# INVERSE OF FUZZY SUBSET FUNCTION WITH FUZZY SUBGROUPOIDS 

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#### Abstract

In ordinary topological space the concept of set function plays an important role in determination of continuity of a function. The concept of this function is carried to fuzzy structure by L. A. Zadeh. Let the ordered semigroup is taken as T . Let T be the fuzzy subset characterized by an arbitrary mapping $\mathrm{f}: \mathrm{T} \rightarrow[0,1]$ where $[0,1]$ is the real numbers with inverse of fuzzy subset function along with fuzzy subgroupoid definition and its properties.


## Keywords

Inverse function, fuzzy subset, fuzzy subgroupoid, fuzzy subgroup, Homomorphism.

## I. Introduction

In the year of 1965, the great Mathematician L. A. Zadeh [28] proposed the hypothesis of fuzzy set. It has given a helpful numerical device to portraying the way of behaving of frameworks that exact numerical examination by old style techniques and instruments. Various fields viz, man-made consciousness, software engineering, the executive's science, Activity Exploration, Mechanical technology, Example acknowledgment and others fuzzy hypothesis has been found. Dib and Galhum [4] characterized fuzzy subgroupoids and fuzzy semigroups. We start with the idea of fuzzy subgroupoid presented by Rosenfield.

In this paper we concentrate on reverse fuzzy subset capability and fuzzy subgroupoid. We will give the meaning of opposite fuzzy subset with fuzzy subgroupoids.

## II. Preliminaries

Here, we give some definition and proposition related to fuzzy subset function in this section which will be used for the next development.

## Definition: 2.1

Let $\mathrm{g}: \mathrm{A} \rightarrow \mathrm{B}$ be a function from a set A into B and $\alpha \in \mathrm{I}^{\mathrm{y}}$, the fuzzy subset of A be $\mathrm{g}^{\prime}(\alpha)$ characterized by $\left[g^{-1}(\alpha)\right](a)=(\alpha[g(a)])$, for all $a \in A$.

## Definition: 2.2

Let $\eta \in I^{x}$ and a fuzzy subset of $Y$ is $f(\eta)$. Then, at the point,

$$
\mathrm{f}[\eta(\mathrm{~b})]=\left\{\begin{array}{c}
\sup [\eta(\mathrm{a})], \text { if } \mathrm{a} \in \mathrm{f}^{-1}(\mathrm{~b}) \neq \varnothing \\
0, \text { if } f^{-1}(b)=0
\end{array}\right.
$$

## Example 2.3

Let $P=\left\{a_{1}, a_{2}, a_{3}, a_{4}\right\}$ and
$Q=\left\{b_{1}, b_{2}, b_{3}, b_{4}\right\}$ and
$\mathrm{g}: \mathrm{P} \rightarrow \mathrm{Q}$ is defined by

$$
\mathrm{g}\left(\mathrm{a}_{1}\right)=\mathrm{b}_{1},
$$

$\mathrm{g}\left(\mathrm{a}_{2}\right)=\mathrm{b}_{3}$,
$g\left(a_{4}\right)=b_{3}$.
If $\alpha=\left\{\left(b_{1}, 0\right),\left(\left(b_{2}, 1\right),\left(b_{3}, 0.2\right),\left(b_{4}, 0.7\right)\right\}\right.$ then
(i) $\quad \mathrm{g}^{-1}\left[\alpha\left(\mathrm{a}_{1}\right)\right]=\alpha\left[\mathrm{g}\left(\mathrm{a}_{1}\right)\right]=\alpha\left(\mathrm{b}_{1}\right)=0$
(ii) $\quad \mathrm{g}^{-1}\left[\alpha\left(\mathrm{a}_{2}\right)\right]=\alpha\left[\mathrm{g}\left(\mathrm{a}_{2}\right)\right]=\alpha\left(\mathrm{b}_{2}\right)=1$
(iii) $\mathrm{g}^{-1}\left[\alpha\left(\mathrm{a}_{3}\right)\right]=\alpha\left[\mathrm{g}\left(\mathrm{a}_{3}\right)\right]=\alpha\left(\mathrm{b}_{2}\right)=1$
(iv) $\mathrm{g}^{-1}\left[\alpha\left(\mathrm{a}_{4}\right)\right]=\alpha\left[\mathrm{g}\left(\mathrm{a}_{4}\right)\right]=\alpha\left(\mathrm{b}_{3}\right)=0.2$

## III. Inverse fuzzy subgroupoid

## Definition: 3.1

A fuzzy set $y: T \rightarrow[0,1]$ is called fuzzy subgroupoid of $T$.
If $y(a b) \geq \min (y(a), y(b))$. Let $(T,$.$) be a groupoid [T is a set, closed under the binary operation ' { }^{\prime}$ '].

## Definition: 3.2

A fuzzy set $y: I \rightarrow[0,1]$ is called an Inverse fuzzy subgroupoid of $I$.
If $\mathrm{f}^{-1}[\mathrm{y}(\mathrm{ab})] \geq \min [y[f(\mathrm{a})], \mathrm{y}[f(b)]$ where (I, .) be a groupoid.
i.e, $f^{-1}[y(a b)]=y[f(a b)]$ by fuzzy subset function

$$
\geq \min \{y[f(a)], y[f(b)]\}
$$

## Proposition: 3.3

Consider $\theta \in[0,1]$, the $\theta$ - level set $[y]_{\theta}=\{a \in I / y(a) \geq \theta\}$ is a subgroup if $y$ is an inverse fuzzy subgroupoid.

## Proof:

Let y be an inverse fuzzy subgroupoid on $\mathrm{I} . \mathrm{a}, \mathrm{b} \in[\mathrm{y}]_{\theta}$. Then $\mathrm{y}[\mathrm{f}(\mathrm{a})] \geq \theta$ and $\mathrm{y}[\mathrm{f}(\mathrm{b})] \geq \theta$ and to prove $[y]]_{\theta}$ is an inverse fuzzy subgroupoid.
$\mathrm{f}^{-1}[y(\mathrm{ab})]=y[f(\mathrm{ab})]$
$\geq \min [y(f(a)), y(f(b)] \geq \theta$
$\geq y(f(\mathrm{a}))$ or $\mathrm{y}(f(\mathrm{~b}))$. Here $\mathrm{a}, \mathrm{b} \in[y]_{\theta}$

## Definition 3.4

Let $y: K \rightarrow[0,1]$, is known as a fuzzy subgroup of $K$ if and only if
(i) $\quad y(a b) \geq \min [y(a), y(b)]$ for all $a, b \in K$
(ii) $\quad y\left(a^{-1}\right) \geq y(a)$ for all $a \in K$
where K is a group.

## Proposition: 3.5

The intersection of inverse fuzzy subgroupoids is an inverse fuzzy subgroupoid.

## Proof:

If $y_{i}^{-1}$ is an Inverse fuzzy subgroupoid.
Then $\cap_{i} y_{i}^{-1}(a b) \geq \min \left(\cap_{i} y_{i}^{-1}(a), \cap_{i} y_{i}^{-1}(b)\right)$
Therefore $y_{i}^{-1}$ is an Inverse fuzzy subgroupoid.

## Proposition: 3.6

If $y^{-1}$ is an inverse fuzzy subgroupoid of a group $K$, is finite. Then $y^{-1}$ is a fuzzy subgroup of $K$.

## IV. Homomorphic image and Pre-image of Inverse fuzzy subgroupoid.

## Definition: 4.1

Let $\mathrm{g}: \mathrm{A} \rightarrow \mathrm{B}$ be a Homomorphism from the Inverse subgroupoid S into the subgroupoid Q such that,

$$
\begin{aligned}
g^{-1}(a b) & =g(a b) \\
& =g(a) \cdot g(b) \\
g^{-1}(a b) & =g(a) \cdot g(b)
\end{aligned}
$$

$\therefore \mathrm{g}(\mathrm{P})$ is a subgroupoid of Q .

## Proposition: 4.2

If fuzzy subgroup has a homomorphic image or preimage then it will be an inverse fuzzy subgroup.

## Proposition: 4.3

$$
\text { If } y^{-1}(a b)=y\left(e_{1}\right) \text { then } y^{\prime}(b)=y(a)
$$

## Proof:

$$
\begin{aligned}
y(b) & =y\left(b a^{-1} a\right) \\
& \geq \min \left\{y\left(b a^{-1}\right), y(a)\right\} \\
& \geq \min \left\{y\left(e_{1}\right), y(a)\right\} \\
\therefore & y(b) \geq y(a)
\end{aligned}
$$

Similarly,
We can prove the other inequality.

$$
\begin{gathered}
y(a) \geq y(b) \\
\therefore y(b)=y(a)
\end{gathered}
$$

## V. Conclusion

In the above, we have characterized inverse of a fuzzy set capability, fuzzy subgroupoid, fuzzy subgroup by utilizing backwards capability. We have depicted converse fuzzy set by utilizing a fuzzy subgroupoid as well as the other way around. We have expressed how the homomorphic picture and pre-picture of fuzzy subgroupoid turns into a converse fuzzy subgroupoid.

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