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Mathematical Modeling of Slip Velocity Effects in Pulsatile Flow Through Stenosed Arteries

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Abstract: This study presents a mathematical model to analyze the slip velocity effects in pulsatile blood flow through stenosed arteries, providing insights into hemodynamic behavior under pathological conditions. The model considers a non-Newtonian fluid representing blood, flowing through an artery with a localized stenosis, characterized by parameters such as stenosis severity (δ), shape (β), and asymmetry (γ). By incorporating slip boundary conditions at the arterial wall, the model accounts for partial slip between the blood and the arterial wall, which significantly influences flow characteristics. The governing equations for pulsatile flow, derived from the Navier-Stokes equations, are solved using appropriate boundary conditions to compute velocity profiles, wall shear stress, and resistance to flow. The results highlight that the slip length parameter (λ) alters the velocity distribution, volumetric flow rate and wall shear stress which are critical factors in understanding atherosclerotic plaque formation, progression, and the effects of therapeutic interventions. This modeling framework enhances the comprehension of blood flow dynamics in stenosed arteries, providing a potential tool for predicting clinical outcomes and optimizing treatment strategies in cardiovascular diseases.

Keywords: Mathematical modeling, slip velocity, pulsatile flow, stenosed arteries,Navier-Stokes equations, stenosis severity, shape parameter, asymmetry parameter, boundary conditions, velocity profile, wall shear stress, resistance to flow, arterial wall slip

I. INTRODUCTION

The study of blood flow dynamics in stenosed arteries is crucial for understanding the physiological and pathological processes associated with cardiovascular diseases. Stenosis, a condition characterized by the narrowing of arteries due to the buildup of atherosclerotic plaques, significantly alters blood flow patterns, leading to complex hemodynamic phenomena. Mathematical modeling of blood flow through stenosed

arteries, particularly under pulsatile conditions driven by the heart's rhythmic pumping, provides a powerful tool for analyzing these complex behaviors. Traditional models often assume no-slip boundary conditions at the arterial wall; however, the slip velocity effect, which considers partial slip between the blood and the vessel wall, can provide more accurate representations of real physiological conditions. This effect is especially relevant in smaller arteries or under certain pathological conditions where the boundary layer behavior deviates from the classical no-slip condition. By incorporating slip boundary conditions and modeling the blood as a non-Newtonian fluid, this study aims to explore how slip velocity influences velocity profiles, wall shear stress, and resistance to flow in pulsatile blood flow through stenosed arteries. Understanding these effects is critical for predicting disease progression, assessing the risk of plaque rupture, and optimizing treatment strategies such as stenting or other vascular interventions.

Das and Sarkar (2014) developed a mathematical model of blood flow through a stenosed tube, incorporating slip velocity at the arterial wall. Their model uses a non-Newtonian fluid approach, and they analyze the impact of slip conditions on flow characteristics, such as velocity profiles and wall shear stress. Their results indicate that slip conditions significantly affect the flow dynamics near the stenosis region, providing insights into the physiological impacts on blood flow under pathological conditions. **Singh and De (2015)** investigated the influence of slip conditions on blood flow in an inclined stenosed artery, using a mathematical model based on non-Newtonian fluid dynamics. They examined the flow behavior under varying slip parameters and inclination angles, showing how these factors affect velocity distribution, resistance to flow, and shear stress. Their study is relevant for understanding blood flow in inclined arteries under different physiological conditions. **Chakravarty and Majhi (2016)** focused on a two-phase model for blood flow in stenosed arteries, considering both core (red blood cells) and plasma phases. They enhanced the model's accuracy by including slip effects at the arterial wall, demonstrating the importance of slip velocity in reducing flow resistance and improving blood flow rates. Their work is crucial for designing better diagnostic tools and treatment options for arterial diseases. **Pradhan and Tripathy (2017)** presented a mathematical model of blood flow through an overlapping stenosed artery, taking into account slip velocity effects at the wall. Their model suggests that slip conditions can help reduce the adverse effects of overlapping stenosis by minimizing flow resistance and wall shear stress, contributing to understanding the complex interplay between overlapping stenosis and slip effects in blood flow dynamics. **Mishra and Das (2018)** developed a mathematical model using the Casson fluid model to simulate blood flow through a stenosed artery with slip effects. They explored the influence of different parameters, such as stenosis severity and slip coefficient, on blood flow behavior. Their findings reveal that slip effects can significantly lower the flow resistance and pressure drop, which is critical for clinical applications in managing stenosis-related conditions. **Srivastava and Saxena (2018)** used the Herschel-Bulkley fluid model to investigate blood flow through a stenosed artery, incorporating slip effects at the wall. Their model captures the non-linear rheological behavior of blood more accurately, providing insights into flow characteristics like plug flow, velocity profiles, and wall shear stress under varying slip conditions. Their study is particularly relevant for understanding hemodynamics in patients with stenosis. **Mandal and Ghosh (2019)** analyzed

oscillatory blood flow in a stenosed artery, considering slip conditions at the arterial wall. Their mathematical model shows that slip effects significantly impact the amplitude and phase of oscillatory flow, which is critical for understanding blood flow behavior under pulsatile conditions. Their research is valuable for applications in cardiovascular diagnostics and therapeutic strategies. **Gupta and Jain (2019)** provided a mathematical analysis of blood flow through stenosed arteries using a Newtonian fluid model with slip effects. They revealed that incorporating slip boundary conditions helps in better predicting flow characteristics, such as velocity and shear stress distributions. Their findings can aid in developing more accurate computational models for blood flow in stenosed arteries. **Chakraborty and Biswas (2020)** explored non-Newtonian blood flow through an asymmetric stenosed artery with slip effects. Their mathematical model emphasizes the asymmetric nature of stenosis and how slip conditions can affect flow separation and recirculation zones. Their study has implications for understanding the hemodynamic factors that contribute to plaque progression and arterial remodeling. **Islam and Mahmud (2020)** developed a mathematical model for pulsatile blood flow through a stenosed tube, considering slip boundary conditions. They provide insights into how slip effects alter the pulsatile nature of blood flow, impacting factors like pressure drop and wall shear stress. The findings by Islam and Mahmud are useful for designing medical devices and improving treatments for vascular diseases involving stenosis. **Kumar and Yadav (2021)** analyzed the effects of slip and heat transfer on blood flow through an overlapping stenosed artery. They employed a mathematical model to understand how thermal gradients and slip conditions affect hemodynamics in stenosed arteries. Their research is significant for developing therapeutic strategies that consider both mechanical and thermal factors in treating vascular diseases. **Singh and Tyagi (2021)** focused on modeling the effects of wall slip on blood flow through arteries with multiple stenoses. Their mathematical model shows that slip effects can reduce the overall resistance to flow in arteries with multiple stenoses, providing a potential pathway for improving clinical outcomes in patients with complex arterial conditions. **Raj and Das (2022)** presented a mathematical model of nanoparticle-enhanced blood flow through stenosed arteries, considering slip effects. They demonstrate how nanoparticles and slip conditions can synergistically reduce flow resistance and enhance drug delivery efficacy. Their research is valuable for the development of targeted drug delivery systems for cardiovascular diseases. **Joshi and Prasad (2013)** modeled non-Newtonian blood flow through stenosed arteries under magnetic fields, incorporating slip effects. Their mathematical model reveals that slip conditions and magnetic fields can jointly influence hemodynamic factors such as velocity profiles and shear stress, providing a basis for advanced therapeutic techniques like magneto-hydrodynamic drug delivery. **Sharma and Kumar (2024)** developed a mathematical model of pulsatile blood flow through an elastic stenosed tube, including slip effects and a porous medium. Their model highlights the role of slip conditions in reducing flow resistance and enhancing perfusion in porous vascular environments. Their study is relevant for designing biomedical devices and therapeutic interventions that consider elastic properties and porous media in vascular tissues.

II. STENOSIS GEOMETRY

The geometry of the stenosed artery is usually modeled with a constriction that changes the radius as a function of axial distance. A common form used for such models is:

$$R(x) = R_0\left[1 - \delta\frac{e^{-\beta(x-x_0)^2}}{1+\gamma e^{-\beta(x-x_0)^2}}\right] \quad (1)$$

where:

$R(x)$ is the radius of the artery at a distance x,

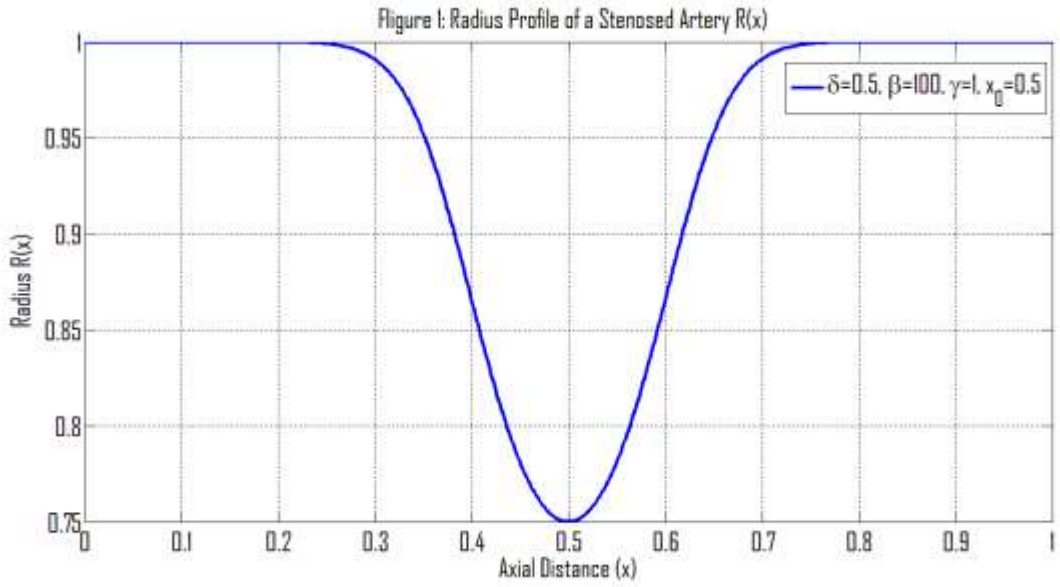
R_0 is the radius of the healthy artery,

δ is the stenosis severity parameter

β is the Shape Parameter

γ is the Asymmetry or Broadening Parameter

x_0 is the location of the stenosis.



The figure (1) shows the radius profile of a stenosed artery $R(\boldsymbol{x})$ along the axial distance x of the artery. The x-axis represents the axial distance (x) along the length of the artery, while the y-axis represents the radius $\boldsymbol{R(x)}$ of the artery at each point. The profile depicts a region of stenosis, where the artery is narrowed, characterized by a smooth, symmetrical reduction in radius around the center at $x_0 = 0.5$. The parameters used for this stenosis model are $\delta = 0.5, \beta = 100, \gamma = 1$ and $x_0 = 0.5$. Here, δ controls the severity of the stenosis, β affects the sharpness or steepness of the narrowing, γ influences the symmetry or broadening of the stenosis profile, and x0x_0x0 is the location of the stenosis center. The plot shows that the artery radius reduces to a minimum near $x_0 = 0.5$ and then gradually returns to its normal radius, $R_0 = 1$, indicating the localized nature of the stenosis. This model helps visualize how parameters affect the geometry and potential impact of arterial stenosis on blood flow dynamics.

III. GOVERNING EQUATIONS

For pulsatile flow through a stenosed artery, the governing equations are the Navier-Stokes equations in cylindrical coordinates(r, θ, z), considering the axial symmetry and incompressibility of the fluid (blood).

3.1. Continuity Equation:

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{\partial v_z}{\partial z} = 0 \quad (2)$$

3.2. Momentum Equations:

Radial Direction:

$$\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{v_z}{r^2} + \frac{\partial^2 v_r}{\partial z^2}\right) \quad (3)$$

Axial (z) Direction:

$$\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2}\right) \quad (4)$$

where:

v_r and v_z are the radial and axial velocity components, respectively.

p is the pressure.

ρ is the fluid density.

$\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, with μ being the dynamic viscosity.

3.3. Boundary Conditions:

(i) No-Slip Condition at the Wall (Traditional):

$$v_z(r = R(z), z, t) = 0, v_z(r = R(z), z, t) = 0 \quad (5)$$

(ii) Slip Velocity Condition (Modified): To account for slip at the arterial wall, the Navier slip condition is applied:

$$v_z(r = R(z), z, t) = \lambda\frac{\partial v_z}{\partial r}, v_r(r = R(z), z, t) = 0 \quad (6)$$

where λ is the slip length parameter, which quantifies the amount of slip at the wall.

(iii) Pulsatile Flow Considerations: The pulsatile nature of blood flow is modeled by a time-dependent pressure gradient, which can be represented as:

$$-\frac{\partial p}{\partial t} = A_0 + A_1\cos(\omega t) \quad (7)$$

where:

A_0 is the steady component of the pressure gradient.

A_1 is the amplitude of the oscillatory component.

ω is the angular frequency of the pulsatile flow.

IV. SOLUTION OF THE PROBLEM

To derive an analytical solution, let's consider:

(i) **Axisymmetric Flow:** Flow is symmetric about the axis, hence there are no angular components.

(ii) **Mild Stenosis:** The stenosis is small, allowing us to treat it as a perturbation around a cylindrical geometry.

(iii) **Low Reynolds Number:** Flow is laminar, and inertia terms can be neglected.

(iv) **Small Slip Condition:** The slip length λ\lambdaλ is small compared to the characteristic length scale, R_0.

Under the above assumptions, the Navier-Stokes equations reduce primarily to the axial component for incompressible, laminar flow. The radial velocity component is negligible, and the axial momentum equation becomes:

$$\frac{\partial v_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r}\right) \quad (8)$$

We seek a solution of the form

$$v_z(r,t) = V_s(r) + V_p(r,t) \quad (9)$$

where $V_s(r)$ represents the steady flow solution and $V_p(r,t)$ represents the pulsatile part of the solution.

The steady-state solution satisfies:

$$\frac{\partial^2 v_s}{\partial r^2} + \frac{1}{r}\frac{\partial v_s}{\partial r} = -\frac{A_0}{\nu} \quad (10)$$

This is a standard form of the Poisson equation in cylindrical coordinates. The general solution is:

$$V_s(r) = -\frac{A_0}{4\nu}r^2 + C_1 ln\, r + C_2 \quad (11)$$

Applying the boundary conditions:

At $r = 0, V_s(r)$ should be finite, implying $C_1 = 0$.

At $= R_0$, using the slip boundary condition:

$$V_s(R_0) = \left(\lambda\frac{dV_s}{dr}\right)_{r=R_0} \quad (12)$$

Substituting the general solution and solving for C_2.

$$-\frac{A_0}{4\nu}{R_0}^2 + C_2 = \lambda\left(-\frac{A_0}{2\nu}R_0\right)$$

$$C_2 = \frac{A_0}{4\nu}{R_0}^2 - \frac{\lambda A_0}{2\nu}R_0$$

$$V_s(r) = -\frac{A_0}{4\nu}r^2 + \left(\frac{A_0}{4\nu}{R_0}^2 - \frac{\lambda A_0}{2\nu}R_0\right)$$

$$V_s(r) = \frac{A_0}{4\nu}\left({R_0}^2 - r^2\right) - \frac{\lambda A_0}{2\nu}R_0 \quad (13)$$

The pulsatile part of the solution satisfies:

$$\frac{\partial V_p}{\partial t} = \nu\left(\frac{\partial^2 V_p}{\partial r^2} + \frac{1}{r}\frac{\partial V_p}{\partial r}\right) + \frac{A_1}{\rho}cos(\omega t) \quad (14)$$

Solution of equation (14) with boundary conditions (5) and (6) has been given by

$$V_p(r,t) = -\left[\frac{A_1}{4\nu\rho}(\quad^2 - r^2) + \lambda\frac{A_1R_0}{2\nu\rho}\right]cos(\omega t + \phi) \quad (15)$$

Combining the steady and pulsatile components:

$$v_z(r,t) = V_s(r) + V_p(r,t)$$

$$v_z(r,t) = \frac{A_0}{4\nu}\left({R_0}^2 - r^2\right) - \frac{\lambda A_0}{2\nu}R_0 - \left[\frac{A_1}{4\nu\rho}\left({R_0}^2 - r^2\right) + \lambda\frac{A_1R_0}{2\nu\rho}\right]cos(\omega t + \phi) \quad (16)$$

This represents the analytical solution for the axial velocity $v_z(r,t)$ of pulsatile flow through a stenosed artery with slip velocity effects, under simplified assumptions. The solution involves a steady-state part due to the constant pressure gradient and a time-dependent part due to the oscillatory pressure gradient.

V. PERFORMANCE MEASURES

5.1. Volumetric Flow Rate(Q): The volumetric flow rate $Q(t)$ is defined as the integral of the axial velocity $v_z(r,t)$ over the cross-sectional area of the artery:

$$Q(t) = \frac{\pi A_0{R_0}^4}{8\nu} - \frac{\pi\lambda A_0{R_0}^3}{2\nu} - \left(\lambda\frac{\pi A_1{R_0}^3}{2\nu\rho} + \frac{\pi A_1{R_0}^4}{8\nu\rho}\right) \quad (17)$$

5.2. Wall shear Stress: The wall shear stress $\tau_w(t)$ at the arterial wall is given by the derivative of the velocity profile with respect to r at $r = R_0$:

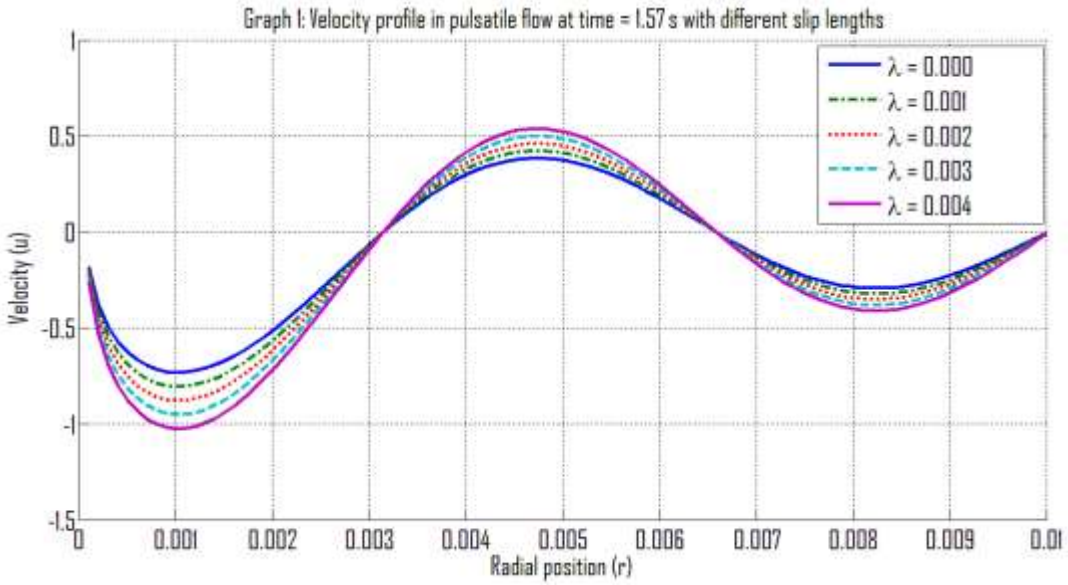
$$\tau_w(t) = \mu\left(\frac{\partial v_z}{\partial r}\right)_{r=R_0} \quad (18)$$

$$\tau_w(t) = \mu\left[-\frac{A_0}{2\nu}R_0 - \frac{A_1}{2\nu\rho}R_0 cos(\omega t + \phi)\right]$$

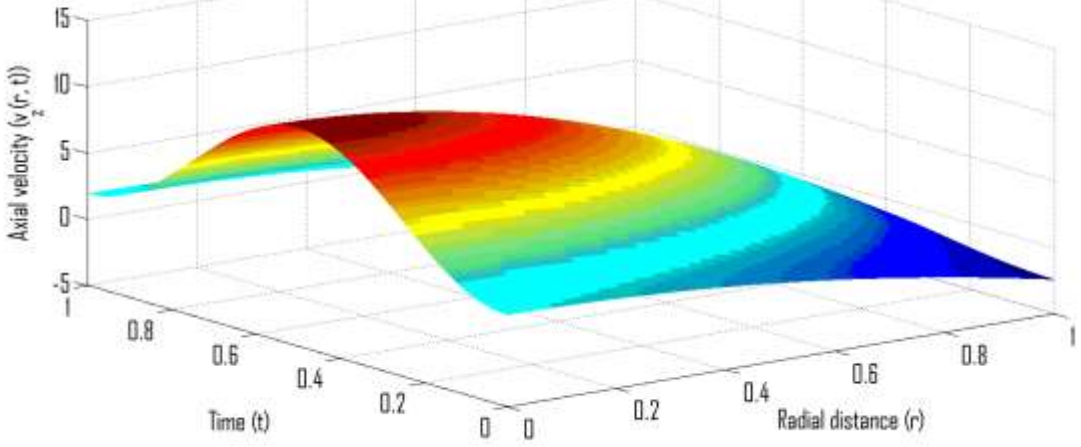
Since $\nu = \frac{\mu}{\rho}$, this simplifies further to:

$$\tau_w(t) = -\frac{\rho A_0R_0}{2} - \frac{A_1R_0}{2}cos(\omega t + \phi) \quad (19)$$

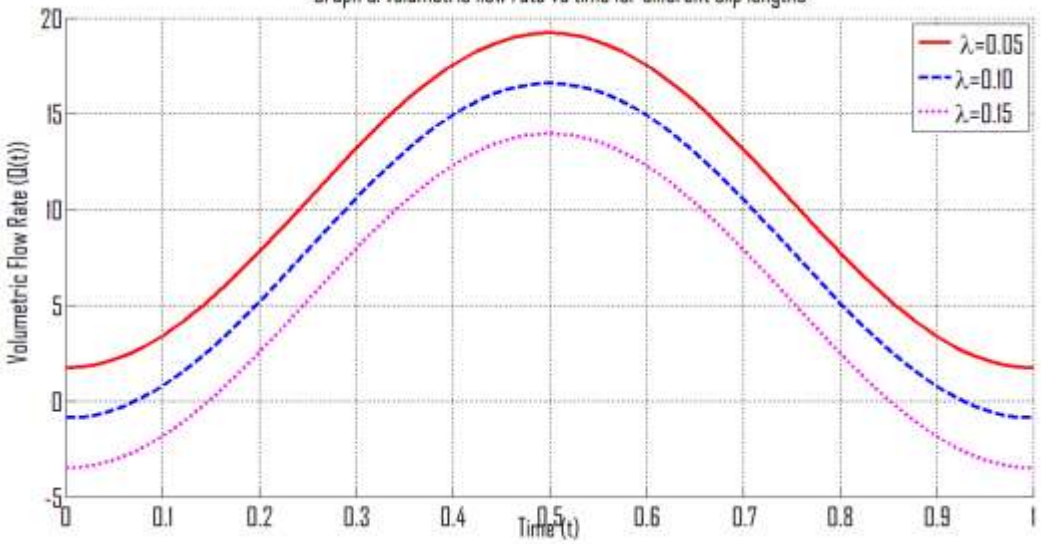
VI. RESULTS AND DISCUSSION



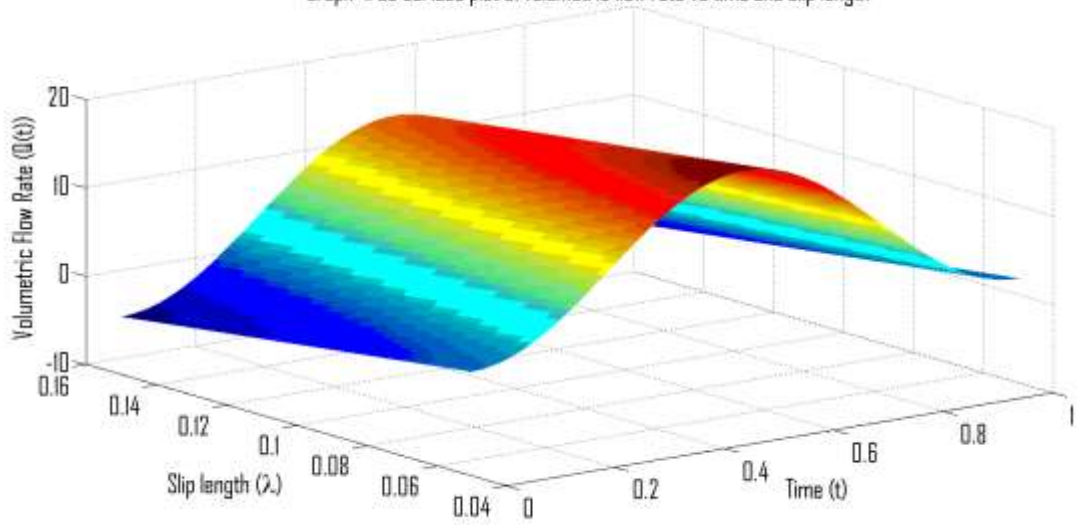
Graph 2: 3D Surface plot of velocity profile $v_z(r, t)$



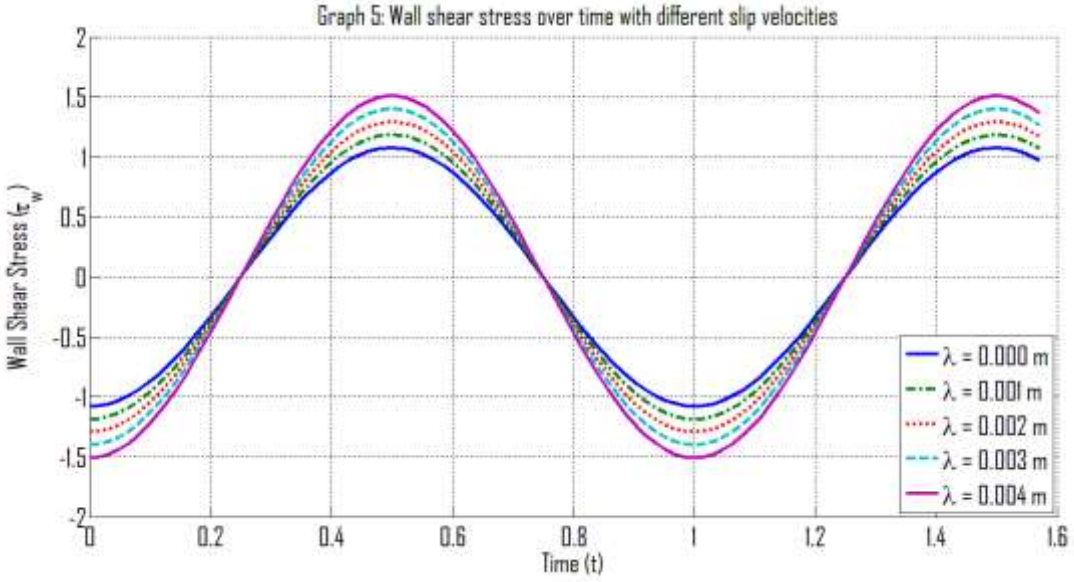
Graph 3: Volumetric flow rate vs time for different slip lengths



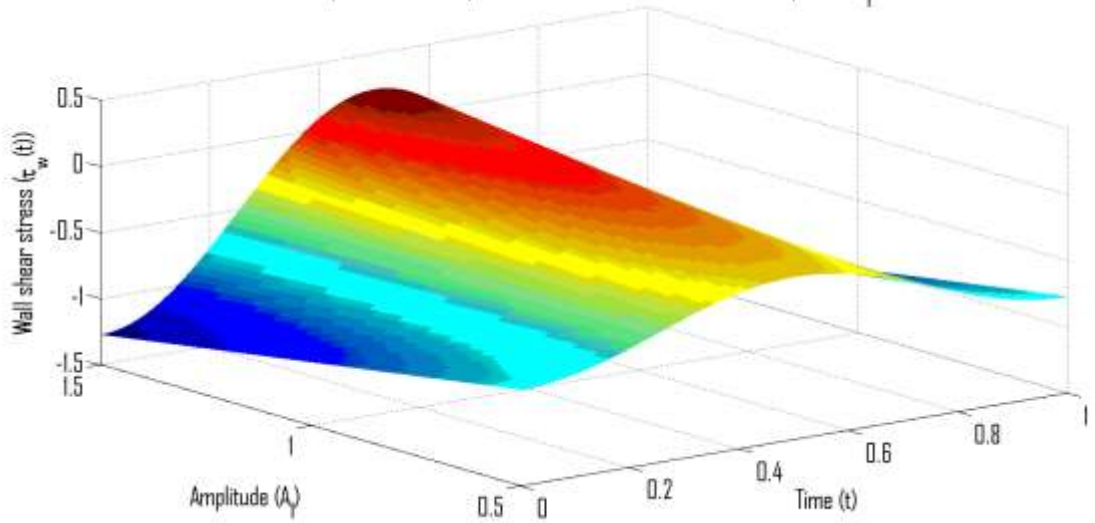
Graph 4: 3D surface plot of volumetric flow rate vs time and slip length



Graph 5: Wall shear stress over time with different slip velocities



Graph 6: 3D surface plot of wall shear stress vs time and amplitude (A_γ)



The graph (1) shows the velocity profile in pulsatile flow for different slip lengths (λ) at a specific time point (1.57 seconds) within a circular tube, such as an artery. The x-axis represents the Radial Position (r) within the artery from the centerline to the wall, while the y-axis shows the Velocity (u) of the fluid (e.g., blood) at each radial position. Each curve corresponds to a different slip length value, ranging from $\lambda = 0$ to $\lambda = 0.004$. The slip length parameter represents the relative motion between the fluid and the artery wall, with larger values indicating more slippage. The curves illustrate how increasing slip length affects the velocity distribution across the radial positions, with higher slip lengths resulting in less pronounced peaks and troughs in velocity due to reduced frictional resistance at the wall. The varying line styles distinguish the different slip length values, highlighting the impact of slip conditions on the flow profile in pulsatile motion.

The surface plot in graph (2) reveals that the velocity is highest near the center $(r = 0)$ and decreases towards the wall $(r = 1)$, consistent with a parabolic velocity profile typical of laminar flow. Additionally, the plot shows an oscillatory pattern over time, reflecting the pulsatile nature of the flow.

Variations in color represent different velocity magnitudes, with warmer colors (red/yellow) indicating higher velocities and cooler colors (blue) indicating lower velocities, revealing how the axial velocity changes both radially and temporally.

The graph (3) shows the volumetric flow rate $Q(t)$ as a function of time (t) for different slip lengths (λ) in a fluid flow system, such as blood flow in an artery with slip boundary conditions at the wall. The x-axis represents time (t), covering one cycle of the pulsatile flow, while the Y-axis represents the volumetric flow rate $Q(t)$, which is the volume of fluid passing through a cross-section per unit time. The three curves correspond to different slip lengths ($\lambda = 0.05$, $\lambda = 0.1$, and $\lambda = 0.15$), represented by solid, dashed, and dotted lines, respectively. The graph reveals that the volumetric flow rate increases with a decrease in slip length. The curve for $\lambda = 0.05$ (solid red line) shows the highest flow rate, indicating less resistance at the wall, while the curve for $\lambda = 0.15$ (dotted magenta line) shows a lower flow rate due to higher resistance. All curves exhibit an oscillatory pattern, which aligns with the pulsatile nature of the flow. This indicates how variations in slip length affect the flow rate, with smaller slip lengths resulting in higher peak flow rates and lower slip lengths reducing the oscillatory amplitude.

The surface plot in graph (4) reveals that the flow rate is highest at lower slip lengths (λ) and decreases as the slip length increases, highlighting how greater resistance (higher slip length) reduces the flow rate. The color gradient from blue (lower values) to red (higher values) represents different magnitudes of flow rates, emphasizing the combined effects of time-dependent pulsations and varying slip lengths on the volumetric flow rate. This visualization effectively demonstrates the interaction between the temporal variations of the flow and the impact of slip conditions on fluid dynamics.

The graph (5) illustrates the wall shear stress $(\boldsymbol{\tau_w})$ over time for different slip velocities (λ) in a pulsatile flow scenario. The x-axis represents time (t) in seconds, while the y-axis shows the Wall Shear Stress $(\boldsymbol{\tau_w})$**.** The wall shear stress is a measure of the tangential force per unit area exerted by the fluid on the wall of a vessel or artery. Each curve corresponds to a different slip length, ranging from $\lambda = 0$ to $\lambda = 0.004$, with different line styles used to distinguish between the slip lengths. As the slip length increases, the magnitude of the wall shear stress also increases, both in positive and negative peaks. This indicates that higher slip lengths result in less frictional resistance between the fluid and the wall, allowing for a higher maximum shear stress during the pulsatile flow cycle. The graph demonstrates that the wall shear stress oscillates with time due to the pulsatile nature of the flow, with slip effects modulating the amplitude of these oscillations.

The plot in graph (6) reveals that the wall shear stress varies both with time and amplitude, showing a pulsating pattern over time. Higher amplitudes (A_1) lead to greater variations in shear stress, as indicated by the color gradient from blue (lower values) to red (higher values). This gradient demonstrates how increased oscillation amplitude amplifies the peaks and troughs of wall shear stress, indicating stronger

interactions between the flow and the vessel wall under higher oscillatory forces. The plot provides valuable insights into how the oscillatory pressure amplitude influences the shear forces acting on the walls of the vessel.

VII. CONCLUDING REMARKS

The mathematical modeling of slip velocity effects in pulsatile flow through stenosed arteries provides valuable insights into the complex hemodynamic interactions that occur under pathological conditions. This study highlights the significant impact of incorporating slip boundary conditions, which allow for partial slip between the blood and the arterial wall, offering a more realistic depiction of blood flow dynamics compared to traditional no-slip models. The results demonstrate that slip velocity affects critical flow characteristics, including velocity profiles, wall shear stress, and resistance to flow, which are vital parameters in understanding the development and progression of atherosclerotic plaques. The findings suggest that higher slip lengths can reduce frictional resistance at the artery walls, potentially altering the forces that contribute to plaque formation and rupture. This model serves as a valuable tool for predicting clinical outcomes, assessing cardiovascular risk, and optimizing therapeutic strategies, such as stent design and placement. Future studies can build upon this model by incorporating more complex artery geometries, varying fluid properties, or patient-specific conditions to further enhance its applicability in clinical settings. Overall, the inclusion of slip effects in the modeling of stenosed arteries offers a more comprehensive understanding of cardiovascular hemodynamics, contributing to better diagnostic and treatment approaches for vascular diseases.

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