JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

"A STUDY ON SYSTEM OF LINEAR EQUATIONS- IN LINEAR ALGEBRA"

J. RAVI KUMAR (Assistant Professor in Department of Mathematics)

Nishitha Degree College, Nizamabad.

ABSTRACT:

In a Linear Algebra, Augmented matrix is a matrix obtained by appending the columns of two given matrices. Augmented matrix is a manner of calculating the unknown variables of a system of linear equations. It provides a concept of estimating unknown variables with aid of relating acquainted data. The main goal of this research is to constitute a Augmented matrix is derived from the system of Linear equations.

KEYWORDS:

Linear equation, System of Linear equations, Matrix, Gauss Elimination, Elementary row operation, Pivot Position, Pivot Column.

System of Linear Equations:

A linear equation in the variables $x_1, x_2, \ldots, x_n$ is an equation that can be written in the form

$$a_1 x_1+a_2x_2+\ldots+a_nx_n=b$$

Where the coefficients $a_1, a_2, \ldots, a_n$ and b are real or complex numbers.

The equations $2x_1-7x_2=3$ and $3x_1+x_2=2\sqrt{5}$ are both linear

But $5x_1+3x_2=x_1x_2$ and $x_1=3\sqrt{x_2}-4$ are not linear because the presence of x_1x_2 in the first equation and $\sqrt{x_2}$ in the second equation.

A system of Linear equations is a collection of one or more linear equations involving the variables $x_1, x_2, \ldots, x_n$.

$$a_{11}x_1+a_{12}x_2+\ldots\ldots+a_{1n}x_n=b_1$$

$$a_{21}x_1+a_{22}x_2+\ldots\ldots+a_{2n}xn=b_2$$

. . .

. . .

. . .

$$a_{n1}x_1+a_{n2}x_2+\ldots\ldots+a_{nn}x_n=b_n$$

The system of equations can be written in matrix form

$$A=\begin{bmatrix} a11 & a12 \dots a1n \\ a21 & a22 \dots a22 \\ \vdots & \vdots \\ \vdots & \vdots \\ an1 & an2 \dots ann \end{bmatrix}; \quad X=\begin{bmatrix} x1 \\ x2 \\ \vdots \\ xn \end{bmatrix}; \quad B=\begin{bmatrix} b1 \\ b2 \\ \vdots \\ bn \end{bmatrix}$$

Augmented Matrix: The Matrix which contains all the columns of 'A' matrix and 'B' has its last column is known as Augmented Matrix and it is denoted by [A B].

Pivot Position: A Pivot Position in a matrix is a location in matrix that corresponding to a leading '1' is the reduced echelon form of the matrix.

Pivot Column: The Pivot Column is a column of matrix that containing at Pivot Position.

Example: Consider System of Equations:

$x_1 + 2x_2 + 3x_3 = 4$

$4x_1 + 5x_2 + 6x_3 = 7$

$7x_1 + 8x_2 + 9x_3 = 1$

The system of equations can be written as

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 1 \end{bmatrix}$$

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad X=\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}, \quad b=\begin{bmatrix} 4 \\ 7 \\ 1 \end{bmatrix}$$

Augmented Matrix

$$[A \quad b] \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 7 & 8 & 9 & 1 \end{bmatrix}$$ be a Augmented Matrix,

Row Reducing the Augmented matrix to echelon form.

$$\sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 7 & 8 & 9 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 4R_1$

$R_3 \rightarrow R_3 - 7R_1$

$$\sim\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -6 & -12 & -27 \end{bmatrix}$$

$R_2 \rightarrow R_2/-3$

$R_3 \rightarrow R_3/-6$

$$\sim\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 27/6 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\sim\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 3/2 \end{bmatrix}$$

$R_1 \rightarrow R_1 - 2R_2$

$$\sim\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 3/2 \end{bmatrix}$$

We get, $\begin{bmatrix} \mathbf{1} & 0 & -1 & -2 \\ 0 & \mathbf{1} & 2 & 3 \\ 0 & 0 & 0 & 3/2 \end{bmatrix}$

Therefore, Pivot Columns are 1^{st} and 2^{nd}.

Statements:

- There are four columns in an augmented matrix and three variables in the system (x_1, x_2, x_3).
- There are two pivot columns 1^{st} and 2^{nd}, the variable x_1 and x_2 corresponding to pivot columns are called **Basic Variables,** and the other variable x_3 is called **Free Variable.**

Note:

If a linear System is **Consistent**, then the solution set contains either

i. Unique Solution
ii. Infinitely many solutions
iii. No Solutions

Unique Solution: If number of Pivot Columns equal to number of variables.

(Or)

When there are no free variable.

Infinitely many solutions: If number of Pivot Columns less than the number of variables.

(Or)

There is atleast one free variable.

No Solution: If the last column of an augmented matrix is pivot column then it has No solution.

Problem 1 :

Let A=$\begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix}$, $\tilde{b}$=$\begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix}$ define T: $R^3 \rightarrow R^3$ T($\tilde{x}$)= $A.\tilde{x}$ .Find a Vector $\tilde{x}$, whose image under T is $\tilde{b}$. Also determine whether $\tilde{x}$ is unique.

Solution:

Given that

Let A=$\begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix}$, $\tilde{b}$=$\begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix}$

The mapping T: $R^3 \rightarrow R^3$ defined by T($\tilde{x}$)= $A.\tilde{x}$. Also given the image of T is $\tilde{b}$.

To find a vector $\tilde{x}$ and also verify $\tilde{x}$ is unique.

Let $\tilde{x}$=$\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$ and

$1x_1+0x_2-2x_3=-1$

$-2x_1+1x_2+6x_3=7$

$3x_1-2x_2-5x_3=-3$

We solve the system of equations

$$A\,\tilde{x} = \tilde{b}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix}\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}=\begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix}$$

Consider the augmented matrix

$$[A\ \tilde{b}\ ] \sim \begin{bmatrix} 1 & 0 & -2 & -1 \\ -2 & 1 & 6 & 7 \\ 3 & -2 & -5 & -3 \end{bmatrix}$$

Row reducing to echelon form

$R_2 \rightarrow R_2+2R_1$

$R_3 \rightarrow R_3-3R_1$

$$\sim \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3+2R_2$

$$\sim \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 5 & 10 \end{bmatrix}$$

$R3 \rightarrow \frac{R3}{5}$

$$\sim \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 \rightarrow R_1+2R_3$

$R_2 \rightarrow R_2-2R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Therefore the number of pivot columns and number of variables is 3

Hence the system has a unique solution.

$$\tilde{x}=\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

i. e A. $\tilde{x} = \tilde{b}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore\ x_1=3 \quad x_2=1 \quad x_3=2$

Problem2 :

Let A= $\begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}$, $\tilde{b}$ = $\begin{bmatrix} -2 \\ -2 \end{bmatrix}$ define T: $R^3 \to R^2$ T($\tilde{x}$)= $A.\tilde{x}$ .Find a Vector $\tilde{x}$, whose image under T is $\tilde{b}$. also determine whether $\tilde{x}$ is unique.

Solution:

Given that

Let A= $\begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}$, $\tilde{b}$ = $\begin{bmatrix} -2 \\ -2 \end{bmatrix}$

The mapping T: $R^3 \to R^2$ defined by T($\tilde{x}$)= $A.\tilde{x}$. Also given the image of T is $\tilde{b}$.

To find a vector $\tilde{x}$ and also verify $\tilde{x}$ is unique

Let $\tilde{x}$= $\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$ and

$1x_1-5x_2-7x_3=-2$

$-3x_1+7x_2+5x_3=-2$

We solve the system of equations

$$A\,\tilde{x} = \tilde{b}$$

$$\begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

Consider the augmented matrix

$$[A\ \tilde{b}\,] \sim \begin{bmatrix} 1 & -5 & -7 & -2 \\ -3 & 7 & 5 & -2 \end{bmatrix}$$

$R_2 \to R_2+3R_1$

$$\sim \begin{bmatrix} 1 & -5 & -7 & -2 \\ 0 & -8 & -16 & -8 \end{bmatrix}$$

$R2 \to \frac{R2}{-8}$

$$\sim \begin{bmatrix} 1 & -5 & -7 & -2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

$R_1 \to R_1 + 5R_2$

$$\sim \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

Therefore The number of pivot columns 2 and number of variables is 3.

Hence the system has infinitely many solutions

i. e A. $\tilde{x} = \hat{b}$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$X_1+3x_3=3$

$X_2+2x_3=1$

$x_1=3-3x_3$; $x_2=1-2x_3$

$$\therefore \tilde{x} = \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 3-3x3 \\ 1-2x3 \\ x3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}$$

$\therefore \tilde{x} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$ is the solution and is not unique.

We get infinitely many solutions by giving various values of x_3

CONCLUSION

After the whole study and work on our paper, we are able to propose the new method with regard to exactness; even the proposed method delivers better accuracy among the mentioned formulas for which our new method is the best. According to these methods examined, it was found that the value of the propound method is very nearly to the exact value of the several problems. That's why we have agreed that this new method is extremely effective as well as perhaps correct to provide a good accuracy rather than the other existing augmented matrix formulas owing to calculating unknown values of the given data.

REFERENCES

1. David C Lay , Linear Algebra and its Applications 4e

2. S Lang, Introduction to Linear Algebra

3.Gilbert Strang, Linear Algebra and its Applications

4.Stephen H Friedberg et al, Linear Algebra

5.Kuldeep Singh, Linear Algebra

6.Sheldon Axler, Linear Algebra Done Right