



Thermal radiation and non-uniform heat source/sink on MHD convective flow of a viscous incompressible fluid over a permeable stretching sheet

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Abstract

This paper explores the influence of a non-uniform heat source/sink in a flow of couple stress fluid by stretching sheet in the MHD convective flow of a viscous incompressible flow. The numerical results of magneto hydrodynamic liquid flowing across a stretching sheet at its stagnation point while also experiencing viscous dissipation, variable magnetic field, thermal radiation, uniform heat source, Joules dissipation and non-uniform heat source/sink. The governing partial differential equations that regulate physical phenomena are converted to non-linear ordinary differential equations via partial differential equations by using the appropriate similarity transformations. Later, resolved by numerically using the Runge-Kutta fourth order with efficient shooting method by using Mathematica. Results of the effect of a non-dimensional parameters like Velocity ratio parameter (λ), Source and Radiation parameter, Magnetic parameter (M), Grashaff number, Eckart Number (Ec), Chemical reaction parameter (γ), Porosity (k), Prandtl Number (Pr), Space dependent parameter, Temperature dependent parameter etc. On the velocity, temperature and concentration profiles for their numerous values were analyzed and depicted graphically and also given in tabular form. The outer-stream velocity is varying linearly with the coordinate parallel to the boundary due to the effect of stagnation point. The porous parameter increases with the increase in the velocity.

Keywords: Convective flow, Permeable stretching sheet, Stagnation point, Magnetohydrodynamics, Stretching Sheet.

1. Introduction

The investigations of heat transfer on viscous incompressible fluid flow over a permeable sheet which is continuously stretching in nature is of tremendous significance under engineering and manufacturing sections like

fibre production, polymer sheets extrusion, crystal growing, etc. Further, the production of the quality sheet through manufacturing processes in the polymer industry is largely dependent on the cooling rate. As a consequence, significant efforts have been taken by a variety of researchers in the last decades to change the flow kinematics in order to ensure a better control of cooling rate. With and without slip regime at the surface, many researchers have been investigated the permeable stretching sheet. Crane [1] studies focus is on how a viscous fluid with electrical conductivity moves across a stretched layer of fluid while transferring heat. Carragher et. al. [2] investigated the heat transfers of the stretched sheet that emerges from the slit and strikes a medium when at rest, the speed of the sheet depends on how far it is from the slit. A power of a distance from the slit, determines temperature difference between sheet and its surroundings. For the both medium and large Prandtl values, rate of heat transfer is determined. Heat transfer of viscoelastic fluids across the boundary layer, a viscous dissipation and non-uniform heat source/sink are present on the stretching sheet authors [3].

Singh and Madhab [4] was studied the Fluid flow across a moving variable surface with MHD heat transfer through free convection in the Porous Medium. Madhukeshet. al. [5] studied the model of non-Fourier heat flow for the flow of Hybrid water-based nanofluid across the curved stretching sheet. Prasannakumara et. al. [6] worked on the impact of a Williamson nanofluid flow's chemical reaction, heat and mass transmission over a stretching sheet. Tawade et. al. [7] described the 2-D boundary layer problem of steady laminar flow of an nanofluid using a linear stretching sheet to transfer heat from a Casson fluid. Ramesh et. al. [8]. investigated the flow over a non-isothermal stretching sheet at the non-Newtonian fluid's stagnation point when it is incompressible.

Bharat kumar et. al. [9] described the MHD mixed convective flow of the Eyring-Powell fluid as it controls the heat transfer by convection over a layered stretching sheet.

The problem of Sisko nanofluid flow over the nonlinear stretching sheet under the influence of nonlinear thermal radiation and chemical reaction investigated by Prasannakumara et. al. [10]. The temperature and magnetic-field-dependent Ferro-fluid's linear and nonlinear stability under imposed time-periodic gravity modulation investigated by Aanam et. al. [11]. Wubshet Ibrahim and Bandari Shanker [12] studied the Heat transfer through a stretched sheet enclosed in an porous non-Darcian medium. The flow of MHD in a laminar state of Maxwell fluid that is upper-convected over isothermal porous of stretching surface has been studied by Guled et. al. [13] using the optimal homotopy analysis (OHAM) approach. Hayat et. al. [14] studied about the non-uniform heat source/sink of thermal layered stretching cylinder fluid flow. The boundary layer flow in a nanofluid carried by linearly stretched sheet, which is investigated by Abel et. al. [15].

The effect of thermal radiation, non-uniform heat sources, and irregular heat sinks on the flow and transmission of heat through the stretching sheet enclosed in non-Darcian porous medium are discussed by W. Ibrahim and B. Shankar [16]. Over a stretched sheet, electrically conducting micropolar nanofluids flow while being impacted by the thermal radiation and non-uniform heat source/sink examined by Pal Dulal and Mandal Gopinath [17]. The stretching sheet with the non-uniform source for an UCM fluid, combining with external magnetic field which is examined by Agadi et. al. [18]. Bharatkumar et. al. [19] investigated the MHD combined convection flow of Eyring-Powell fluid's, which controls the convective movement of heat over the stretching sheet with multiple layers.

Inspired by the importance of an incompressible fluid flow, an objective here is set out to accomplish the solution of non-uniform heat source/sink of MHD boundary layer flow, and heat and mass transfer over stretchable sheet. Existing literature reveals that the influence of thermally radiative nonuniform heat source/sink of MHD boundary

layer flow over a horizontal stretching sheet has not been considered yet. Hence, results obtained from this study are novel and genuine and will fill the gap in the literature.

1. Mathematical formulation

Consider incompressible 2-D laminar constant flow hydro magnetic viscous flow that conducts electricity across a stretching sheet. The slit from which sheet is drawn serves as the system's entry point. The axis frame is taken in this coordinate along with the route of the uninterrupted extending plane.

Taking the velocity of the stretching sheet into account to be $U_w(x) = bx$ and the free stream flow's velocity to be $U_\infty(x) = ax$, where a and b are positive constants and x is the coordinate along the stretching plane.

The flow is carried out at $y \geq 0$, where y is the perpendicular to the stretching sheet coordinate. Let T_w and C_w be the wall surface temperature and nanoparticle concentration of the Nanofluid at the stretching layer and let T_∞ and C_∞ be the ambient temperature and concentration.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta_t(T - T_\infty) + g\beta_c(C - C_\infty) - \frac{\nu}{k}u - \frac{\sigma B_0^2}{\rho}u + U_\infty \frac{\partial U_\infty}{\partial x} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{Q}{\rho C_p}(T - T_\infty) + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{\rho C_p} u + \frac{q'''}{\rho C_p} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} - K_r(C - C_\infty) \quad (4)$$

Where u and v represents components of the velocity along the coordinate axes, Temperature T , concentration of species is C , Kinematic viscosity is ν , Thermal diffusivity is α , Brownian motion coefficient is D_B , using radiation of Roseland approximation, radiative heat flux q_r is given by

$$q_r = \frac{-4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (5)$$

Where average absorption coefficient k^* , Stefan-Boltzmann constant σ^* . We assume that the flow's internal temperature variations are not very significant. The expression T^4 , by Taylor series expansion, expanding the powers of T^4 and T_∞ while higher-order terms are ignored then we get,

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4$$

hence we get,

$$q_r = \frac{-4\sigma^* T_\infty^3}{3k^*} \frac{\partial T}{\partial y}$$

Then,

$$\frac{\partial q_r}{\partial y} = \frac{-16\sigma^* T_\infty^3}{3\rho c_p k^*} \frac{\partial^2 T}{\partial y^2} \quad (6)$$

The corresponding boundary conditions are

$$u = bx; v = 0; T = T_w; C = C_w \quad \text{at } y = 0$$

$$u = U_\infty = ax, T = T_\infty; C = C_\infty \quad \text{as } y \rightarrow \infty \quad (7)$$

the similarity transformations, which satisfy the continuity equation, then,

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \quad (8)$$

Such that

$$u_1 = bxf'(\eta), v_1 = -\sqrt{vb}f(\eta); \eta = \sqrt{\frac{b}{v}}y, \quad (9)$$

$$\theta = \frac{T-T_\infty}{T_w-T_\infty}, \quad \phi = \frac{C-C_\infty}{C_w-C_\infty}$$

By Substituting Eq's 6, 8 and 9 in eq's 2-4. The resulting coupled nonlinear differential equations are as follows.

$$f''' + ff'' + Gr\theta + Gm\phi - \left(M - 1 + \frac{1}{k}\right)(f')^2 + \lambda^2 = 0 \quad (10)$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + Pr[Ec[(f'')^2 + M(f' - 1)^2] + S\theta + f\theta'] + [A^*f' + B^*\theta] = 0 \quad (11)$$

$$\phi'' + Sc[f\phi' - \gamma\phi] = 0 \quad (12)$$

The parameters that are present in the above equations are Gr is a Thermal Grashoff Number and Gm be the Mass Grashoff number, $1/k$ is the Porosity parameter, λ is velocity ratio parameter, R is a radiation, Pr is a Prandtl number, Ec is the Eckert number, M is a magnetic parameter, S_c is Schmidt number, γ is a chemical reaction rate parameter and the S is the Source parameter and given as follows.

$$Pr = \frac{\vartheta}{\alpha}, \quad M = \frac{\sigma B_0^2}{\rho b}, \quad \frac{K}{\vartheta} = k, \quad R = \frac{4\sigma^* T_\infty^3}{k^* k}, \quad \lambda = \frac{a}{b},$$

$$Ec = \frac{b^2 x^2}{(T - T_\infty) C_p}, S = \frac{Q}{\rho c_p b}, Sc = \frac{\vartheta}{D_B}, \gamma = \frac{k_r}{b}$$

The transformed boundary conditions are

$$f(\eta) = 0, f'(\eta) = 1, \theta(\eta) = 1, \phi(\eta) = 1 \text{ as } \eta \rightarrow 0$$

$$f'(\eta) \rightarrow \lambda, \theta(\eta) \rightarrow 0, \phi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (13)$$

Where η is a similarity variable, dimensionless stream velocity is the $f(\eta)$, Temperature profile is $\theta(\eta)$, Concentration is $\phi(\eta)$ and their prime denotes a differentiation with respect to η . Physical Quantities are Local skin friction coefficient is C_f , mass transfers Sh , Nusselt Number N_u , coefficients from the plate respectively are given by,

$$C_f = \frac{\tau_w}{\rho U_w^2} \Rightarrow C_f \sqrt{Re} = f''(0)$$

$$N_u = \left(\frac{x q_w}{(T - T_\infty)} \right)_{y=0} \Rightarrow \frac{N_u}{\sqrt{Re}} = -\theta'(0), Sh = \frac{-x \left(\frac{\partial c}{\partial y} \right)_{y=0}}{(C_w - C_\infty)} \Rightarrow \frac{Sh}{\sqrt{Re}} = -\phi'(0) \quad (14)$$

Where $Re = \sqrt{\frac{bx^2}{\vartheta}}$ is the local Reynold's number.

2. Numerical Solution

Equation No. (10) -(12) are non-linear ODE's along with Equation No. (13) i. e. boundary conditions are solved by using the Mathematica, Runge-Kutta Fourth Order with efficient Shooting Method. For this, the aforementioned equations are transformed into the following first-order ODEs are:

$$f(1) = f, f(2) = f', f(3) = f''; \theta(1) = \theta, \theta(2) = \theta';$$

$$\phi(1) = \phi, \phi(2) = \phi'$$

$$f''' = -f(1)f(3) - Gr \theta - Gm \phi + \left[M - 1 + \frac{1}{k} \right] [f(2)]^2 - \lambda^2$$

$$\theta'' = \frac{-Pr}{\left[1 + \frac{4R}{3} \right]} [Ec(f(3))^2 + M(f(2) - 1)^2 + S\theta(1) + f(1)\theta(2)] - [A^*f(2) + B^*\theta(1)]$$

$$\phi'' = Sc[\gamma\phi(1) - f(1)\phi(2)]$$

Along with the boundary conditions,

$$f(0) = 0, f(1) = 1, \theta(0) = 1, \phi(0) = 1$$

$$f(2) \rightarrow \lambda, \theta(1) \rightarrow 0, \phi(1) \rightarrow 0$$

3. Results and Discussion:

The role of the various parameters is involved in coupled partial differential equations such as Velocity ratio parameter, Source parameter, radiation parameter, Magnetic parameter, Schmidt number, Prandtl number etc.

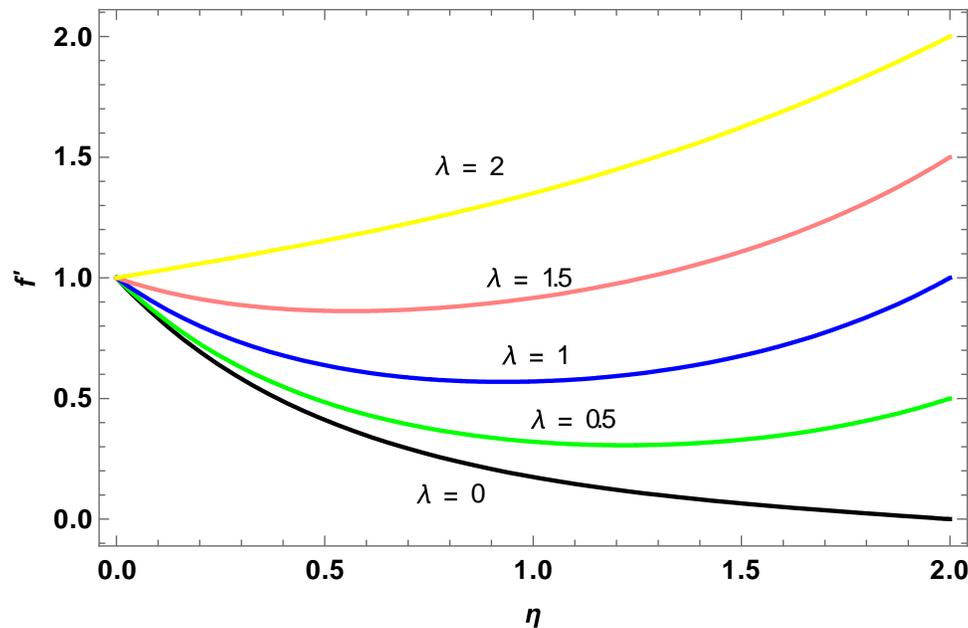


Figure 1: Effect of velocity ratio parameter (λ) on velocity

Figure.1. Depicts the differences in the velocity profile for various velocity ratio parameter values. From these figures is clear that the higher than free stream velocity of stretched surface velocity; nonetheless, the flow of the velocity rises and falls the thickness of the boundary layer. (i. e., $\lambda > 1$).

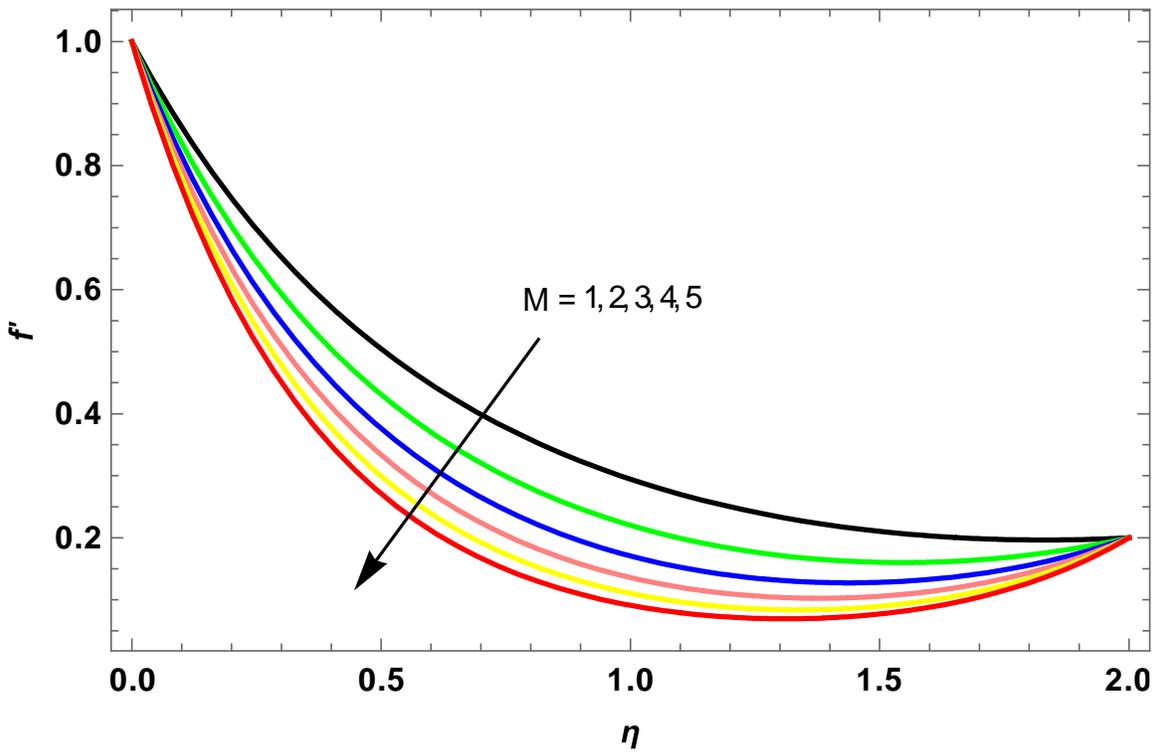


Figure 2: Magnetic parameter (M) effects on velocity

Figure 2. Demonstrates effect of the Magnetic parameter (M) on velocity. Here, value of magnetic parameters are taken $M = 1, 2, 3, 4, 5$ which shows the values of magnetic parameter decreases with velocity.

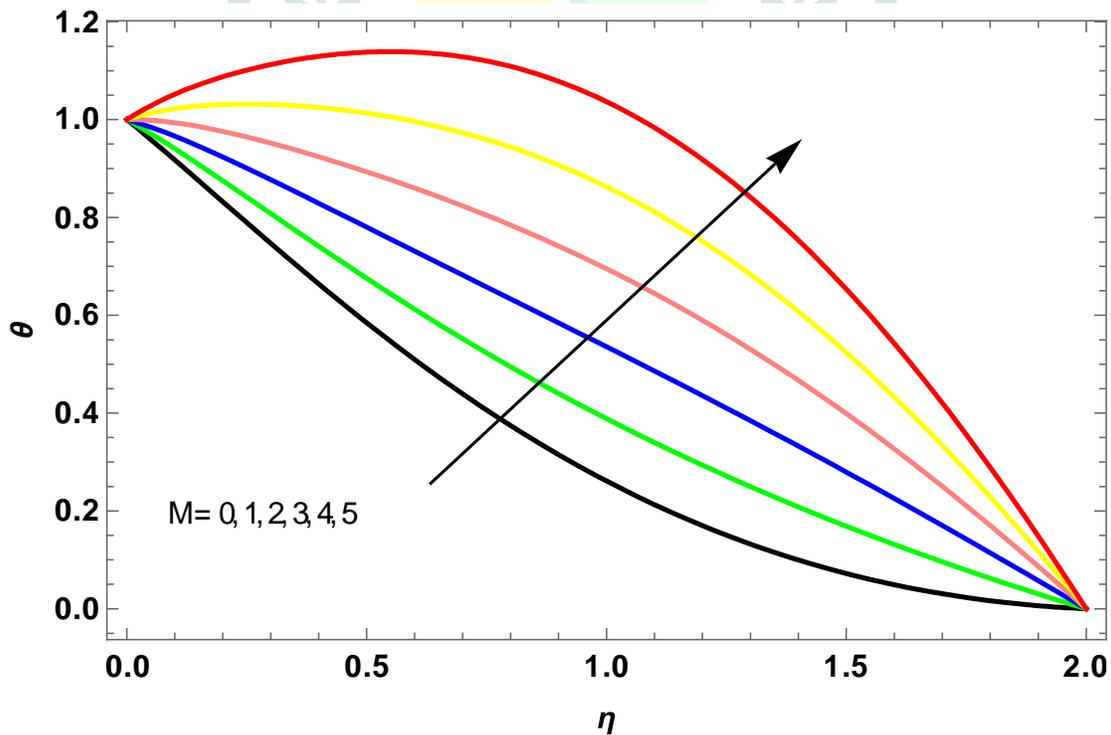


Figure 3: Magnetic parameter (M) Effects on Temperature

Figure.3. Shows the effect of Temperature distribution for various Magnetic parameter values (M). In above figure, increased the value of magnetic parameter, decreases with the temperature.

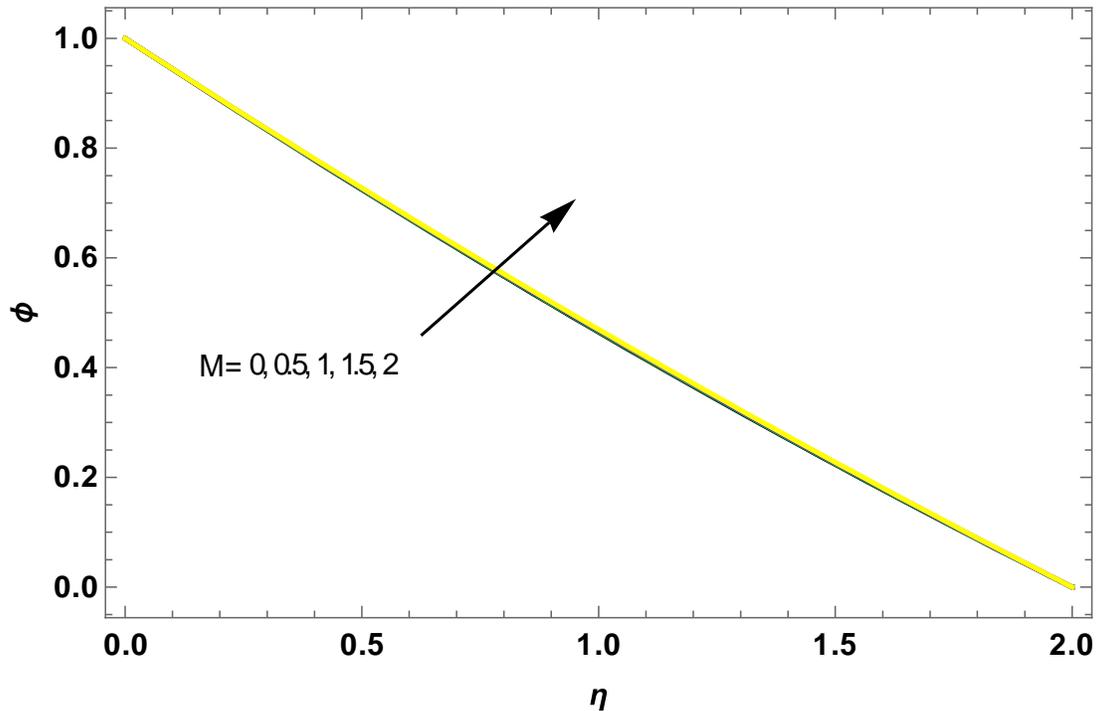


Figure 4: Magnetic parameter (M) effects on concentration

Figure 4. Shows the effect of a concentration for various Magnetic parameter values (M). Increases the value of a magnetic parameter decreases in a value of Concentration profile.

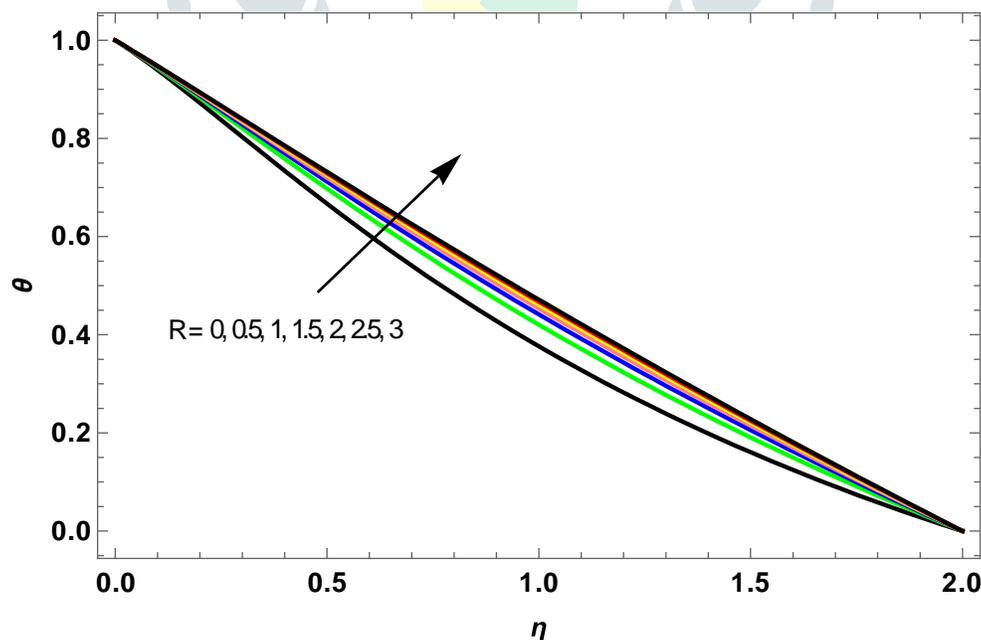


Figure 5: Impact of Radiation (R) on Temperature

Figure 5. Shows the graph it can be conclude that as the radiation parameter is increased, momentum and thermal boundary layer thickness increases.

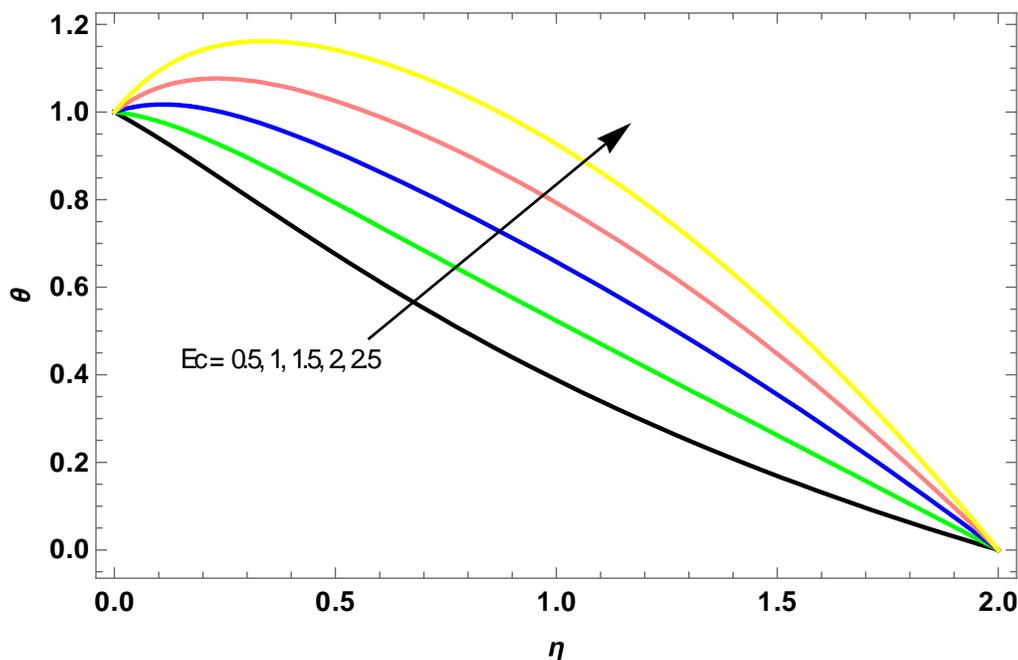


Figure 6: Temperature Effects of Eckert Number (Ec)

Figure 6. Demonstrates variation of a Eckert number (Ec) on the temperature distribution can be viewed and demonstrates that temperature enhances as Ec increases. According to the definition of Ec it is directly proportional to square of velocity. Consequently, a faster rate of sheet stretching with larger levels of Ec and thus a large enhancement through the motion of particles adjacent to the surface, can increase the temperature of the fluid, especially around the sheet.

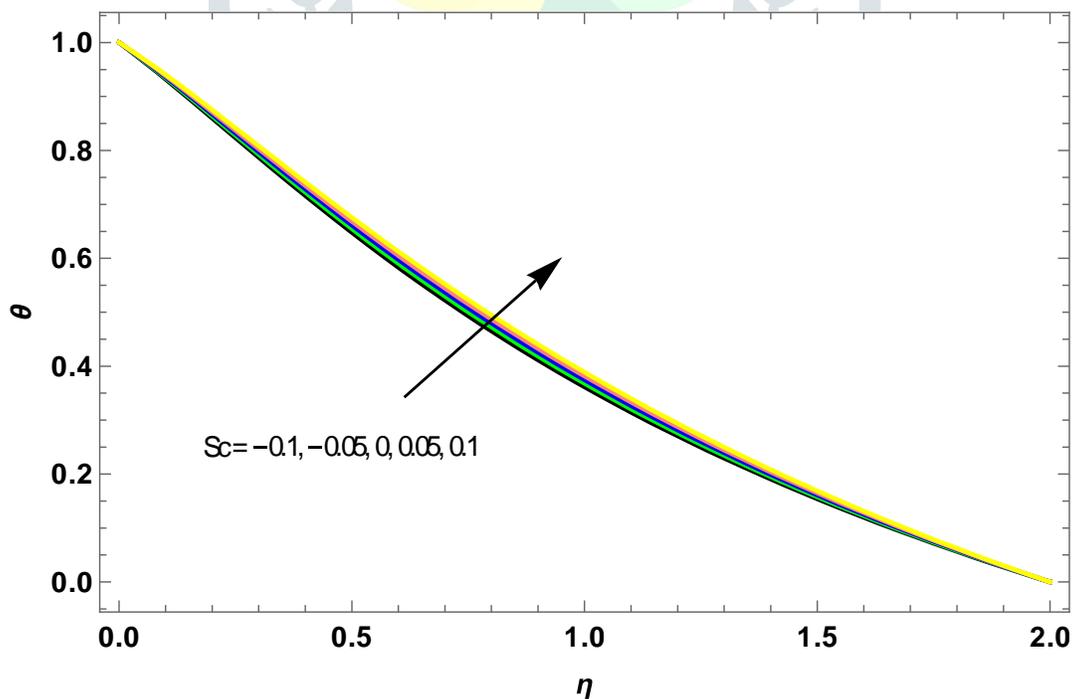


Figure 7: Effect of source parameter (S) on temperature

Figure 7. Demonstrates effect of source parameter on the temperature. Here the figure indicates the source parameter increases with increase in temperature.

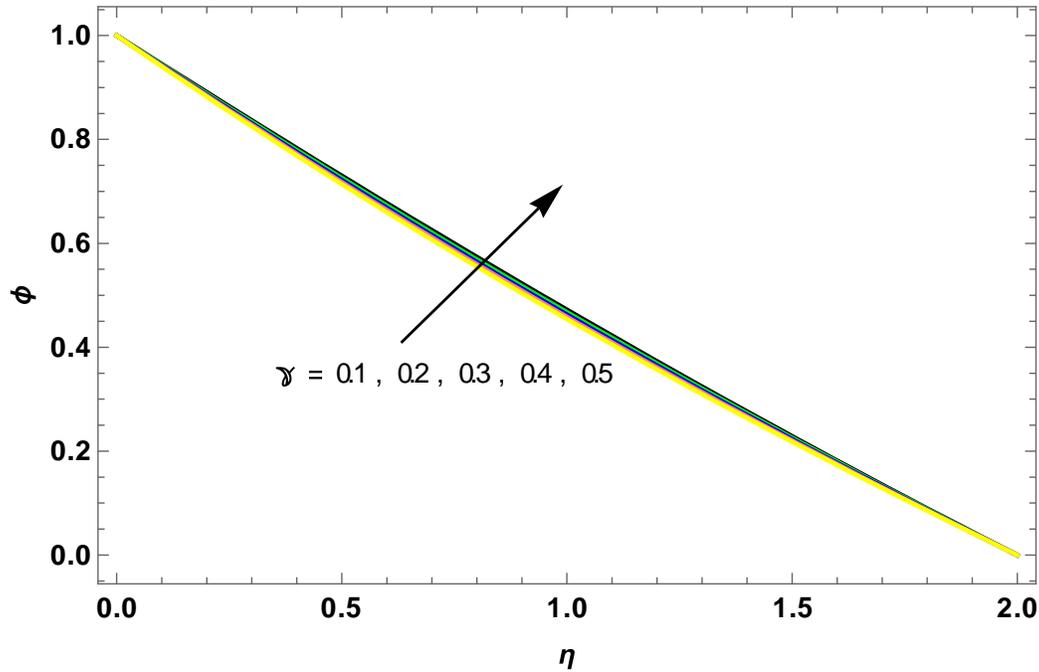


Figure 8: Effect of Chemical reaction parameter (γ) on concentration

Figure 8. Demonstrates effect of chemical reaction on concentration, the graph indicates the parameter of chemical reaction increases with the increase in temperature.

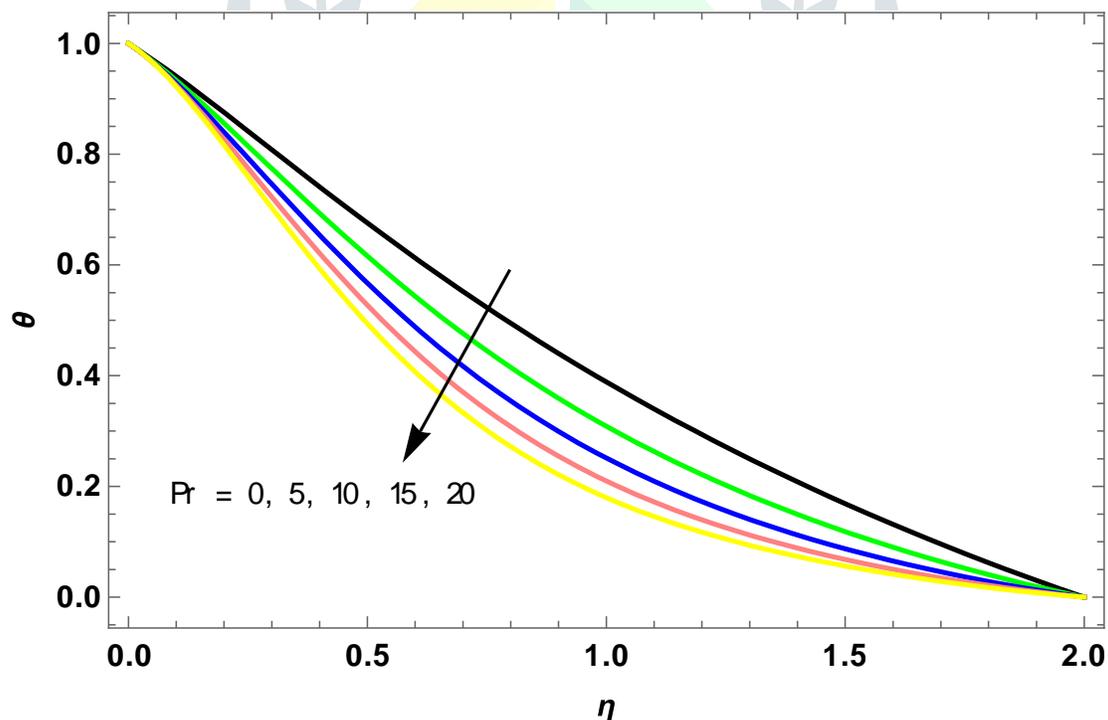


Figure 9: Impact of Prandtl number (Pr) on Temperature

Figure 9. Shows effect of a Prandtl number on the temperature. Here we have selected the arbitrarily as (Pr from 0 to 20). The physical reason behind this phenomenon is that greater value of the Pr relatively lower thermal conductivity and as consequence reduction of the thermal boundary layer thickness and a decrement in the heat transfer rate over the boundary surface which causes to decrease temperature profile significantly.

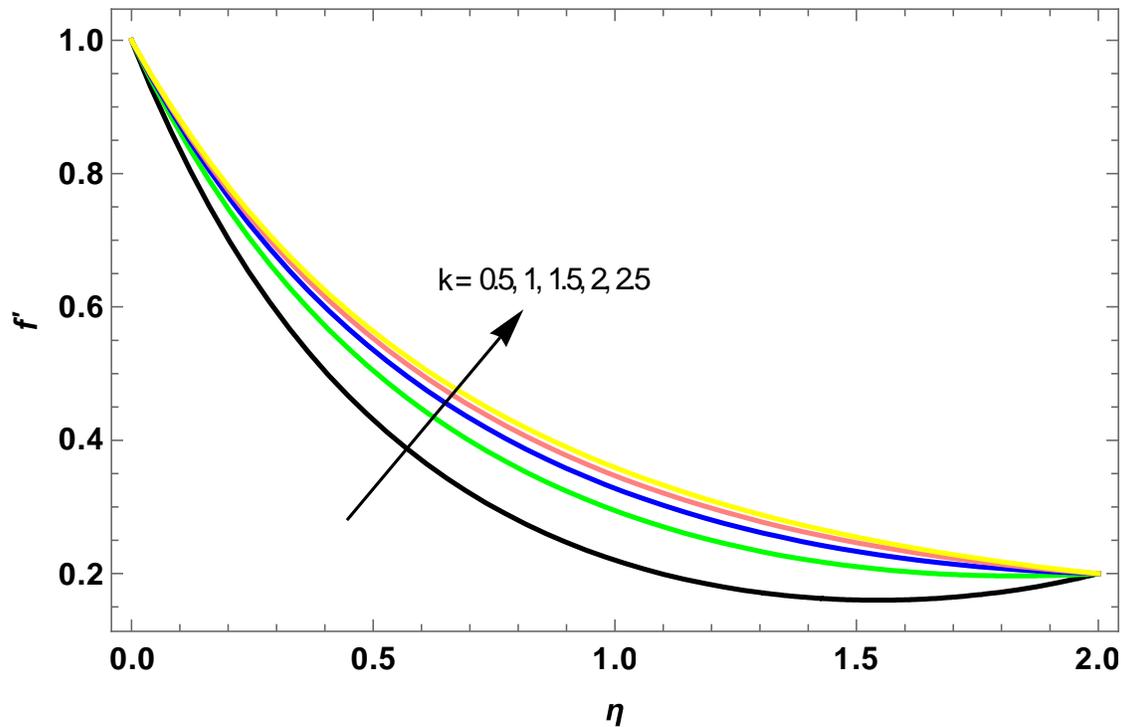


Figure 10: Impact of Porous parameter (k) on Velocity

Figure 10. Demonstrates how a porous parameter affects velocity. The value of the porous parameter increases on velocity.

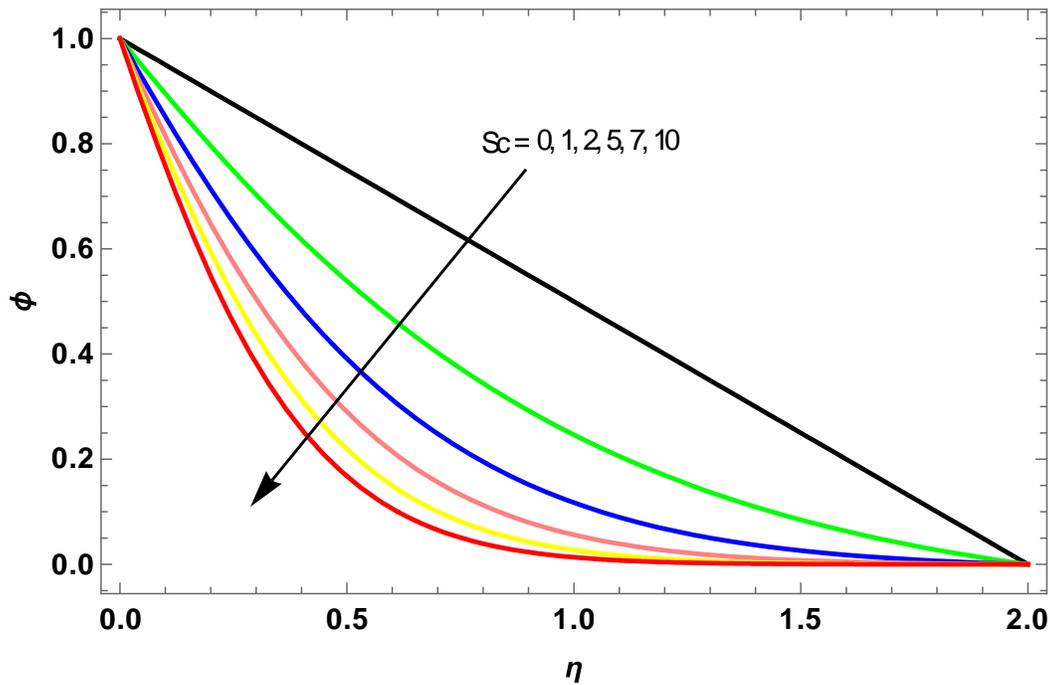


Figure 11: Impact of Schmidt number (Sc) on concentration profile

Figure 11. Shows the Schmidt number's (Sc) effects on concentration, as the chemical reaction parameter is increased Sc decreases, the concentration profile reduces and the concentration boundary layer thickness also reduces because of the higher values of chemical reaction parameter, it produces a reduction in molecular diffusivity.

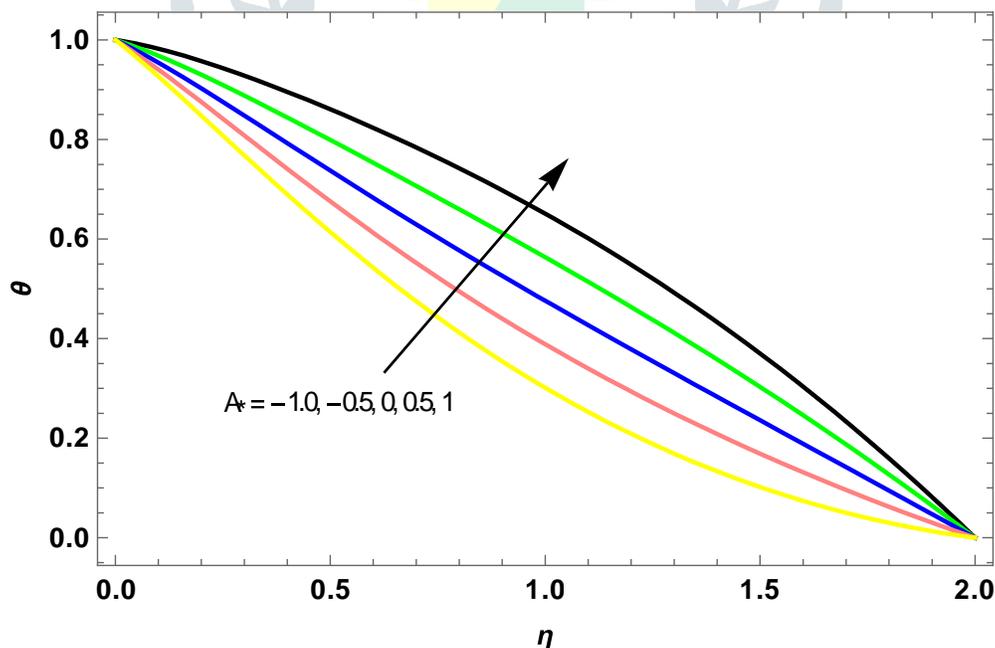


Figure 12: Effect of space dependent parameter (A^*) on temperature profile

Figure 12. Demonstrates the effect of temperature profile for the different values of the heat generator parameter (B^*) in the figure, here as the heat generator parameter value increases, decreases in the temperature profile.

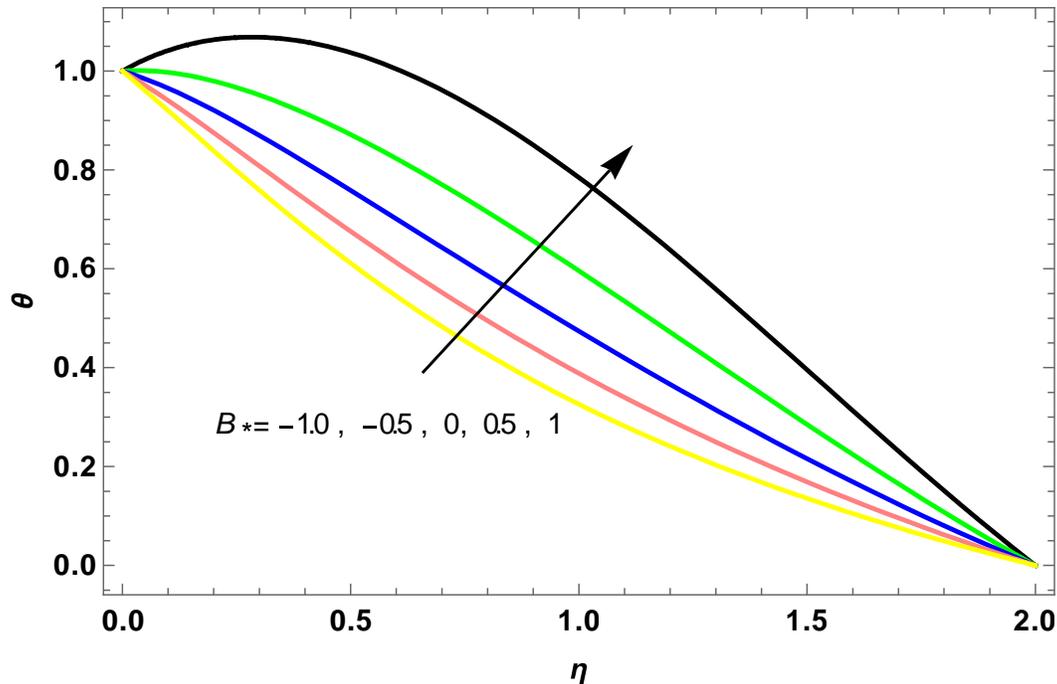


Figure 13: Impact of temperature dependent parameter (B^*) on temperature profile

Figure 13. Demonstrates the effect of temperature profile for different values of the heat generator parameter (A^*) in the figure, it observed that as heat generator parameter value increases, the temperature profile decreases.

Conclusion:

1. When Velocity profiles decreases, increases in the magnetic parameter.
2. The distribution of temperature in the flow zone is increased as a result of viscous dissipation.
3. Increase in the dissipation parameter and heat source/sink improves thermal boundary layer's thickness and rate of heat transmission.
4. When the Prandtl number rises up, temperature distribution falls down.
5. With increasing Sc concentration decreases for increase in a Schmidt number which reduces boundary layer thickness.
6. Temperature rising with the increasing of space (A^*) and temperature dependent (B^*) parameters for heat source/sink.

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