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Logotropic Model Of The Universe in Bianchi Type-III Space Time

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Abstract:

Bianchi type-III cosmological model with logotropic equation of state has been investigated in general theory of relativity. We have considered the model in the presence of single dark fluid with logotropic equation of state (EoS)

The general solutions of the Einstein's field equations for Bianchi type-III space time have been derived under the assumption that the scalar expansion θ is proportional to the shear scalar σ^2. The kinematic and physical properties of the model are also discussed.

Keywords:

Bianchi type-III space time, Logotropic equation of state, Single Dark Fluid.

Introduction:

The Universe has undergone transformation through four main eras, which are vacuum energy era (or, Planck era), radiation era & matter dominated era and the dark energy era (or, de Sitter era) according to Binney and Tremaine [1]. The universe initially passes through a phase of early inflation [2] which is called the vacuum energy era (Planck era). After that, the universe goes through the radiation era followed by the matter-dominated era, at a temperature below 103 K approximately [3]. Finally, the dark energy era sees the universe going through a phase of late inflation [4]. The singularity problems are taken care of in the early inflation [2] whereas late inflation explains the accelerating expansion pattern of the universe [5-13]. The cosmological inferences and data sets like Atacama Cosmology Telescope (ACT) [14], Planck 2015 results- XIII [15], are indicative about late time inflation of the universe and helps researchers to determine important cosmological parameters such as the Hubble constant H and also the deceleration parameter q. Some other problems arisen by the CMBR (Cosmic Microwave Background Radiation) study [16, 17], WMAP (Wilkinson Microwave

Anisotropy Probe) [18], observations of galaxies at low red-shift [19, 20] etc., it has been found that the universe is governed by some mysterious components. This unknown component of the energy is referred to as dark energy which has negative pressure along with positive energy density. It is believed that about 73% of our universe is Dark energy, about 23% comprises of Dark matter and the rest 4% is occupied by baryonic matter. Therefore, the study of the role of dark energy has become an interesting topic for researchers all over the world [21-27]. Einstein's cosmological constant Λ describes a model with cold dark matter (CDM) which is generally called the ΛCDM model. In 1919, Einstein put forth the idea of gravitation in the theory of general relativity which formed the basis for defining a model of the universe. Later, many researchers studied gravitation in different contexts. Some significant theories of gravitation are Brans-Dicke theory [28], Scalar-tensor theories [29], Saez and Ballester [30], Vector-tensor theory [31], These theories of gravitation were helpful in investigating the accelerating expansion nature of the universe. Researchers used various equations of state in relativity and cosmology, which is dealt with the relationship between matter, temperature, energy density and pressure. Many researchers like Ivanov [32], Sharma and Maharaj [33], Thirukkanesh and Maharaj [34], Feroze and Siddiqui [35], Varela et al. [36] etc. investigated cosmological models with linear and non-linear equation of state. Few proposed a model of the universe with quadratic equation of state. Dark energy universe with various equations of state were already discussed by authors like, Bamba et al [24], Nojiri and Odintsov [37, 38], Nojiri et al. [39], Nojiri and Odintsov [40] and Capozziello et al. [41,42,43].

In this paper, we introduce a new model using logotropic equation of state that may be able to provide a solution to the cosmological problems. We intend to describe the universe with a logotropic equation of state (EoS) $p = D\log(\frac{\rho}{\rho_P})$, where D is the logotropic temperature, ρ is the rest mass density and ρ_P is the reference density taken as Planck's density. This equation of state was found to be mentioned in astrophysics by (McLaughlin, D. E., & Pudritz, R. E. 1997) to describe the internal organisation as well as the properties of clouds and clumps on the surface of the universe. The logotropic equation of state is an interesting concept for solving the mysteries of dark energy. The logotropic temperature has been considered to be strictly positive (D > 0) from a thermodynamic angle. Chavanis, P. H. [44,45,46,47], has discussed whether the universe is logotropic.

Field Equations & their solutions:

We consider the Bianchi type III line element as

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2ax}dy^2 + C^2dz^2 \tag{2.1}$$

where A, B, C are functions of cosmic time t and a is a constant.

The Einstein field equations, in natural limits ($8\pi G = 1$ and $c = 1$), are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \tag{2.2}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar and T_{ij} is the energy momentum tensor.

Also the conservation equation of the energy momentum tensor is

$$T^{ij}_{;j}=0 \tag{2.3}$$

The energy momentum tensor for the single dark fluid is given by

$$T_{ij}=(p+\rho)u_i u_j - p g_{ij} \tag{2.4}$$

where ρ is the energy density, p is the pressure and u^i is the four velocity vector satisfying $g_{ij}u^i u_j=1$.

We assume an equation of state (EoS), $p=D\log(\rho/\rho_P)$ (2.5)

in the logotropic form, where D is the logotropic constant, D>0, ρ is the rest-mass density and ρ_P is the reference density taken as Planck's constant.

In a co-moving coordinate system, the Einstein field equations (2.2) for the metric (2.1) with the help of equation (2.4) reduce to the following set of equations

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{a^2}{A^2}=\rho \tag{2.6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{a^2}{A^2}=-p \tag{2.7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=-p \tag{2.8}$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{B}}{B}+\frac{\dot{B}\dot{C}}{BC}=-p \tag{2.9}$$

$$a\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=0 \tag{2.10}$$

where overhead dot $(\cdot)$ denote differentiation with respect to time t.

The energy conservation equation $T^{ij}_{;j}=0$ leads to the following simple expression

$$\dot{\rho}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)(p+\rho)=0 \tag{2.11}$$

We define the spatial volume V and the average scale factor $a(t)$ for Bianchi type VI$_0$ space time respectively as

$$V=a^3(t)=A^2C \tag{2.12}$$

The mean Hubble parameter H is given by

$$H=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \tag{2.13}$$

The scalar expansion θ and the shear scalar σ^2 are given by

$$\theta=3H \tag{2.14}$$

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^2-3H^2\right) \tag{2.15}$$

The average anisotropy parameter Δ is given by

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{2.16}$$

where $H_i, i=1,2,3$ represent the directional Hubble parameters in x,y,z directions respectively and $\Delta=0$ corresponds to isotropic expansion.

Integrating equation (2.10) we get

$$A=Bc_1 \tag{2.17}$$

Where c_1 is a constant of integration.

Now, to find solutions to the Einstein's field equations, we have considered that the expansion scalar θ is proportional to the shear scalar σ^2 . This condition leads to

$$B=C^m \tag{2.18}$$

where $m\neq 0$ is constant

The field equations reduce to

$$m^2\frac{\dot{C}^2}{C^2}+(m+1)\frac{\ddot{C}}{C}=-p \tag{2.19}$$

$$\frac{-a^2}{C^{2m}}+(3m^2-2m)\frac{\dot{C}^2}{C^2}+2m\frac{\ddot{C}}{C}=-p \tag{2.20}$$

$$\frac{-a^2}{C^{2m}}+(m^2+2m)\frac{\dot{C}^2}{C^2}=\rho \tag{2.21}$$

Subtracting equation (2.20) from equation (2.19) we get :

$$\ddot{C}=\frac{a^2}{m-1}C^{1-2m}-2m\frac{\dot{C}^2}{C} \tag{2.22}$$

The above non-linear differential equation is converted into the linear form by proper substitution and solution is obtained as

$$C = (\alpha t + \beta)^{1/m} \qquad (2.23)$$

Where $\alpha \neq 0$ and β is constant of integration.

From equations 2.18 and 2.23 we get

$$B = (\alpha t + \beta) \qquad (2.24)$$

$$A = (\alpha t + \beta) \qquad (2.25)$$

Without loss of generality Taking suitable values of α and β as $\alpha = 1$ and $\beta = 0$

Substituting in equation (2.13) we get mean Hubble parameter as

$$H = (\frac{2m+1}{3m})\frac{1}{t} \qquad (2.26)$$

The expansion scalar θ and the shear scalar σ^2 are given by

$$\theta = 3H = (\frac{2m+1}{m})\frac{1}{t} \qquad (2.27)$$

$$\sigma^2 = \frac{(m-1)^2}{3m^2t^2} \qquad (2.28)$$

The average anisotropy parameter Δ from equation 2.16 is given by

$$\Delta = \frac{2(m-1)^2}{(2m+1)^2} \qquad (2.29)$$

And $\Delta = 0$ when $m = 1$

Physical Discussion:

Now, from equation (2.21), we obtain the energy density as

$$\rho = (-a^2 + \frac{m+2}{m})\frac{1}{t^2} \qquad (3.1)$$

Also, from equation (2.5), we obtain the pressure as

$$p = D[\log \frac{(-a^2 + \frac{m+2}{m})\frac{1}{t^2})}{\rho_P}] \qquad (3.2)$$

The energy density parameter is given by

$$w = \frac{p}{\rho} = \frac{D[\log \dfrac{(-a^2 + \frac{m+2}{m})\frac{1}{t^2})}{\rho_P}]}{(-a^2 + \frac{m+2}{m})\frac{1}{t^2}} \tag{3.3}$$

Conclusion :

A study of Bianchi type III cosmological model with a logotropic equation of state has been done. The physical and kinematic parameters which play an important role in the discussion of cosmological models are obtained.

It is noted that spatial volume of the universe increases with time.

The energy density, pressure, average Hubble parameter, expansion scalar and shear scalar are infinite at $t = 0$ and approaches 0 as $t \to \infty$.

The average anisotropy parameter is constant and becomes zero at m = 1, which shows early inflation and late time acceleration. This is compatible with the results of modern cosmology. (Riess et al. 1998 and Perlmutter et al. 1999).

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