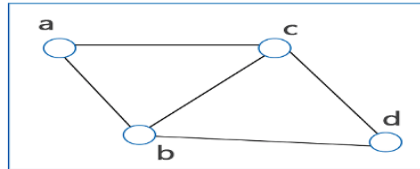
Cycle Graph: A graph will be known as the cycle graph if it completes a cycle. It means that for a cycle graph, the given graph must have a single cycle. With the help of symbol C_n , we can denote a cycle graph with n vertices. The diagram of a cycle graph is described as follows:



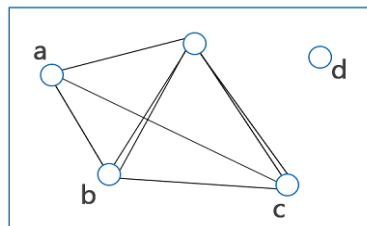
Complete Graph: A graph will be known as the complete graph if each pair of vertices is connected with the help of exactly one edge. A simple graph will be a complete graph if there are n numbers of vertices which are having exactly one edge between each pair of vertices.



Planer Graph: A graph will be known as the planer graph if it is drawn in a single plane and the two edges of this graph do not cross each other. In this graph, all the nodes and edges can be drawn in a plane. The diagram of a planer graph is described as follows:



Non-planer graph: A given graph will be known as the non-planer graph if it is not drawn in a single plane, and two



edges of this graph must be crossed each other. The diagram of a non-planer graph is :

APPLICATIONS OF GRAPH THEORY: -

Graph Theory is used in vast area of science and technologies. Some of them are given below:

1. Computer Science

In computer science graph theory is used for the study of algorithms like: Dijkstra's Algorithm Prim's Algorithm. & Kruskal's Algorithm, Graphs are used to define the flow of computation. Graphs are used to represent networks of communication. Graphs are used to represent data organization. Graph transformation systems work on rule-based in-memory manipulation of graphs. Graph databases ensure transaction-safe, persistent storing and querying of graph structured data. Graph theory is used to find shortest path in road or a network. In Google Maps, various locations are represented as vertices or nodes and the roads are represented as edges and graph theory is used to find the shortest path between two nodes.[6][7]

2. Electrical Engineering: In Electrical Engineering, graph theory is used in designing of circuit connections. These circuit connections are named as topologies. Some topologies are series, bridge, star and parallel topologies.

3. Linguistics: In linguistics, graphs are mostly used for parsing of a language tree and grammar of a language tree. Semantics networks are used within lexical semantics, especially as applied to computers, modeling word meaning is easier when a given word is understood in terms of related words. Methods in phonology (e.g. theory of optimality, which uses lattice graphs) and morphology (e.g. morphology of finite - state, using finite-state transducers) are common in the analysis of language as a graph.[7]

4. Physics and Chemistry

- In physics and chemistry, graph theory is used to study molecules.
- The 3D structure of complicated simulated atomic structures can be studied quantitatively by gathering statistics on graph-theoretic properties related to the topology of the atoms.
- Statistical physics also uses graphs. In this field graphs can represent local connections between interacting parts of a system, as well as the dynamics of a physical process on such systems.

- Graphs are also used to express the micro-scale channels of porous media, in which the vertices represent the pores and the edges represent the smaller channels connecting the pores.
- Graph is also helpful in constructing the molecular structure as well as lattice of the molecule. It also helps us to show the bond relation in between atoms and molecules, also help in comparing structure of one molecule to other. [7]

5. Computer Network

- In computer network, the relationships among interconnected computers within the network, follow the principles of graph theory.
- Graph theory is also used in network security.
- We can use the vertex coloring algorithm to find a proper coloring of the map with four colors.
- Vertex coloring algorithm may be used for assigning at most four different frequencies for any GSM (Grouped Special Mobile) mobile phone networks.[3] [7]

CONCLUSION:

The main aim of this paper is to present the importance of graph theory principles, different types of graph and ideas in various fields of computer science and technology. The researchers can use these concepts to model and optimize different process in fields like Computer science, computer networks, Electrical Engineering, physics and Chemistry, etc.

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