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A MULTIPLE SET WITH BOND FUNCTION ANDHOMOLOGOUS COLLECTION OF MULTIPLE SET

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Abstract

In this paper we discussed about multiset or mset theory and its results and we defined a new function called bond function and some results shown related to bond function in mset or multiset theory also defined homologous collection of multiset theory.

Index item-multiset, mset, submset, power full mset, wholesubmset, partial whole submset, full submset, Bond function, Homologus collection of multiple set.

Keywords : *multiset, Bond function, Homologus collection etc.*

I. Introduction

In a classical set theory,a set is a 'well defined collection of distinct objects', so Crisp set is a set where any member of that set appears only one time butmultiset or mset[4]is a set where we can allow reiteration of any member. Let us suppose if we take H_20, here is two hydrogen atom and one oxygen atom that is here multiset is {H, H, O} i.e. {<H,2>,<O,1>} where value of count function [4] of H and O are 2 and 1 respectively.Again, if we take prime factors of 36.As $36=2^2.3^2$.So 2 comes two times and 3 also comes two times here. Therefore, relation between two members in a crisp set theory is either equal or they are different. But it is absolutely differ from modern life or science. In modern life there are infinitely many DNA, many atoms bounded together in a molecule. These atoms or DNA are same type but separate. So in multiset theory three possible relation may arise, maybe they are different or may be identical or may be same but separate. So, it is clear that multiple set can be viewed in chemistry and other branches of mathematics and in multiple settheory is very important for us to build many theories and results which are very important and related to our life.

Actually, the word 'multiset' was first disclosed by N.G. De Bruijn in a privet correspondence with Knuth. Multiset theory was first introduced in 1971 by V.G. Cerf and after that many more like Yager (1987,1986), Blizard (1989) ([1],[2],[3]), Girish & John (2009,2011,2012) ([4],[5],[6],[7]) have lots of contributions in multiset theory.

Girish & John have given the ideas of multiset topologies, relations and functions on multisets in a new way.Idea of multigroup and soft multi group came in the studies of S.K Nazmul and S.K.Samanta in 2013 ([8]).In the year 2020 P.A.Ejegwa and A.M. Ibrahim([10]) has given the concept of multigroupoid and its properties.So it is clear that this topic multiring is very important for us and it will be more useful in future.

II. PRELIMINARIES

Definition 2.1.1. ([5]) Anmset or multiset U taken from a crisp set Y is represented by a function C_U (count function) defined as $C_U: U \longrightarrow \mathbb{N} \cup \{0\}$, where ℕ is a set of Natural numbers.

Let U taken from the set Y = $\{u_1, u_2, \ldots\ldots\ldots u_n\}$ as U = $\{r_1/u_1, r_2/u_2, \ldots\ldots\ldots\ldots, r_n/u_n\}$ where r_i is number of reiteration of $u_i$where i = 1,2,….., n and written $u_i \epsilon^{r_i} U$. $[U]_u$ denotes u belongs to U. That crisp set Y from which U constructed is called domain set.

$[Y]^n$ denotes the space of all such multisets, such that maximum possible perpetration of any element of that multiset is n where n $\epsilon\mathbb{N}$**.**

$[Y]^\infty$ denotes the set of all multisets over a domain Y, such that there is no upper limit on the number of perpetrations of an element in a multiset.

Definition 2.1.2. ([5]) Let U = $\{r_1/u_1, r_2/u_2, \ldots\ldots\ldots\ldots, r_p/u_n\}$ is a multiset drawn from Y ,where u_i is reiterate $r_i$times then support set of V denoted by V^* is a subset of Y and V= $\{u_i \epsilon Y ; C_U(u_i) > 0\}$.

Let U be a multiset drawn from Y then Card U = $\sum C_U(u)$, uϵY. And $[U]_u$ denotes the cardinality of an element u in U.

Definition 2.1.3. ([5]) For any two multisets S and T from the multiset space $[Y]^n$, we have the following operations:

(i) S=T if C_S (u) = C_T (u),for all u ϵY.

(ii) S ⊆ T if C_S (u) ≤ C_T (u),for alluϵ Y.

(iii) M=S U T if C_M (u) = max {C_S (u) , C_T (u)} for all u ϵ Y.

(iv) M= S ∩ T if C_M (u) = min {C_S(u), C_T (u)} for all u ϵ Y.

(v) M= S ⊕ T if C_M (u) = min {C_S(u) + C_T (u), n} for all u ϵ Y.

(vi) M=S ⊖ T if C_M (u) = max {C_S(u) - C_T (u), 0} for all u ϵ Y.

Suppose S and T are two msets of $[Y]^n$.

Definition 2.1.4. ([5]) S is a whole submset of T if there value of count function is same for all u in S,that is C_S (u) = C_T (u) for all u in S.

Definition 2.1.5.([5]) S is a partial whole submset of T ,if there exist at least one element u in S such that C_S (u) = C_T (u) and for other elements v of S, C_S(v)< C_T(v).

Definition 2.1.6.([5]) S is a full submset of T if support set of S and support set of T equal and C_S (u) ≤ C_T (u) for all u in S.

Example : Let T = {2/m,3/n} , S = {2/m}, P = {2/m,1/n} , Q = {1/m,2/n} ,

Then S is a Whole submset of T whereas P is a partial whole submset and Q is a full submset of S.

Note . Empty set is not partial whole mset or full submset of any non empty set but it is a whole submset of any set.

Definition 2.1.7.([5]) Let S is a mset from a mset space $[Y]^n$. Power whole mset of S is denoted by PW(S) is the set of all whole submsets of S.

Definition 2.1.8.([5]) Let S is a mset from a mset space $[Y]^n$.Power full mset of S is denoted by PF(S) is the set of all full submsets of S.

Definition 2.1.9.([5]) Let Sis a mset from a mset space $[Y]^n$. Power mset P(S) is the set of all submsets of S. If Tϵ P(S) and T is empty then T ϵ^1 P(S) and if T is non empty then T ϵ^r P(s) where r = $\prod_u \binom{[S]_u}{[T]_u}$,where u ϵ T.Power set of a mset S is the root set or support set of power mset and is denoted by $P^*(S)$.

2.2.1. Operations of collection of multisets.

Suppose $S_1, S_2, \ldots\ldots\ldots$ are msets taken from mset space $[Y]^n$ then ,

(i) $\cup_{i\epsilon I} S_i = \{C_{US_i}(u)/u : C_{US_i}(u) = \max\{C_{S_i}(u) : u \in Y\}$

(ii) $\cap_{i\epsilon I} S_i = \{C_{\cap S_i}(u)/u : C_{\cap S_i}(u) = \min\{C_{S_i}(u) : u \in Y\}$

Definition 2.2.2. ([5]) Let S_1 and S_2 be two msets taken from a set Y ,then $S_1 \times S_2$ of S_1 and S_2 is defined as $S_1 \times S_2 = \{(k/u, l/v)/kl : u\epsilon^k S_I, v\epsilon^l S_2\}$.

Definition 2.2.3. ([5])

(i) An mset relation R on an mset S is reflexive if m/u R m/u for all m/u in S.

(ii) An mset relation R on an mset S is symmetric if m/u R n/v implies n/v R m/u.

(iii) An mset relation R on an mset is transitive if m/u R n/v , n/v R p/w implies m/u R p/w.

Definition 2.2.4. ([5]) An mset relation γ is called an mset function if for any member p/u in dom γ ,there is exactly one r/v in Ran γ such that (p/u,r/v) is in γ and the pair comes as the product of C_1 (u,v) and C_2(u,v).

III. MAIN DEFINITIONS AND RESULTS

3.1.1. BondFunction:-

Letus suppose, $S=\{<x_1,s_1>,<x_2,s_2>,<x_3,s_3>,\ldots\ldots<x_r,s_r>\}$ is a multiset where reiteration of $x_i = s_i$,where s_i is a natural number and i= 1,2,3……r. Let $M=\{s_1,s_2,\ldots\ldots s_r\}$,then we take a function B: M→(0,1) such as $B(s_i)=\frac{s_i}{s_{i+1}}$ where $s_i \in M$ and i=1,2……r.

This function is called bond function.

For example let us suppose if we take CH_4,then the multiset S={<C,1>,<H,4>} and the set S can be written with bond function as, S_B={<C,1/2>,<H,4/5>}

Homologouscollection of multiple set:-

3.1.2. A collection of multiple sets μ is called homologouscollection of multiple set if they follow a general formula.

Some example:-

1. C_nH_{2n} is the general formula of Alkene[11]. Now if we take n=1, then it becomes CH_2 that is Methylene.

Hence the multiset with bond function

M_{B_1}={<C,1/2>,<H,2/3>}.

If we take n=2 it will have C_2H_4 i.e., Ethylene.

Hence the multiset with bond function is,

M_{B_2}={<C,2/3>,<H,4/5>}.

If we take n=3, then it becomes C_3H_6i.e. Propylene.

hence the multiple set with bond function is,

M_{B_3}={<C,3/4>,<H,6/7>}.

Therefore, µ is the collection of all these M_{B_1}, M_{B_2},M_{B_3} ………

Here members of µ follows the general formula C_nH_{2n}. Therefore, here µ is a homologous collection of multiple sets.

2. C_nH_{2n+2} is the general formula for Alkane group[11].

Now if we take n=1, it becomes CH_4 i.e. Methane. Here the multiple set with bond function is,

M_{B_1}={<C,1/2>,<H,4/5>}.

If we take n=2,it becomes C_2H_6 i.e. Ethane.Here the multiple set with bond function is,

M_{B_2}={<C,2/3>,<H,6/7>}.

And this type of collections also follows a general rule and therefore form a homologouscollection of multiple sets.

3. We know that the general formula for of Alcohol series is $C_nH_{2n+1}OH$ [11]. In this case if we take n=1 then we have CH_3OH which is methyl Alcohol.

Here the multiple set with bond function

M_{B_1}={<C,1/2>,<H,3/4>,<OH,1/2>}

If we take n=2 then the formula becomes C_2H_5OH which is Ethyl Alcohol or Ethanol and therefor multiple set with bond function

M_{B_2}={<C,2/3>,<H,5/6>,<OH,1/2>}

And therefore these type of multiple sets forms homologous collection of multiple sets .

4. We know that cellulose is an organic compound and thegeneral formula here $(C_6H_{10}O_5)_n$[11].

Take n=1 we get $C_6H_{10}O_5$ and multiple set with bond function M_{B_1}={<C,6/7>,<H,10/11>,<O,5/6}.

If we take n=2 then we have $(C_6H_{10}O_5)_2$ and in this case multiple set we haveM_{B_2}={<C,12/13>,<H,20/21>,<O,10/11>}.

So,these types of cellulose forms also a homologous collection of multiple sets.

3.1.2.Definition:-β cut set for bond multiple set.

Suppose M_B is a multiple set with bond function B.Then β cut set for M_B isM_{B_β}={x_i∈ M_B; B(x_i) > β}. Where β ∈ (0,1) that is we have neither 0 cut nor 1 cut. Also, β cut set is a crisp set.

Theorem:-1.If α > β then $M_{B_\alpha} \subseteq M_{B_\beta}$.

Proof-Let x_i ∈ M_{B_α}

⇨ B(x_i) > α

⇨ B(x_i) > β as α>β

⇨ x_i ∈ M_{B_β}

Therefore $M_{B_\alpha} \subseteq M_{B_\beta}$.

2. $M_{B_{\alpha+\beta}} \subseteq M_{B_\alpha} U M_{B_\beta}$.

Proof- As α + β > α therefore$M_{B_{\alpha+\beta}} \subseteq M_{B_\alpha}$.

and also $M_{B_\alpha} \subseteq M_{B_\alpha} U M_{B_\beta}$.

So ,$M_{B_{\alpha+\beta}} \subseteq M_{B_\alpha} U M_{B_\beta}$.

3.$M_{B_\alpha} \cap M_{B_\beta} \supseteq M_{B_{\alpha+\beta}}$

Proof- Ifα>β, then$M_{B_\alpha} \subseteq M_{B_\beta}$ $M_{B_\alpha} \cap M_{B_\beta} = M_{B_\alpha} \supseteq M_{B_{\alpha+\beta}}$.

And if β > α $M_{B_\beta} \subseteq M_{B_\alpha}$

$M_{B_\alpha} \cap M_{B_\beta} = M_{B_\beta} \supseteq M_{B_{\alpha+\beta}}$

And if β = α then obviously Theorem hold by Theorem 1.

Hence the proof.

IV. CONCLUSION

In this paper we defined Bond function with some results and Homologous collection of multiple set which are going to be very important in real life and in every subject in the future.

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