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WATER JETS ISSUING IN AIR: ACCOUNTING FOR THE EFFECT OF VISCOSITY

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Abstract

A jet of liquid is issued from a nozzle in the atmosphere or viscous air in many situations. The equation of trajectory of a water jet in air considering viscosity effect is different from that without viscosity effect. The viscosity of air does not influence the trajectory path of a water jet considerably. The equations for path of trajectory and maximum height of water jet on accounting for the effect of viscosity of air have been modeled. The maximum height attained by a water jet with and without consideration of viscosity of air is approximately equal for a particular initial angle and an initial velocity. There is a considerable loss of energy of water jet due to diffusion in the surrounding atmosphere, air entrainment into jet stream, wind velocity influence, etc. in open field.

Keywords: Trajectory, maximum height, water jet, viscosity, initial angle, initial velocity

I. INTRODUCTION

A water jet is a stream of water discharged into fluid issuing from a nozzle under pressure and takes curved motion. Water jet technology is successfully used in many fields of engineering such as construction engineering, building rehabilitation, rock and concrete fragmentation, pavement maintenance, material engineering, manufacturing operations, chemical process engineering, architectural profile, etc. Viscosity is one of the most important properties of environmental fluid (here air), which influences the fluid motion. The wall jet is defined as a jet of fluid, impinging tangentially (or at an angle) on a boundary, surrounded by stationary (or moving) fluid (Rajaratnam, 1967). A simple plane wall jet is generally defined as a plane jet coming out of a slot tangentially to a flat plate, submerged in a large stagnant mass of the same fluid (Rajaratnam, 1972). Rajaratnam and Pani (1974a) found that the dimensions of the nozzle producing wall jet in the major and minor axis directions roughly the same, like a square, circle, or rectangle for which the aspect ratio is not very different from unity. They studied the wall jets issuing from circular, square, triangular, elliptical, and rectangular nozzles and it was found that in all these cases, the velocity distribution was (reasonably) similar for longitudinal distances greater than about 10 times the nozzle height. Beltaos and Rajaratnam (1974b) studied the impinging circular turbulent jets normally on a plane smooth surface. They used two jet nozzles of internal diameters of 23.4 mm and 6.4 mm. According to Jirka (2007), jet discharging into water is different from that discharging into air in respect of transition from initially jet-like to final plume-like conditions, bottom interaction in the presence of cross flow, central line buoyancy, etc. Guha et al. (2010) reported that high-speed water jets diffuse in the surrounding atmosphere by the processes of mass and momentum transfer. Air is entrained into the jet stream and the entire process contributes to jet spreading and subsequent pressure decay. Huang et al. (2020) found that the central velocity of the circular water is highest, whereas that of the elliptical water jet is the lowest, and those square, triangular and cross-shaped jets are in the middle. Considerable research has been done on the plane and high pressure water jets, whereas, water jets issuing in air accounting its viscosity effect have received little attention. This paper presents a mathematical model study of a jet impinging in viscous air. The decay of the velocity of the water jet is caused due to the viscosity of air along with acceleration due to gravity. The model has been developed to predict the trajectory path of the jet and the decay of the maximum height of the water jet.

II. RESEARCH METHODOLOGY

A free jet is a particular case which has the influence of acceleration due to gravity in the vertical direction. In viscous air, acceleration is also influenced by the viscous force in both X and Y directions. As the horizontal acceleration also exists in viscous medium contrary to free jet, the horizontal velocity component V_x does not retain its initial value throughout the flight. The present mathematical model has been developed assuming jet spraying in air as a typical smaller cube of unit side (Fig. 1).

If we choose a reference frame with the positive y-axis vertically upward, we may take vertical acceleration (acceleration due to gravity, - g) along with the horizontal acceleration $(-\nu V_x)$, and vertical acceleration $(-\nu V_y)$ produced by kinematic viscosity (ν) of the air. At time t = 0, the jet begins the flight with velocity V from the origin, which makes an angle θ with the positive x-direction.

Force F_x required to move the surface of water in x direction is provided by necessary shear resistance of air due to viscosity. According to Newton's law of viscosity

$$F_x = \mu A \frac{V_x}{y} \tag{1}$$

Where,
A = Area of the moving surface
μ = Dynamic viscosity of air
$\frac{V_x}{y}$ = velocity gradient in the y-direction

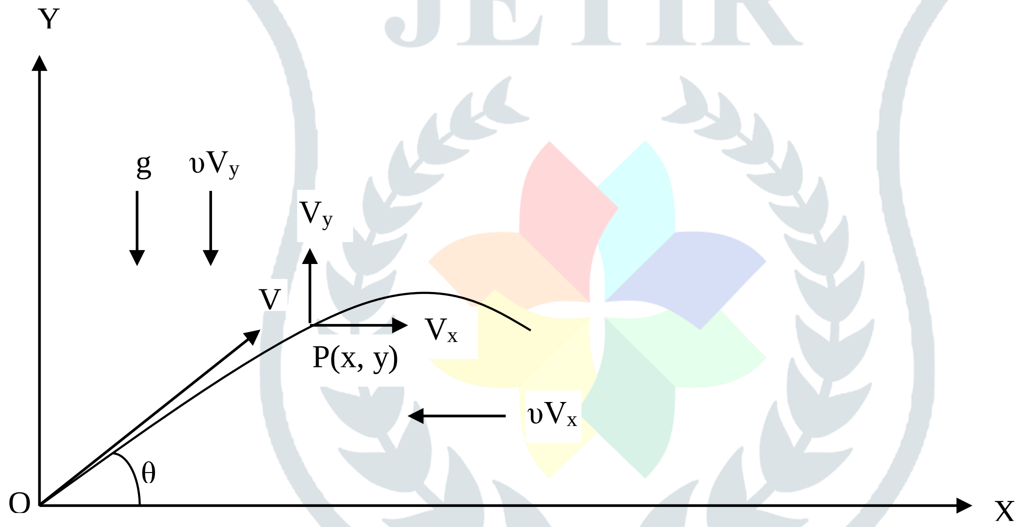


Fig. 1 Definition sketch of jet trajectory in air

EQUATION OF MOTION PARALLEL TO X-AXIS

According to Newton's second law of motion in X direction, one can write

$$-F_x = m\frac{dV_x}{dt} \tag{2}$$

Using Eqs. (1) and (2),

$$m\frac{dV_x}{dt} = -\mu A\frac{V_x}{y} \tag{3}$$

The mass, m of air of a cube of unit side moving relative to jet spray; hence the mass, m equals to the mass density, ρ of air, y is one of the sides of the square (here it equals to unity) and hence, area A for the square element also equals to unity. Therefore, Eq. (3) may be written as

$$\frac{dV_x}{dt} = -\nu V_x \tag{4}$$

The negative sign of $-\nu V_x$ indicates that the horizontal acceleration acts along negative X direction (Fig. 1).
Integrating Eq. (4), one may write

$$\int\frac{dV_x}{V_x} = -\nu\int dt + k_1 \tag{5}$$

Eq. (5) becomes

$$\ln V_x = -\nu t + k_1 \tag{6}$$

Initially, when t = 0; V_x = V Cosθ; hence, k_1 = VCosθ (where θ is initial angle of jet with horizontal) and Eq. (6) becomes

$$\ln\left(\frac{V_x}{V\cos\theta}\right) = -\nu t \quad (7)$$

Eq. (7) may be written as

$$\frac{dx}{dt} = V_x = V\cos\theta\, e^{-\nu t} \quad (8)$$

Integrating equation (8), one may write

$$x = \frac{V\cos\theta}{-\nu} e^{-\nu t} + k_2 \quad (9)$$

Initially when t = 0; x = 0; hence, $k_2 = \frac{V\cos\theta}{\nu}$ and Eq. (9) becomes

$$x = \frac{V\cos\theta}{\nu}(1 - e^{-\nu t}) \quad (10)$$

EQUATION OF MOTION PARALLEL TO Y-AXIS

Force F_y required to move the surface of water in y direction, is given by

$$F_y = mg + \mu A \frac{V_y}{x} \quad (11)$$

According to Newton's second law of motion in Y direction, one can write

$$F_y = -m\frac{dV_y}{dt} \quad (12)$$

Equating Eqs. (11) and (12), one can write

$$\frac{dV_y}{dt} = -(g + \nu V_y) \quad (13)$$

Here, negative sign of $-(g + \nu V_y)$ indicates vertical acceleration acts along negative Y direction (Fig. 1).
Integrating Eq. (13), one may write

$$\int \frac{dV_y}{g + \nu V_y} = -\int dt + K_3 \quad (14)$$

Eq. (14) becomes

$$\frac{1}{\nu}\ln\left(g + \nu V_y\right) = -t + k_3 \quad (15)$$

Initially, when t = 0; V_y = V Sinθ; hence $K_3 = \frac{1}{\nu}\ln(g + \nu V\sin\theta)$ and Eq. (15) becomes

$$\frac{1}{\nu}\ln\left(\frac{g + \nu V_y}{g + \nu V\sin\theta}\right) = -t \quad (16)$$

One may write Eq. (16) as

$$\frac{dy}{dt} = V_y = \frac{1}{\nu}[(g + \nu V\sin\theta)e^{-\nu t} - g] \quad (17)$$

Integrating equation (17), one may get

$$y = \frac{1}{\nu}\left(\frac{g}{\nu} + V\sin\theta\right)(1 - e^{-\nu t}) - \frac{gt}{\nu} \quad (18)$$

Eliminating t from Eq. (10) and (18), one may write

$$y = \left(\frac{g}{\nu} + V\sin\theta\right)\frac{x}{V\cos\theta} + \frac{g}{\nu^2}\log\left(1 - \frac{\nu x}{V\cos\theta}\right) \quad (19)$$

Expanding Eq. (19), one may write

$$y = \frac{gx}{\nu V\cos\theta} + \frac{V\sin\theta x}{V\cos\theta} - \frac{\nu x g}{\nu^2 V\cos\theta} - \frac{g}{\nu^2}\frac{\nu^2 x^2}{2V^2\cos^2\theta} - \cdots \quad (20)$$

For non-viscous fluid, υ = 0, one may write Eq. (20) as

$$y = (\tan\theta)x - \frac{gx^2}{2(V\cos\theta)^2} \quad (21)$$

Eq. (19) represents the trajectory path of water jet in viscous medium (air), whereas Eq. (21) is the trajectory path of water jet in non-viscous medium. Eqs. (19) and (21) reveal that the equation of trajectory of the jet in air considering viscosity effect is different from that without viscosity effect.
The jet will obtain the maximum height, when $\frac{dy}{dt} = 0$. On differentiating Eq. (18), one may get

$$t = \frac{1}{\nu}\ln\left(1 + \frac{\nu V\sin\theta}{g}\right) \quad (22)$$

Substituting the value of t from Eq. (22) in Eq. (18), one may get

$$y_{max} = \frac{V\sin\theta}{\nu} - \frac{g}{\nu^2}\ln\left(1 + \frac{\nu V\sin\theta}{g}\right) \quad (23)$$

Eq. (23) gives the maximum height attained by jet accounting viscosity effect of air.
Eq. (23) may be expanded as

$$y_{max} = \frac{V\sin\theta}{\nu} - \frac{V\sin\theta}{\nu} + \frac{1}{2}\frac{V^2\sin^2\theta}{g} - \frac{1}{3}\frac{\nu V^2\sin^3\theta}{g^2} + \cdots \quad (24)$$

For non-viscous fluid, υ = 0, Eq. (24) may be written as

$$y_{max} = \frac{V^2 Sin^2\theta}{2g} \quad (25)$$

Eq. (25) represents the trajectory path of water jet without viscosity effect of air.

III. EXPERIMENTAL DESCRIPTION

The experiments were conducted in an open field. Water pressures were varied two times with the help of pressure valve to get two different average initial velocities of the water jet (exit diameter 5 mm). Firstly, Volume of water collected in a measuring bucket and time taken for the same was noted with the help of stop watch. Discharge was calculated using volume of water collected in bucket and time recorded by stop watch and further velocity was calculated using discharge and cross-section area of bucket. The collection of volume, evaluation of discharge and subsequently evaluation of velocity were made in three sets at a particular pressure. The average of the velocities was 4.084 m/s. Initial angles of water jet were measured with the help of Set Squares. The centres of maximum heights of water jet were decided by eye estimation and were measured with the help of metallic tape from horizontal ground level for initial angles, $\theta = 30^0$, 60^0 and 90^0. Similar procedures were repeated at other water pressure to get second average velocity 4.623 m/s and the maximum heights were measured as per the procedures stated above for all above initial angles. The velocities and maximum heights for $\theta = 30^0$, 60^0 and 90^0 are presented in Table 1.

IV. RESULTS AND DISCUSSION

It was observed that the theoretical trajectory paths of water jet with and without considerations of viscosity effect of air almost coincide for each initial angle with a particular initial velocity. A typical Fig. 2 has been presented showing the almost coinciding trajectory paths of water jet with and without considerations of viscosity effect of air using Eqs. (19) and (21); kinematic viscosity of air, $\upsilon = 1.5 \times 10^{-5}$ m^2/s (at temperature 20^o C and 1 atmospheric pressure); V = 4.623 m/s; and $\theta = 30^o$. Thus, viscosity of air does not influence the trajectory path of a water jet considerably. Fig. 2 and Table 1 show that the theoretical y_{max} reached by a water jet are approximately equal with and without consideration of viscosity of air for particular initial angle and initial velocity, this may be due to too small numerical value of the kinematic viscosity of air. Figs. 3(a), 3(b), and Table 1 show a trend of increasing of y_{max} in both theoretically and experimentally as θ increases from 30^0 to 90^0 and also reflects that there are considerable variations between the theoretical and experimental y_{max} of water jets for a particular initial angle and initial velocity. Table 1 also reveals that with the increase of initial velocity of jet from 4.084 m/s to 4.623 m/s, there is constant rate of increase of theoretical y_{max} of jet by 28.14 %, for each particular initial angle. Thus, one may conclude that theoretical y_{max} of jet may increase with constant rate for particular initial angle on increasing the initial velocity. But Table 1 also reveals that there is considerable variability ranging from 0 to 22.62 % in experimental y_{max} of jet for initial angles ranging from 30^0 to 90^0 on increasing the initial velocity from 4.084 m/s to 4.623 m/s. The reasons for variations in theoretical and experimental results may be the diffusion of the high-speed turbulent water jets in air and in the surrounding atmosphere by the processes of mass and momentum energy transfer, and the air entrainment into the jet stream contributing to jet spreading and subsequent pressure decay. One cannot ignore also the considerable influence of wind velocity and its direction in an open field experiments. This variability trends also indicates significant role of Reynold's number.

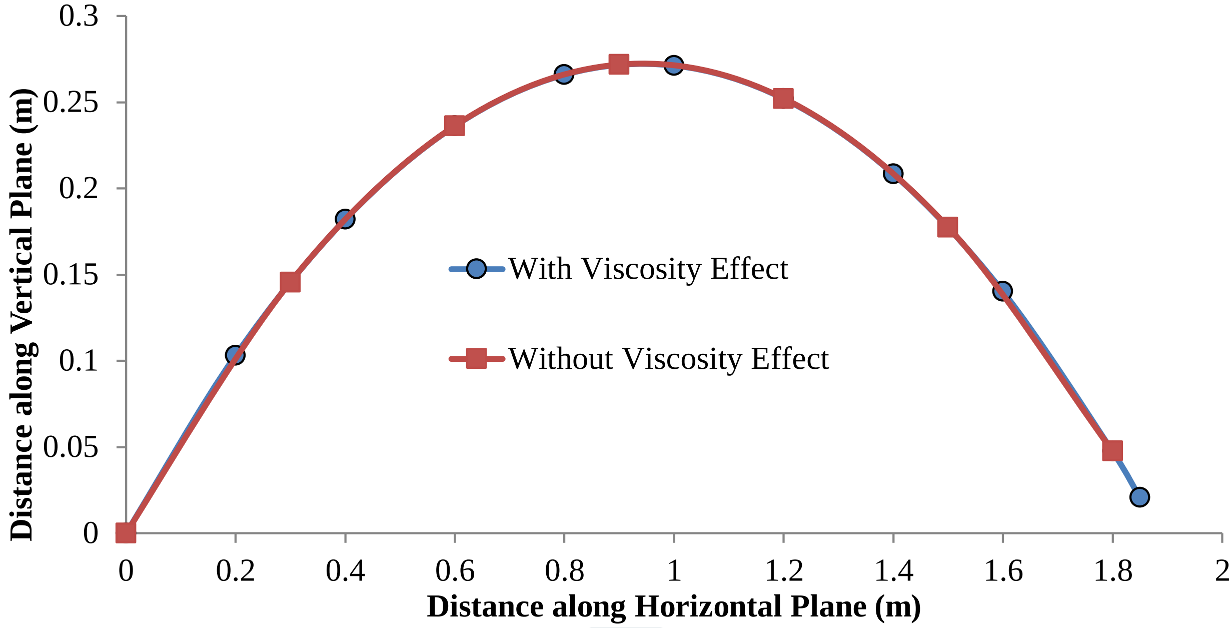
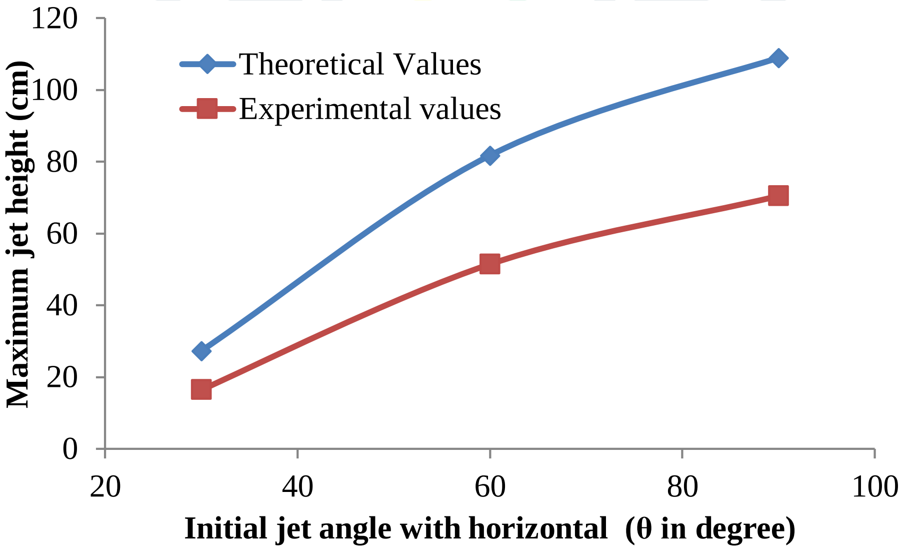


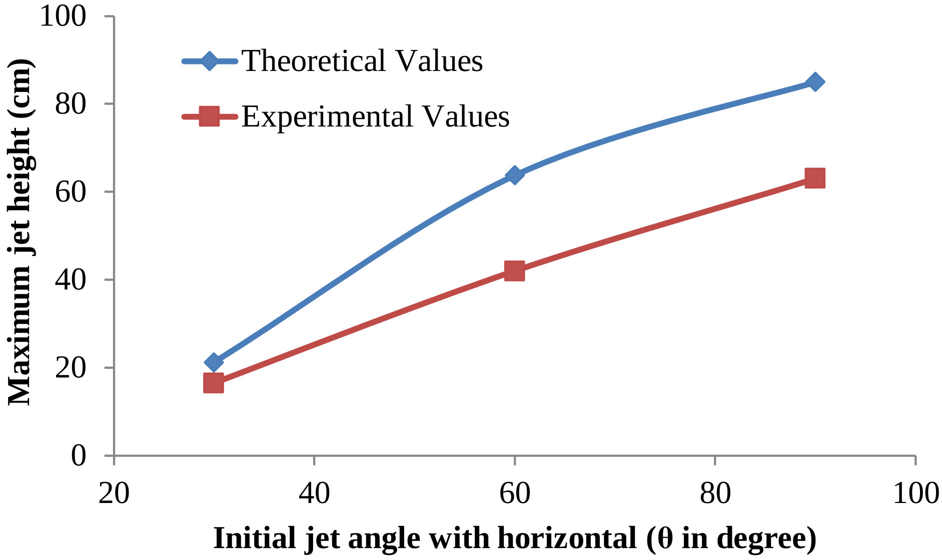
Fig. 2 Typical trajectory paths of water jet with and without viscosity effect of air at θ = 30 Degree

Table 1: Theoretical y_{max} with or without viscosity effect of air and experimental data

Velocity V (m/s)	Kinematic viscosity ν (m^2/s)	θ	y_{max} using modeled Eq. (23) (with viscosity effect of air) (cm)	y_{max} using modeled Eq. (25) (without viscosity effect of air) (cm)	y_{max} from experimental data (cm)	% variation in y_{max} of Col (4) or Col (5) for same θ	% variation in y_{max} of Col (7) for same θ
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
4.084	1.5 x 10^{-5}	30°	21.25267	21.25262	16.50	28.14 %	0 %
4.623	1.5 x 10^{-5}	30°	27.23247	27.23238	16.50		
4.084	1.5 x 10^{-5}	60°	63.75721	63.75786	42.00	28.14 %	22.62 %
4.623	1.5 x 10^{-5}	60°	81.69738	81.69774	51.50		
4.084	1.5 x 10^{-5}	90°	85.01031	85.01048	63.00	28.14 %	11.90 %
4.623	1.5 x 10^{-5}	90°	108.92971	108.93032	70.50		



(a) For Velocity 4.623 m/s



(b) For Velocity 4.084 m/s

Fig. 3 Variations of Theoretical and Experimental y_{max} (cm) with θ (degree) for Velocities (a) 4.623 m/s & (b) 4.084 m/s

V. CONCLUSIONS

- The equations of trajectory of a water jet in air considering viscosity effect are different from that without viscosity effect.
- The theoretical trajectory paths of water jet with and without considerations of viscosity effect of air almost coincide for a particular initial angle and an initial velocity.
- The viscosity of air does not influence the trajectory path of a water jet considerably.
- The theoretical maximum heights attended by a water jet are approximately equal with and without consideration of viscosity of air for a particular initial angle and an initial velocity.
- The theoretical maximum heights of jet may increase with constant rate for a particular initial angle on increasing the initial velocity.
- There is considerable variation between the experimental and theoretical values in respect of maximum heights of jet. The possible reasons for variations may be the diffusion of the high-speed turbulent water jets in air in the surrounding atmosphere; the air entrainment into the jet stream; the influence of wind velocity and its direction in an open field experiments; significant role of Reynold's number, etc.

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