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INVENTORY MODEL FOR DETERIORATING ITEMS UNDER TWO PARAMETRIC PARETO TYPE–I DETERIORATION AND PARTIALLY BACKLOGGED SHORTAGES

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Abstract: This research paper is based on economic order quantity model for deteriorating items under two parameter pareto type I distribution. In the initial phase of purchase, time to ameliorate following two parameter Weibull distribution is considered. Shortages are allowed to occur and partially backlogged. This model aims in minimising the total cost while finding the optimal time interval and optimal order quantity. The optimal solution of the model is illustrated with numerical example and sensitivity analysis along with graphical representation is done to study the effects of various parameters on the deciding variables and to demonstrate the model.

Keywords: Deterioration; Weibull Distribution; Pareto Type I Distribution; Partially Backlogged Shortages

I. INTRODUCTION

Deterioration means damage, spoilage, dryness, vaporization, etc. The deterioration of inventory items in the inventory system occurs due to one or many factors such as storage condition, weather condition including the nature of the product under study. The deterioration is usually a function of total amount of inventory on hand. This is one of the crucial factors that affect the inventory system. Also in real life, almost all the physical commodities in stock experience deterioration over time, pragmatic examples spoilage as in food stuffs, fruit, vegetables, physical depletion as in pilferage or evaporation of volatile liquids such as gasoline and alcohol, decay in radioactive substance, degradation as in photographic films. In common use, the term shortage may refer to a situation where most people are unable to find a desired good at an affordable price. If the demand exceeds the stock in inventory shortage occurs**.** If the shortage is in semi-finished products, raw materials etc. then the production of final output will suffer. In the case of finished goods shortage means to lose potential customers, and they may go to other competing firms to get their demand fulfilled. Sometimes shortages being voluntary in the company’s policy in order to stimulate the demand. In all this cases the shortage cost is expressed per unit of lost sale.

Several publishers have addressed the importance of the shortages and deterioration phenomenon in field applications. Consequently, many deterioration models have been developed. Ghare and Schrader [5] developed a model for an exponentially decaying inventory. Goyal and Giri [6] published trends in modelling of deteriorating inventory. Ruxian Li, Hongjie Lan and Jhon R. Mawhinney [13] reviewed on deteriorating inventory study. Kirtan Parmar, Indu Aggarwal and U.B. Gothi [7] formulated order level inventory model for deteriorating items under varying demand conditions. Devyani Chatterji and U.B. Gothi [3] have presented EOQ model for deteriorating items under two and three parameters weibull distribution and constant IHC with partially backlogged shortages. Ankit Bhojak and U.B. Gothi [1] developed inventory models for ameliorating and deteriorating items with time dependent demand and IHC. Nandgopal Rajeswari and Thirumalaisamy Vanjikkodi [10] presented an inventory model for items with two parameter weibull distribution deterioration and backlogging. Srinivasa Rao, Begum and Murthy [15] obtained optimal ordering policies of inventory model for deteriorating items having generalized pareto lifetime. Singh N., Vaish, and Singh, S. R. [16] considered an EOQ model with pareto distribution for deterioration, trapezoid type demand and backlogging under trade credit policy. Inventory Model pertaining to deteriorating items under two component mixture of Pareto lifetime and selling price dependent demand is developed by Vijayalaxmi, Srinivasa Rao Nirupama Devi [22]. Pooja D. Khatri and U.B. Gothi [12] developed an EPQ model for non – instantaneous weibully decaying items with ramp type demand and partially backlogged shortages. R. Uthayakumar and K. V., Geetha [14] studied a replenishment policy for non-instantaneous deteriorating inventory systems with partial backlogging. Milu Acharya and Smrutirekha Debata [9] obtained an inventory model for deteriorating items with time dependent demand under partial backlogging. Vinod Kumar Mishra and Lal Sahab Singh [23] developed deteriorating inventory model with time dependent demand and holding cost with partial backlogging. U. B. Gothi and Ankit Bhojak [19] presented Inventory model for ameliorating and deteriorating items under time dependent demand and IHC with partial backlogging. Ankit Bhojak and U. B. Gothi [2] developed production inventory system under weibull amelioration, pareto type I deterioration and exponentially time based demand under fully backlogged shortages.

Dye, Hsieh and Ouyang [4] find an optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. Nita Shah and Kunal Shukla [11] developed a deteriorating inventory model for waiting time partial backlogging when demand and deterioration rate are constant. U.B. Gothi and Ankit Bhojak [18] developed Inventory system with deterioration of items under pareto type I distribution, power demand pattern and partial backlogging. U.B. Gothi, Kareeshaa Shah and Santosh Rohida [20] developed an EOQ model for weibully distributed deteriorating items with the effect of permissible delay in payments and partially backlogged shortages. Tadj, Sarhan and Gohary [8] obtained an optimal control of an inventory system with ameliorating and deteriorating items. Mishra, Raju, U.K. Misra and G. Misra [17] have published their work on optimal control of an inventory system with variable demand and ameliorating / deteriorating items. U.B. Gothi, Pooja D. Khatri developed [21] An Inventory Model for Deteriorating Items in a Single Warehouse Starting with Lead Time, Price Dependent Demand and Shortages.

In the present literature, we have developed the inventory model by considering an EOQ model for deteriorating items along with partially backlogged shortages. for different time periods, Weibull distribution with two parameters and Pareto type - I distribution with two parameters are considered for deterioration rate. The probability density function of two parameter pareto type–I distribution is given by $f(t)=\frac{\theta_2}{\mu}\left(\frac{t}{\mu}\right)^{-\theta-1}$;$t \geq \mu$ where θ_2 and μ are parameters with positive real values. The instantaneous rate of deterioration θ (t) of the non – deteriorated inventory at time t, can be obtained from $\theta(t)=\frac{f(t)}{1-F(t)}$, where $F(t)=1-\left(\frac{t}{\mu}\right)^{-\theta}$ is cumulative distribution function for the two parameter Pareto type - I distribution. The instantaneous rate of deterioration of the on hand inventory is $\theta(t)=\frac{\theta_2}{t}$. In our model inventory holding cost is considered as a linear function of time. The Pareto family of distribution is often applied in economics, finance and actuarial science to measure size; for example, income, loss or claim, severity estimation and fitting from data. Pareto distribution is useful for modelling and predicting tools wide variety of socioeconomic contexts. There is a definite advantage in focusing discussion on one specific field of application, the size distribution of income. It was that context that Pareto introduced the concept in his well-known economics text. The developed model is illustrated by means of a numerical illustration with its sensitivity analysis.

II. ASSUMPTIONS

The following assumptions are considered to develop the model:

1. The Inventory system involves only one item and one stocking point.
2. Replenishment rate is infinite.
3. Lead time is Zero.
4. The deterioration occurs when the item is effectively in stock.
5. The deteriorated items are not replaced during the given cycle.
6. Time horizon period is considered to be infinite.
7. Shortages are allowed to occur and unsatisfied demand is partially backlogged at a rate $e^{-\delta(T-t)}$, where the backlogging parameter δ is a positive constant.
8. Inventory holding cost is a linear function of a time and it is $C_h = h + rt \ (h\,,\ r > 0)$.
9. The demand rate is $R(t) = -\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}$ following power demand pattern over time period $[0\,,\ T]$ where d is demand size and n is a finite positive index.
10. The amelioration rate is derived from Weibull distribution with two parameters and it is given by, $A(t)=\alpha\beta t^{\beta-1}$; $0 \leq t \leq t_1$ where α and β are the positive constants and α is very small.
11. The deterioration rate is

$$\theta(t)=\begin{cases}\theta_1 & ;0 \leq t \leq \mu \\ \frac{\theta_2}{t} & ;\mu \leq t \leq t_1\end{cases}$$

12. Unit amelioration cost, deterioration cost**,** purchase cost, ordering cost and shortage cost per unit are known and constants.
13. Total Inventory cost is continuous real function which is convex to the origin.
14. Terms having higher power of θ_1, α and δ are ignored as they are very very small.

III. NOTATIONS

The mathematical model is developed using the following notations:

1. *Q (t)* : Inventory level of the product at time t.
2. *R (t)* : Demand rate varying over time.
3. *A (t)* : Amelioration rate at any time t.

4. $\theta(t)$: Deterioration rate.
5. A : Ordering cost per order during the cycle period.
6. C_h : Inventory holding cost per unit per unit time.
7. C_a : Amelioration cost per unit.
8. C_d : Deterioration cost per unit time.
9. C_p : Purchase cost per unit $(C_p > C_a)$.
10. C_s : Shortage cost per unit.
11. C_l : Opportunity cost due to lost sale per unit.
12. Q_1 : Inventory level at time t=0.
13. Q_2 : Inventory level at time t=μ.
14. Q_3 : Inventory level at time t=T.
15. T : Duration of a cycle.
16. TC : Total cost per unit time.

IV. MATHEMATICAL FORMULATION AND SOLUTION

At the initial stage cycle starts with inventory level of Q_1 units. The total time is distributed into three time intervals. In the first time interval $[0,\mu]$ the inventory level decreases due to constant deterioration rate and demand rate *R (t)* and reaches to Q_2. In the second time interval $[\mu,t_1]$ the inventory level goes down to zero due to effects of Pareto type – I deterioration rate and demand rate *R (t)*. Shortages are allowed to occur during the third time interval $[t_1,T]$. Unsatisfied demand is partially backlogged and the rest is lost. The above developed inventory model is presented graphically in Fig. 1.

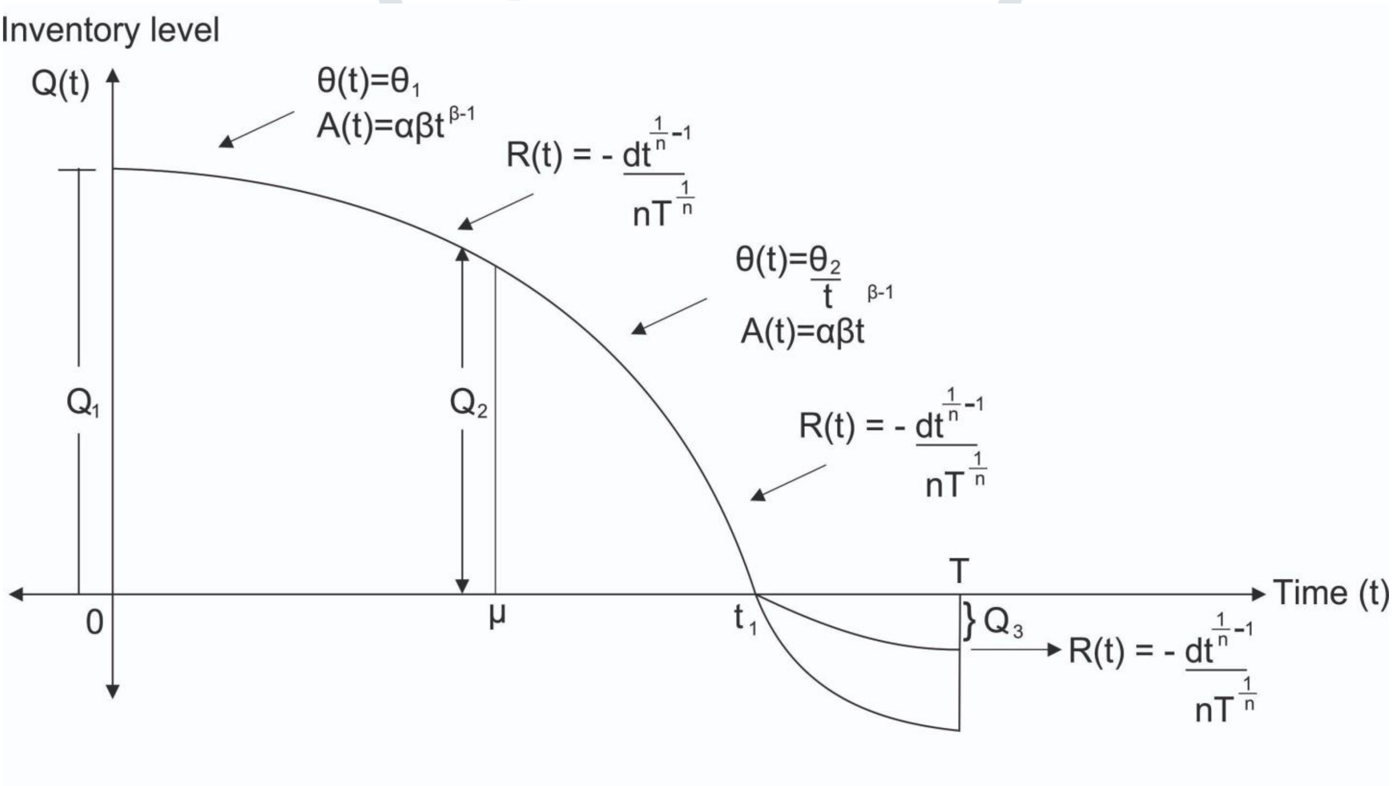

Figure 1: Graphical Representation of the Inventory System

Differential equations to obtain *Q (t)* pertaining to the situations as described above are given by

$$\frac{dQ(t)}{dt} + \left(\theta_1 - \alpha\beta t^{\beta-1}\right) Q(t) = -\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \quad (0 \le t \le \mu) \qquad (1)$$

$$\frac{dQ(t)}{dt} + \left(\frac{\theta_2}{t} - \alpha\beta t^{\beta-1}\right) Q(t) = -\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \quad (\mu \le t \le t_1) \qquad (2)$$

$$\frac{dQ\,(t)}{dt} = -\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\, e^{-\delta(T-t)} \qquad (t_1 \le t \le T) \tag{3}$$

Under the boundary conditions $Q(0)=Q_1, Q(t_1)=0$, the solutions of equations (1) to (3) are,

$$\left\{ Q\,(t) = Q_1\,(1 - \theta_1 t + \alpha t^{\beta}) - \frac{d}{T^{\frac{1}{n}}} \left\{ \begin{array}{l} t^{\frac{1}{n}} - \frac{n}{n+1}\,\theta_1\, t^{\frac{1}{n}+1} \\ + \frac{n\alpha\beta}{1+n\beta}\, t^{\frac{1}{n}+\beta} \end{array} \right\},\ (0 \le t \le \mu) \right\} \tag{4}$$

$$\left\{ \begin{array}{l} Q(t) = \dfrac{Q_2 + \dfrac{d}{T^{\frac{1}{n}}\,(1+n\theta_2)} \left\{ \mu^{\frac{1}{n}} + \dfrac{n\alpha\beta}{1+n(\theta_2+\beta)}\, \mu^{\frac{1}{n}+\beta} \right\}}{\mu^{-\theta_2} + \alpha\mu^{-\theta_2+\beta}} \\ \left(t^{-\theta_2} + \alpha t^{-\theta_2+\beta}\right) - \dfrac{d}{T^{\frac{1}{n}}\,(1+n\theta_2)} \left\{ t^{\frac{1}{n}} + \dfrac{n\alpha\beta}{1+n(\theta_2+\beta)}\, t^{\frac{1}{n}+\beta} \right\} \\ ,(\mu \le t \le t_1) \end{array} \right\} \tag{5}$$

$$\left\{ Q(t) = -\frac{d}{T^{\frac{1}{n}}} \left[\begin{array}{l} (1-\delta T)\,(t^{\frac{1}{n}} - t_1^{\frac{1}{n}}) \\ + \frac{\delta}{n+1}\,(t^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}) \end{array} \right] \right\},\ (t_1 \le t \le T) \tag{6}$$

Substituting $Q(\mu) = Q_2$ in equation (4), we get

$$Q_1 = \frac{Q_2 + \frac{d}{T^{\frac{1}{n}}} \left\{ \mu^{\frac{1}{n}} - \frac{n}{n+1}\,\theta_1\, \mu^{\frac{1}{n}+1} + \frac{n\alpha\beta}{1+n\beta}\, \mu^{\frac{1}{n}+\beta} \right\}}{1 - \theta_1\mu + \alpha\mu^{\beta}} \tag{7}$$

Substituting $Q\,(t_1) = 0$ in equation (5), we get

$$Q_2 = \left\{ \begin{array}{l} \dfrac{\mu^{-\theta_2} + \alpha\mu^{-\theta_2+\beta}}{t_1^{-\theta_2} + \alpha t_1^{-\theta_2+\beta}}\ \dfrac{d}{T^{\frac{1}{n}}\,(1+n\theta_2)} \\ \left\{ t_1^{\frac{1}{n}} + \dfrac{n\alpha\beta}{1+n\,(\theta_2+\beta)}\, t_1^{\frac{1}{n}+\beta} \right\} - \dfrac{d\,\xi}{T^{\frac{1}{n}}\,(1+n\theta_2)} \end{array} \right\} \quad \text{where } \xi = \mu^{\frac{1}{n}} + \frac{n\alpha\beta}{1+n(\theta_2+\beta)}\,\mu^{\frac{1}{n}+\beta} \tag{8}$$

Substituting $Q\,(T) = -Q_3$ in equation (6), we get

$$Q_3 = \frac{d}{T^{\frac{1}{n}}} \left[(1-\delta T) \right] (T^{\frac{1}{n}} - t_1^{\frac{1}{n}}) + \frac{\delta}{n+1}\,(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}) \tag{9}$$

V. COST COMPONENTS

The total cost per replenishment cycle consists of the following cost components:

1. Operating Cost (OC)

The operating cost over the period $[0\ ,\ T]$ is

$$OC = \mathrm{A} \tag{10}$$

2. Inventory Holding Cost (IHC)

The inventory holding cost for carrying inventory over the period $[0\ ,\ T]$ is

$$IHC = \int_0^{\mu} (h+rt)\ \mathrm{Q}(t)\ \mathrm{dt} + \int_{\mu}^{t_1} (h+rt)\ \mathrm{Q}(t)\ \mathrm{dt}$$

$$IHC = \left[\begin{aligned} &\int_0^{\mu}(h+rt)\left[Q_1\left(1-\theta_1 t+\alpha t^{\beta}\right)-\frac{d}{T^{\frac{1}{n}}}\left[t^{\frac{1}{n}}-\frac{n}{n+1}\theta_1 t^{\frac{1}{n}+1}+\frac{n\alpha\beta}{1+n\beta}t^{\frac{1}{n}+\beta}\right]\right]\mathrm{dt} \\ &+\int_{\mu}^{t_1}(h+rt)\left[\eta\left(t^{-\theta_2}+\alpha t^{-\theta_2+\beta}\right)-\frac{d}{T^{\frac{1}{n}}(1+n\theta_2)}\left(t^{\frac{1}{n}}+\frac{n\alpha\beta}{1+n(\theta_2+\beta)}t^{\frac{1}{n}+\beta}\right)\right]\mathrm{dt}\end{aligned}\right]$$

$$IHC = \left[\begin{aligned} &\mathrm{h}\left[\begin{aligned}&Q_1\left(\mu-\frac{\theta_1\mu^2}{2}+\frac{\alpha\mu^{\beta+1}}{\beta+1}\right)\\ &-\frac{d}{T^{\frac{1}{n}}}\left(\frac{\mu^{\frac{1}{n}+1}}{\frac{1}{n}+1}-\frac{n\theta_1}{n+1}\frac{\mu^{\frac{1}{n}+2}}{\frac{1}{n}+2}+\frac{n\alpha\beta}{1+n\beta}\frac{\mu^{\frac{1}{n}+\beta+1}}{\frac{1}{n}+\beta+1}\right)\end{aligned}\right]+r\left[\begin{aligned}&Q_1\left(\frac{\mu^2}{2}-\frac{\theta_1\mu^3}{3}+\frac{\alpha\mu^{\beta+2}}{\beta+2}\right)\\ &-\frac{d}{T^{\frac{1}{n}}}\left(\frac{\mu^{\frac{1}{n}+2}}{\frac{1}{n}+2}-\frac{n\theta_1}{n+1}\frac{\mu^{\frac{1}{n}+3}}{\frac{1}{n}+3}+\frac{n\alpha\beta}{1+n\beta}\frac{\mu^{\frac{1}{n}+\beta+2}}{\frac{1}{n}+\beta+2}\right)\end{aligned}\right] \\ &+\mathrm{h}\left[\begin{aligned}&\eta\left(\frac{t_1^{-\theta_2+1}-\mu^{-\theta_2+1}}{1-\theta_2}+\frac{\alpha t^{-\theta_2+\beta+1}-\mu^{-\theta_2+\beta+1}}{1+\beta-\theta_2}\right)\\ &-\frac{d}{T^{\frac{1}{n}}(1+n\theta_2)}\left(\frac{t_1^{\frac{1}{n}+1}-\mu^{\frac{1}{n}+1}}{\frac{1}{n}+1}+\frac{n\alpha\beta}{1+n(\theta_2+\beta)}\frac{t_1^{\frac{1}{n}+\beta+1}-\mu^{\frac{1}{n}+\beta+1}}{\frac{1}{n}+\beta+1}\right)\end{aligned}\right] \\ &+\mathrm{r}\left[\begin{aligned}&\eta\left(\frac{t_1^{-\theta_2+2}-\mu^{-\theta_2+2}}{2-\theta_2}+\frac{\alpha t^{-\theta_2+\beta+2}-\mu^{-\theta_2+\beta+2}}{2+\beta-\theta_2}\right)\\ &-\frac{d}{T^{\frac{1}{n}}(1+n\theta_2)}\left(\frac{t_1^{\frac{1}{n}+2}-\mu^{\frac{1}{n}+2}}{\frac{1}{n}+2}+\frac{n\alpha\beta}{1+n(\theta_2+\beta)}\frac{t_1^{\frac{1}{n}+\beta+2}-\mu^{\frac{1}{n}+\beta+2}}{\frac{1}{n}+\beta+2}\right)\end{aligned}\right]\end{aligned}\right] \tag{11}$$

3. Deterioration Cost (DC)

The deterioration cost is

$$DC = C_d \left[\int_0^{\mu} \theta_1 \, Q(t) \, dt + \int_{\mu}^{t_1} \frac{\theta_2}{t} \, Q(t) \, dt \right]$$

$$DC = C_d \left[\begin{array}{l} \theta_1 \left(Q_1 \mu - \dfrac{d}{T^{\frac{1}{n}}} \dfrac{\mu^{\frac{1}{n}+1}}{\frac{1}{n}+1} \right) \\ + \theta_2 \left(\eta \left(\dfrac{t_1^{-\theta_2} - \mu^{-\theta_2}}{-\theta_2} + \dfrac{\alpha t_1^{-\theta_2+\beta} - \mu^{-\theta_2+\beta}}{-\theta_2+\beta} \right) \right) \\ - \dfrac{d}{T^{\frac{1}{n}} (1+n\theta_2)} \left(\dfrac{t_1^{\frac{1}{n}} - \mu^{\frac{1}{n}}}{\frac{1}{n}} + \dfrac{n\alpha\beta}{1+n(\theta_2+\beta)} \dfrac{t_1^{\frac{1}{n}+\beta} - \mu^{\frac{1}{n}+\beta}}{\frac{1}{n}+\beta} \right) \end{array} \right]$$

$$, where \ \eta = \frac{\theta_2 + \dfrac{d}{T^{\frac{1}{n}}(1+n\theta_2)} \xi}{\mu^{-\theta_2} + \alpha \mu^{-\theta_2+\beta}} \qquad (12)$$

4. Opportunity Cost due to lost sales (LSC)

The opportunity cost due to lost sales during the period $[t_1 \ , \ T]$

$$LSC = C_l \int_{t_1}^{T} \frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \left(1 - e^{-\delta (T-t)}\right) dt$$

$$LSC = C_l \ \frac{d\delta}{T^{\frac{1}{n}}} \left(T \left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}} \right) - \frac{1}{n+1} \left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1} \right) \right) \qquad (13)$$

5. Shortage Cost (SC)

The shortage cost over the period $[t_1 \ , \ T]$

$$SC = -C_s \int_{t_1}^{T} Q(t) \, dt$$

$$SC = C_s \ \frac{d}{T^{\frac{1}{n}}} \left[\begin{array}{l} \left\{ (1-\delta T) \, t_1^{\frac{1}{n}} + \dfrac{\delta}{n+1} t_1^{\frac{1}{n}+1} \right\} (t_1 - T) \\ + (1-\delta T) \dfrac{T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}}{\frac{1}{n}+1} + \dfrac{\delta}{n+1} \dfrac{T^{\frac{1}{n}+2} - t_1^{\frac{1}{n}+2}}{\frac{1}{n}+2} \end{array} \right] \qquad (14)$$

6. Purchase Cost (PC)

The purchase cost during the period is

$$PC = C_p\ \left[Q_1 + Q_3\right]$$

$$PC = C_p \left[\frac{ \dfrac{\mu^{-\theta_2} + \alpha\mu^{-\theta_2+\beta}}{t_1^{-\theta_2} + \alpha t^{-\theta_2+\beta}} \dfrac{d}{T^{\frac{1}{n}}(1+n\theta_2)} \left\{ t_1^{\frac{1}{n}} + \dfrac{n\alpha\beta}{1+n(\theta_2+\beta)} t_1^{\frac{1}{n}+\beta} \right\} - \dfrac{d\left(\mu^{\frac{1}{n}} + \dfrac{n\alpha\beta}{1+n(\theta_2+\beta)} \mu^{\frac{1}{n}+\beta} \right)}{T^{\frac{1}{n}}(1+n\theta_2)} + \dfrac{d}{T^{\frac{1}{n}}} \left\{ \begin{matrix} \mu^{\frac{1}{n}} - \dfrac{n}{n+1}\theta_1 \mu^{\frac{1}{n}+1} + \\ \dfrac{n\alpha\beta}{1+n\beta} \mu^{\frac{1}{n}+\beta} \end{matrix} \right\} }{1 - \theta_1\mu + \alpha\mu^{\beta}} + \frac{d}{T^{\frac{1}{n}}} \left[(1-\delta T)\right] (T^{\frac{1}{n}} - t_1^{\frac{1}{n}}) + \frac{\delta}{n+1} (T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}) \right] \tag{15}$$

7. Amelioration Cost (AMC)

The amelioration cost over the period $\left[0\ ,\ t_1\right]$ is

$$AMC = C_a \int_0^{t_1} \alpha\beta t^{\beta-1}\ Q(t)\ dt$$

$$AMC = C_a\ \alpha\beta \left[\left\{ Q_1 \frac{\mu^{\beta}}{\beta} - \frac{d}{T^{\frac{1}{n}}} \frac{\mu^{\frac{1}{n}+\beta}}{\frac{1}{n}+\beta} \right\} + \eta \left[\begin{matrix} \dfrac{1}{\beta-\theta_2}\left(t_1^{\beta-\theta_2} - \mu^{\beta-\theta_2}\right) + \alpha \dfrac{1}{2\beta-\theta_2}\left(t_1^{2\beta-\theta_2} - \mu^{2\beta-\theta_2}\right) \\ - \dfrac{d}{T^{\frac{1}{n}}(1+n\theta_2)} \left(\begin{matrix} \dfrac{1}{\frac{1}{n}+\beta}\left(t_1^{\frac{1}{n}+\beta} - \mu^{\frac{1}{n}+\beta} \right) \\ + \dfrac{n\alpha\beta}{1+n(\theta_2+\beta)} \dfrac{1}{\left(\frac{1}{n}+2\beta\right)} \left(t_1^{\frac{1}{n}+2\beta} - \mu^{\frac{1}{n}+2\beta} \right) \end{matrix} \right) \end{matrix} \right] \right] \tag{16}$$

Hence, the total cost per unit time is given by

$$TC = \frac{1}{T}\ \left(OC + IHC + AMC + DC + SC + LSC + PC\right) \tag{17}$$

Our objective is to determine the optimum values μ^*, t_1^* and T^* of μ, t_1 and T respectively so that average total cost is minimum.

Note that optimum values μ^*, t_1^* and T^* are the solutions of the equations

$$\frac{\partial(TC)}{\partial\mu} = 0,\ \frac{\partial(TC)}{\partial t_1} = 0\ \&\ \frac{\partial(TC)}{\partial T} = 0 \tag{18}$$

such that they can satisfy the following sufficient conditions:

$$\left.\begin{array}{l}
\left|\begin{array}{ccc}\frac{\partial^2 TC}{\partial \mu^2} & \frac{\partial^2 TC}{\partial \mu\, \partial t_1} & \frac{\partial^2 TC}{\partial \mu\, \partial T}\\ \frac{\partial^2 TC}{\partial t_1 \partial \mu} & \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1\, \partial T}\\ \frac{\partial^2 TC}{\partial T \partial \mu} & \frac{\partial^2 TC}{\partial T \partial t_1} & \frac{\partial^2 TC}{\partial T^2}\end{array}\right|_{\mu=\mu^*, t_1=t_1^*, T=T^*} > 0 \\
\left|\begin{array}{cc}\frac{\partial^2 TC}{\partial \mu^2} & \frac{\partial^2 TC}{\partial \mu\, \partial t_1}\\ \frac{\partial^2 TC}{\partial t_1 \partial \mu} & \frac{\partial^2 TC}{\partial t_1^2}\end{array}\right|_{\mu=\mu^*, t_1=t_1^*, T=T^*} > 0 \\
\text{and} \\
\left.\frac{\partial^2 TC}{\partial \mu^2}\right|_{\mu=\mu^*, t_1=t_1^*, T=T^*} > 0
\end{array}\right\} \quad (19)$$

The optimal solution of the equations in (19) can be obtained by using appropriate software. The above developed model is illustrated by means of the following numerical example.

VI. NUMERICAL EXAMPLE

We consider the following numerical example to illustrate the above inventory model. We take the values of the parameters A =80, δ =0.0001, h =5, r =3, C_p =15, C_s =3, C_d =1, C_l =8, C_a =7, n =2, d =0.7, α =0.0001, β =5, θ_1 =0.3 and θ_2 =0.8. We obtain the values

μ^* =1.426287283 units, t_1^* =6.236332531 units, T^* =42.76426697 units and optimum average total cost TC =5.877777836 units.

VII. PARTIAL SENSITIVITY ANALYSIS

Sensitivity Analysis works on the principle of "Ceteris Paribus" i.e. observing change in the value of average total cost with respect to one parameter keeping the rest of the parameters constant. It is a technique used to determine how different values of independent variables impact a particular dependent variable and the magnitude of impact under a given set of assumptions. In this section, we study the sensitivity of total cost TC per time unit with respect to the changes in the values of the parameters A, δ, h, r, C_p, C_s, C_d, C_l, C_a, n, d, α, β, θ_1, and θ_2.

This analysis is performed by considering 10% and 20% increase and decrease in the values of each one of the above parameters keeping all other remaining parameters fixed. The results are presented in the form of a table - 1. Graphical Presentation of the Sensitivity Analysis is required to facilitate the process of drawing a conclusion regarding the magnitude of influence each parameter individually holds on the average total cost. It is especially valuable in presenting data in simple, clear and effective manner as well as making trends easily recognizable.

Table – 1 Partial Sensitivity Analysis

Sensitivity Analysis					
Parameters	**% Change**	**μ**	**t_1**	**T**	**TC**
		1.426287283	**6.236332531**	**42.76426697**	**5.877777836**
α	-20	2.985365424	4.711500629	42.51005612	5.960148603
	-10	2.898696537	4.693930460	42.23866106	5.945773740
	10	1.425909405	6.204742661	42.70573312	5.876301386
	20	1.425523472	6.170529672	42.64814601	5.875047444
β	-20	1.416525417	5.265127311	42.08450267	5.899524881
	-10	3.200402922	0.716535894	41.02212365	6.213456330
	10	1.419561044	5.794047706	42.17196455	5.870783666
	20	1.410725016	5.263843769	41.61984502	5.872677844
d	-20	3.256386750	0.735614415	40.58068948	6.207776328
	-10	1.428617937	6.480068752	42.38310297	5.886393463

	10	1.423901830	6.006985055	43.09161398	5.867343333
	20	1.421446354	5.790601316	43.37177425	5.855413234
h	-20	1.428377850	6.461079306	42.60674786	5.854658736
	-10	1.427390493	6.352558779	42.69249216	5.866389418
	10	1.425034656	6.110439585	42.81962022	5.888812314
	20	1.423584197	5.972237841	42.85520916	5.899476051
r	-20	1.429616485	6.593507516	42.77185715	5.856260289
	-10	3.256976202	0.735683786	41.01656068	6.210595579
	10	1.424350205	6.047440002	42.72951875	5.887457107
	20	3.246191361	0.734305760	41.03579167	6.211795150
C_p	-20	1.427265480	6.356203545	41.92389913	5.804288058
	-10	1.426804882	6.297958408	42.34833455	5.841313655
	10	3.245235838	0.734182721	41.37789968	6.238275820
	20	3.237385223	0.733141376	41.73098421	6.265304172
C_d	-20	1.426599438	6.249839409	42.75260427	5.876259246
	-10	1.426443363	6.243106055	42.75846720	5.877019149
	10	3.252695880	0.735129683	41.02589302	6.211162413
	20	1.425975032	6.22266251	42.77567399	5.879291513
δ	-20	1.426273731	6.234973336	42.71637385	5.880035009
	-10	1.426280493	6.235651427	42.74028256	5.878907146
	10	1.426294103	6.237016663	42.78832754	5.876647086
	20	1.426300950	6.237703838	42.81246470	5.875514891
C_l	-20	1.426285990	6.236202794	42.76433791	5.877743860
	-10	1.426286637	6.236267665	42.76430244	5.877760851
	10	1.426287930	6.236397392	42.76423150	5.877794825
	20	1.426288577	6.236462248	42.76419602	5.877811812
C_s	-20	1.421779928	5.818886336	42.87849748	6.012743400
	-10	1.424439498	6.056975662	42.84188261	5.945662730
	10	1.427691097	6.380730393	42.66422715	5.809265345
	20	1.428817563	6.501890280	42.55078650	5.740233655
n	-20	1.420599579	4.999154039	29.45833664	7.684003401
	-10	1.425962391	5.790986253	36.00584758	6.676306047
	10	1.425564704	6.553047827	49.80905454	5.231989689
	20	1.424609346	6.807820468	57.13500794	4.700037347
θ_1	-20	1.768017145	6.496643970	42.36650851	5.838303702
	-10	1.578551771	6.369864691	42.58036840	5.858730285
	10	1.300898344	6.091641166	42.91633046	5.895630466
	20	4.884791523	8.170032442	31.28995467	5.041462740
θ_2	-20	1.256627181	6.769686220	42.50626381	5.829787960
	-10	3.188252115	0.869623065	40.82291937	6.183136469
	10	1.497523571	5.831960693	42.69918853	5.899143172
	20	1.557966794	5.265314082	42.39461025	5.917450193
A	-20	3.225404971	0.731442702	35.94874535	5.794927000
	-10	1.423349004	5.956585249	40.31644471	5.685193279
	10	3.264863757	0.736671718	43.45599489	6.400422120
	20	3.275129679	0.737904035	45.82905534	6.579646622
C_a	-20	1.426770236	6.276887220	42.79220614	5.876911260
	-10	1.426527496	6.256412982	42.77810753	5.877348525
	10	3.253156787	0.735187982	41.02317333	6.211010984
	20	1.425813967	6.197282075	42.73730883	5.878613714

VIII. GRAPHICAL PRESENTATION

Graphical presentation of the various parameters is shown in the following figure.

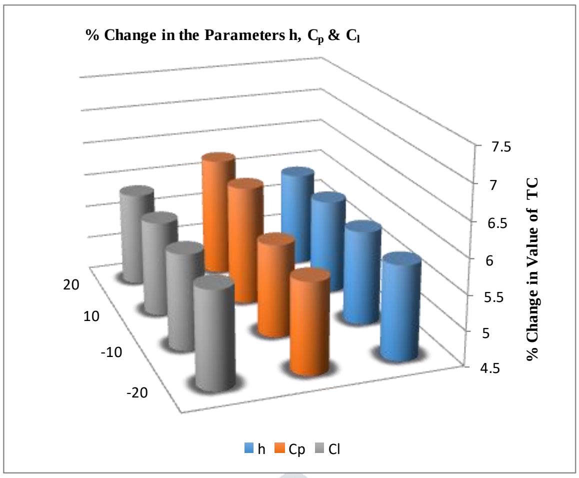

Figure 2: Graphical Representation of h, C_p, C_l

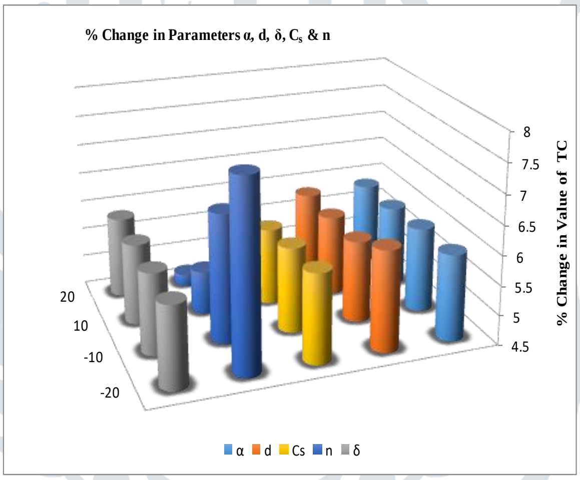

Figure 3: Graphical Representation of α, d, C_s, n, δ

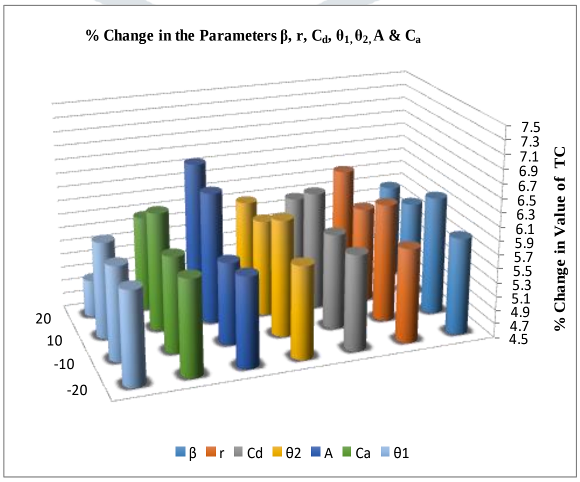

Figure 4: Graphical Representation of β, r, C_d, $θ_2$, A, C_a, $θ_1$

IX. CONCLUSION

From the sensitivity Analysis and the Graphs presented above, we can observe that different parameters affect the average total cost differently and conclusions are as under.

If with the change in parameter value, there is a significant change in the value of average total cost, the average total cost is said to be highly sensitive to the change in parameter. If there is moderate change, the average total cost is said to be moderately sensitive with respect to the parameter. If there is less change, the average total is said to be less sensitive with respect to parameter.

From the above sensitivity analysis, it is observed that the total cost per unit is highly sensitive to the changes in the values of A, β, r, n, θ_2, moderately sensitive to changes in the values of C_p, C_s, C_d, C_a, d and less sensitive to the changes in the values of δ, h, C_l, α, θ_1.

From the Table – 1 it is observed that as the value of the parameters h, C_p, C_l increase the average total cost also increases and the value of the parameters α, d, C_s, n, δ decrease the average total cost also decreases and the value of the parameters A, r, C_d, C_a, β, θ_1, θ_2 fluctuate the average total cost also fluctuate.

REFERENCES

[1] Ankit Bhojak, U.B. Gothi, (2016), Inventory Models for Ameliorating and deteriorating Items with Time Dependent Demand and IHC, International Journal of Management and Social Science Research Review, Vol.1 (2), 85-98.

[2] Ankit Bhojak, U.B. Gothi, (2016), Production Inventory System under Weibull Amelioration, Pareto Deterioration and Exponentially Time Based Demand under Fully Backlogged Shortages, International Journal of Scientific and Innovative Mathematical Research, Vol.4 (5), 26-36.

[3] Devyani Chatterji, U.B. Gothi, (2015), EOQ Model for deteriorating Items Under Two and Three Parameter Weibull Distribution and Constant IHC with Partially Backlogged Shortages, International Journal of Science, Engineering and Technology Research (IJSETR), Vol.4 (10), 3582 – 3594.

[4] Dye, Chung-Yuan, Heish Tsu-Pang, Ouyang Liang-Yuh, (2007), Determining optimal selling price lot size with a Varying rate of deteriorating and Exponential partial backlogging, European Journal of Operational Research, Vol.181 (2), 668- 678.

[5] Ghare P.M, Schrader G.F., (1963), A model for an exponentially decaying inventory, Journal of Industrial Engineering, 238-243.

[6] Goyal S.K., Giri B.C., (2001), Recent trends in modelling of deteriorating inventory, European Journal of Operational Research, 1-16.

[7] Kirtan Parmar, Indu Aggarwal, U.B. Gothi, (2015), order level inventory model for deteriorating item under varying demand condition, Sankhya Vignan, (NSV 11), 20-30.

[8] L. Tadj, A.M. Sarhan, A. EI-Gohary, (2008), Optimal control of an inventory system with ameliorating and deteriorating items, Applied Sciences, Vol.10, 243-255.

[9] Milu Acharya, Smrutirekha Debata, (2014), An Inventory model for deteriorating items with time dependent demand under partial backlogging, International Journal of Advent Technology, Vol. 2, 86-90.

[10] Nandgopal Rajeswari, Thirumalaisamy Vanjikkodi, (2012), An Inventory Model for Items with Two Parameter Weibull Distribution Deterioration and Backlogging, American Journal of Operational Research, Vol.2, 247-252.

[11] Nita H. Shah, Kunal T. Shukla, (2009), Deteriorating Inventory Model for Waiting Time Partial Backlogging, Applied Mathematical Sciences, Vol.3 (9), 421-428.

[12] Pooja D. Khatri, U.B. Gothi, (2020), An EPQ Model for Non – Instantaneous Weibully Decaying Items with Ramp Type Demand and Partially Backlogged Shortages, International Journal of Mathematics Trends and Technology (IJMTT) Vol. 66 (3), 124-131.

[13] Ruxian Li, Hongjie Lan, John R. Mawhinney, (2010), A review on deteriorating Inventory Study, Journal of Service Science and Management, Vol.3, 117-129.

[14] R. Uthayakumar, K.V. Geetha, (2009), A Replenishment Policy for Non-instantons deteriorating Inventory System with Partial Backlogging, Tamsui Oxford Journal of Mathematical Sciences, Vol.25 (3), 313-332.

[15] Srinivasa Rao K., Koushar Jaha Begum, Vivekananda Murthy.M., (2007) "Optimal Ordering policies of inventory model for deteriorating items having generalized Pareto lifetime," Current Science, Vol.93 (10), 1407-1411.

[16] Singh N., Bindu Vaish, S.R. Singh, (2010), An EOQ Model with Pareto Distribution for Deterioration, Trapezoidal Type Demand and Backlogging under Trade Credit Policy, The IUP Journal of Computational Mathematics, Vol.3 (4), 30-53.

[17] Srichandan Mishra, L.K. Raju, U.K. Misra, G. Misra, (2012), Optimal Control of an Inventory System with Variable Demand & Ameliorating / Deteriorating Items," Asian Journal of Current Engineering and Maths, 154 – 157.

[18] U.B. Gothi, Ankit Bhojak, (2016), Inventory System with deterioration of Items under Pareto Type – I Distribution, Power Demand Pattern and Partial Backlogging, Sankhya Vignan, (NSV 12) Vol.1, 47-61.

[19] U.B. Gothi, Ankit Bhojak, (2016), Inventory Model for Ameliorating and Deteriorating Items under Time Dependent Demand and IHC with Partial Backlogging, International Journal of Engineering Science and Computing, Vol.6 (4), 3979-3985.

[20] U.B. Gothi, Kareeshaa Shah, Santosh Rohida, (2019), An EOQ model for weibully distributed deteriorating items with the effect of permissible delay in payments and partially backlogged shortages, International Journal of Advanced Scientific Research and Management, Vol.4 (5), 175-194.

[21] U.B. Gothi, Pooja D. Khatri, (2019), An Inventory Model for Deteriorating Items in a Single Warehouse Starting with Lead Time, Price Dependent Demand and Shortages, International Journal of Research and Analytical Reviews (IJRAR), Vol.6 (2), 500-508.

[22] Vijayalaxmi G., Srinivasa Rao K., Nirupama Devi K., (2014), Inventory Model for deteriorating Items Having Two Component Mixture of Pareto Lifetime and Selling Price Dependent Demand, International Journal of Scientific and Engineering Research, Vol.5 (7), 254-262.

[23] Vinod Kumar Mishra, Lal Sahab Singh, (2011), Deteriorating Inventory Model for Time Dependent and Holding Cost with Partial Backlogging, International Journal of Management Science and Engineering Management, Vol.6 (4), 267-271.