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On Sober $\mathcal{M}_X\, \hat{\mu}\beta R_0$ Spaces in $\mathcal{M}$ - Structures

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Abstract- In this paper, I introduce the concept of weakly ultra-$\mathcal{M}_X\, \hat{\mu}\beta$ -separation of two sets in a m-space using $\mathcal{M}_X\, \hat{\mu}\beta$ -open sets. The $\mathcal{M}_X\, \hat{\mu}\beta$ -closure and the $\mathcal{M}_X\, \hat{\mu}\beta$ -kernel are defined in terms of this weakly ultra-$\mathcal{M}_X\, \hat{\mu}\beta$ -separation. I also investigate some of the properties of the $\mathcal{M}_X\, \hat{\mu}\beta$ -kernel and the $\mathcal{M}_X\, \hat{\mu}\beta$ -closure. It is the aim of this paper to offer some weak separation axioms by utilizing $\mathcal{M}_X\, \hat{\mu}\beta$ - open sets and the $\mathcal{M}_X\, \hat{\mu}\beta$ -closure operator. Also I introduce $Sober$-$\mathcal{M}_X\, \hat{\mu}\beta R_0$. Further, we obtain some characterizations and some properties.

Keywords: $\mathcal{M}_X\, \hat{\mu}\beta$ -closed set, $\mathcal{M}_X\, \hat{\mu}\beta$ -closure, $\mathcal{M}_X\, \hat{\mu}\beta$ -interior, weakly ultra-$\mathcal{M}_X\, \hat{\mu}\beta$ separation, $\mathcal{M}_X\, \hat{\mu}\beta$ -kernel, $Sober$- $\mathcal{M}_X\, \hat{\mu}\beta R_0$.

I. Introduction

In 1950, H. Maki, J. Umehara and T. Noiri [3] introduced the notions of minimal structure and minimal space. Also they introduced the notion of m_X-open set and m_X-closed set andcharacterize those sets using $m_X - cl$ and $m_X - int$ operatorsrespectively. Further they introduced m -continuous functions [11] and studied some of its basic properties. They achieved many important results compatible by the general topology case. Some other results about minimal spaces can be found in [4–11]. For easy understanding of the material incorporated in this paper we recall some basic definitions. For details on the following notions we refer to [4], [3] and [7]. I introduced $\mathcal{M}_X\, \hat{\mu}\beta$ -closed sets [13]. In this paper we introduce the $\mathcal{M}_X\, \hat{\mu}\beta$ -closure and the $\mathcal{M}_X\, \hat{\mu}\beta$ -kernel are defined in m -spaces. I also investigate some of the properties of the $\mathcal{M}_X\, \hat{\mu}\beta$ -kernel and the $\mathcal{M}_X\, \hat{\mu}\beta$ -closure. It is the aim of this paper to offer some weak separation axioms by utilizing $\mathcal{M}_X\, \hat{\mu}\beta$ -open sets and the $\mathcal{M}_X\, \hat{\mu}\beta$ -closure operator. Using this concept I introduce $Sober$-$\mathcal{M}_X\, \hat{\mu}\beta R_0$ in minimal structures.

2. Minimal Structures

In this section, we introduce the $\mathcal{M}$ -structure and define some important subsets associated to the $\mathcal{M}$ -structure and the relation between them.

Definition 2.1:[1][2].A subfamily m_X of the power set $P(X)$ of a non-empty set X is called a minimal structure (briefly, m -structure) on X if $\emptyset \in m_X$ and $X \in m_X$. In this case (X, m_X) is called m -space. Each member of m_X is said to be m_X -open and the complement of an m_X -open set is said to be m_X -closed set and $c(m_X)$ the collection of all m_X -closed sets.

Definition 2.2: [1][2].Let (X, m_X) be a m -space, for a subset A of X , the m_X - closure of A and the m_X -interior of A are defined as follows:
(i) $m_X - cl(A) = \cap \{F : A \subseteq F, F \in c(m_X)\}$
$(ii) m_X - int(A) = \cup \{U : U \subseteq A, U \in m_X\}$
Note that $m_X - cl(A)$ is not necessarily m_X -closed, also $m_X - int(A)$ is not necessarily m_X -open.

Lemma 2.3: [1][2].Let (X, m_X) be a m -space, for a subset A of X , the following hold :
(i). $m_X - int(A)^c = [m\_X - cl(A)]^c$ and $m_X - cl(A^c) = [m_X - int(A)]^c$
(ii). If $A \in c(m_X)$, then $m_X - cl(A) = A$ and if $A \in m_X$,then $m_X - int(A) = A$
(iii). $m_X - cl(\emptyset) = \emptyset$, $m_X - cl(X) = X$, $m_X - int(\emptyset) = \emptyset$ and $m_X - int(X) = X$
(iv). If $A \subseteq B$, then $m_X - cl(A) \subseteq m_X - cl(B)$ and $m_X - int(A) \subseteq m_X - int(B)$
(v). $A \subseteq m_X - cl(A)$ *and* $m_X - int(A) \subseteq A$
(vi). $m_X - cl(m_X - cl(A)) = m_X - cl(A)$ and $m_X - int(m_X - int(A)) = m_X - int(A)$.

Definition 2.4: [1][2]. An m -structure m_X on a non-empty set X is said to have property (β) if the union of any family of subsets belonging to m_X belonging to m_X.

Lemma 2.5:[1].For an m -structure m_X on a non-empty set X , the following are equivalent:
(i) m_X has property (β).
(ii) If $m_X - int(V) = V$, then $V \in m_X$.
(iii) If $m_X - cl(F) = F$, then $F \in c(m_X)$.

Lemma 2.6:[1][2]. Let (X, m_X) be an m -space with property (β).For a subset A of X, the following properties hold:
(i) $A \in m_X$ iff $m_X - int(A) = A$.
(ii) $A \in c(m_X)$ iff $m_X - cl(A) = A$.
(iii) $m_X - int(A) \in m_X$,and $m_X - cl(A) \in c(m_X)$.

Definition 2.7:[3]. Two sets A, B in an m -space (X, m_X) are said to be weakly separated if there are two m_X -open sets U, V such that $A \subseteq U, B \subseteq V$ and $A \cap V = B \cap U = \emptyset$.

3. $\mathcal{M}_X\,\hat{\mu}\beta$ Closed Set, $\mathcal{M}_X\,\hat{\mu}\beta$ Kernel and $\mathcal{M}_X\,\hat{\mu}\beta$ Closure

Definition 3.1: Let (X, m_X) be an m-space. A subset A of X is said to be $m\hat{\mu}\beta$ – closed if $m_X - \hat{\mu}cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in m_X - \beta$ open.

Definition 3.2: Let (X, m_X) be an m-space and let A be the subset of X. Then

(a) The intersection of all $\mathcal{M}_X\,\hat{\mu}\beta$ open subsets of (X, m_X) containing A is called the $\mathcal{M}_X\,\hat{\mu}\beta$ Kernel of A. (ie) $\mathcal{M}_X\,\hat{\mu}\beta_{ker}(A) = \cap \{G \in \mathcal{M}_X\,\hat{\mu}\beta O(X) : A \subseteq G\}$.
(b) Let X be a m space and let $x \in X$. A subset N of X is said to be $\mathcal{M}_X\,\hat{\mu}\beta$ nbhd of x if there exists a $\mathcal{M}_X\,\hat{\mu}\beta$ open set G such that $x \in G \subset N$ which is denoted by $\mathcal{M}_X\,\hat{\mu}\beta\, N(X)$.
(c) The union of all $\mathcal{M}_X\,\hat{\mu}\beta$-open sets that are contained in A is called the $\mathcal{M}_X\,\hat{\mu}\beta$-interior of A and is denoted by $\hat{\mu}\beta_{\mathcal{M}_X} int(A)$.
(d) The intersection of all $\mathcal{M}_X\,\hat{\mu}\beta$-closed sets containing A is called the $\mathcal{M}_X\,\hat{\mu}\beta$ -closure of A and is denoted by $\hat{\mu}\beta_{\mathcal{M}_X} cl(A)$.

Theorem 3.3: Let X be a m -space. Then for any nonempty subset A of X, $\mathcal{M}_X\,\hat{\mu}\beta_{ker}(A) = \{x \in X : \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x\} \cap A \neq \emptyset\}$.

Proof. Let $x \in \mathcal{M}_X\, \hat{\mu}\beta_{ker}(A)$. Suppose that $\hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\}) \cap A = \emptyset$. Then $A \subseteq X - \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\})$ and $X - \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\})$ is $\mathcal{M}_X\, \hat{\mu}\beta$ -open set containing A but not x, which is a contradiction.
Conversely, let us assume that $x \notin \mathcal{M}_X\, \hat{\mu}\beta_{ker}(A)$ and $\hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\}) \cap \neq \emptyset$. Then there exist an $\mathcal{M}_X\, \hat{\mu}\beta$ -open set D containing A but not x and $y \in \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\}) \cap A$. Hence an $\mathcal{M}_X\, \hat{\mu}\beta$ -closed set $X - D,$ contains, and $\{x\} \subset X - D, y \notin X - D$. This is a contradiction to $y \in \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\}) \cap A$. Therefore $x \in \mathcal{M}_X\, \hat{\mu}\beta_{ker}(A)$.

Definition 3.4: In a space X, a set A is said to be *weakly ultra $\mathcal{M}_X\, \hat{\mu}\beta$ -separated* from a set B if there exists an $\mathcal{M}_X\, \hat{\mu}\beta$ -open set G such that $A \subseteq G$ and $G \cap B = \emptyset$ or $A \cap \hat{\mu}\beta_{\mathcal{M}_X}\, cl(B) = \emptyset$.

Definition 3.5: For any point x of a space X, is called
(a) $\mathcal{M}_X\, \hat{\mu}\beta$ -derived (briefly, $\mathcal{M}_X\, \hat{\mu}\beta D(\{x\})$ set of x is defined to be the set. $\mathcal{M}_X\, \hat{\mu}\beta D(\{x\}) = \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\}) - \{x\} = \{y: y \neq x$ and $\{y\}$ *is not weakly ultra* $\mathcal{M}_X\, \hat{\mu}\beta$ *separated from* $\{x\}\}$.
(b) $\mathcal{M}_X\, \hat{\mu}\beta$ -shell (briefly,$\mathcal{M}_X\, \hat{\mu}\beta_{shl}(\{x\})$) of a singleton set $\{x\}$ is defined to be the set. $\mathcal{M}_X\, \hat{\mu}\beta_{shl}(A) = \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\}) - \{x\} = \{y : y \neq x$ *and* $\{x\}$ *is not weakly ultra* $\mathcal{M}_X\, \hat{\mu}\beta$*-separated from* $\{y\}\}$.

Definition 3.6: Let X be a m -space. Then we define

(a) $\mathcal{M}_X\, \hat{\mu}\beta ND = \{x : x \in X$ and $\mathcal{M}_X\, \hat{\mu}\beta D(\{x\}) = \emptyset\}$,
(b) $\mathcal{M}_X\, \hat{\mu}\beta_{Nshl} = \{x : x \in X$ and $\mathcal{M}_X\, \hat{\mu}\beta_{shl}(\{x\}) = \emptyset\}$ and
(c) $\mathcal{M}_X\, \hat{\mu}\beta - \langle x \rangle = \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\}) \cap \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\})$.

Theorem 3.7: Let $x, y \in X$, Then the following conditions hold.
(a) $y \in \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\})$ if and only if $x \in \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{y\})$
(b) $y \in \mathcal{M}_X\, \hat{\mu}\beta_{shl}(\{x\}))$ if and only if $x \in \mathcal{M}_X\, \hat{\mu}\beta D(\{y\})$
(c) $y \in \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\})$ implies $\hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{y\}) \subseteq \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\})$ and
(d) $y \in \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\})$ implies $\mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{y\}) \subseteq \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\})$
Proof. The proof of (a) and (b) are obvious.
(c). Let $z \in \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{y\})$. Then $\{z\}$ *is not weakly ultra-*$\mathcal{M}_X\, \hat{\mu}\beta$ *-separated from* $\{y\}$. So there exists an $\mathcal{M}_X\, \hat{\mu}\beta$ -open set containing z such that $G \cap \{y\} \neq \emptyset$. Hence $y \in G$ and by assumption $G \cap \{x\} \neq \emptyset$. Hence $\{z\}$ *is not weakly ultra-*$\mathcal{M}_X\, \hat{\mu}\beta$ *-separated from* $\{x\}$. So $z \in \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\})$. Therefore $\hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{y\}) \subseteq \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\})$.
(d).Let $z \in \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{y\})$. Then $\{y\}$ *is not weakly ultra-*$\mathcal{M}_X\, \hat{\mu}\beta$ *-separated from* $\{z\}$. So $y \in \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{z\})$. Hence $\hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{y\}) \subseteq \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\})$. By assumption $y \in \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\})$ and then $x \in \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{y\})$ so $\hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\}) \subseteq \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{y\})$. Ultimately $\hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\}) \subseteq \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{z\})$, that is $z \in \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\})$. Therefore $\mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{y\}) \subseteq \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\})$.

Theorem 3.8: Let $x, y \in X$. Then,

(a) for every $x \in X$, $\mathcal{M}_X\, \hat{\mu}\beta_{shl}(\{x\}))$ is degenerate if and only if for all $x, y \in X, x \neq y$, $\mathcal{M}_X\, \hat{\mu}\beta D(\{x\}) \cap \mathcal{M}_X\, \hat{\mu}\beta D(\{y\}) = \emptyset$.
(b) for every $x \in X$, $\mathcal{M}_X\, \hat{\mu}\beta D(\{x\})$ is degenerate if and only if for every $x \in X, x \neq y$, $\mathcal{M}_X\, \hat{\mu}\beta_{shl}(\{x\}))\cap \mathcal{M}_X\, \hat{\mu}\beta_{shl}(\{y\})) = \emptyset$.

Proof. (a) Let $\mathcal{M}_X\, \hat{\mu}\beta D(\{x\}) \cap \mathcal{M}_X\, \hat{\mu}\beta D(\{y\}) \neq \emptyset$. Then there exists a $z \in X$, such that $z \in \mathcal{M}_X\, \hat{\mu}\beta D(\{x\})$ and $z \in \mathcal{M}_X\, \hat{\mu}\beta D(\{y\})$. Then $z \neq y \neq x$ and $z \in \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{x\})$ and $z \in \hat{\mu}\beta_{\mathcal{M}_X}\, cl(\{y\})$, that is $x, y \in \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{z\})$. Hence $\mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{z\})$ and so $\mathcal{M}_X\, \hat{\mu}\beta_{shl}(\{z\}))$ is not a degenerate set.

Conversely, let $x, y \in \mathcal{M}_X\, \hat{\mu}\beta_{shl}(\{z\}))$. Then we get $x \neq y$, $x \in \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{z\})$ and $y \neq z$, $y \in \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{z\})$ and hence z is an element of both $\hat{\mu}\beta_{\mathcal{M}_X} cl(\{x\})$ and $\hat{\mu}\beta_{\mathcal{M}_X} cl(\{y\})$, which is a contradiction.

(b) Obvious.

Theorem 3.9: If $y \in \mathcal{M}_X\, \hat{\mu}\beta - \langle x \rangle$, then $\mathcal{M}_X\, \hat{\mu}\beta - \langle x \rangle = \mathcal{M}_X\, \hat{\mu}\beta - \langle y \rangle$.

Proof. If $y \in \mathcal{M}_X\, \hat{\mu}\beta - \langle x \rangle$, then $y \in \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x\}) \cap \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\})$. Hence $y \in \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x\})$ and $y \in \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\})$ and so we have $\hat{\mu}\beta_{\mathcal{M}_X} cl(\{y\}) \subseteq \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x\})$ and $\mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{y\}) \subseteq \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\})$. Then $\hat{\mu}\beta_{\mathcal{M}_X} cl(\{y\}) \cap \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{y\}) \subseteq \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x\}) \cap \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\})$. Hence $\mathcal{M}_X\, \hat{\mu}\beta - \langle y \rangle \subseteq \mathcal{M}_X\, \hat{\mu}\beta - \langle x \rangle$. The fact that $y \in \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x\})$ implies $x \in \hat{\mu}\beta_{\mathcal{M}_X} cl(\{y\})$ and $y \in \mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\})$ implies $x \in \hat{\mu}\beta_{\mathcal{M}_X} cl(\{y\})$. Then we have that $\mathcal{M}_X\, \hat{\mu}\beta - \langle x \rangle \subseteq \mathcal{M}_X\, \hat{\mu}\beta - \langle y \rangle$. So $\mathcal{M}_X\, \hat{\mu}\beta - \langle x \rangle = \mathcal{M}_X\, \hat{\mu}\beta - \langle y \rangle$.

Theorem 3.10: For all $x, y \in X$, either $\mathcal{M}_X\, \hat{\mu}\beta - \langle x \rangle \cap \mathcal{M}_X\, \hat{\mu}\beta - \langle y \rangle = \emptyset$ or $\mathcal{M}_X\, \hat{\mu}\beta - \langle x \rangle = \mathcal{M}_X\, \hat{\mu}\beta - \langle y \rangle$.

Proof. $\mathcal{M}_X\, \hat{\mu}\beta - \langle x \rangle \cap \mathcal{M}_X\, \hat{\mu}\beta - \langle y \rangle \neq \emptyset$, then there exists $z \in X$, such that $z \in \mathcal{M}_X\, \hat{\mu}\beta - \langle x \rangle$ and $z \in \mathcal{M}_X\, \hat{\mu}\beta - \langle y \rangle$. So by *Theorem 2.9*, $\mathcal{M}_X\, \hat{\mu}\beta - \langle z \rangle = \mathcal{M}_X\, \hat{\mu}\beta - \langle y \rangle = \mathcal{M}_X\, \hat{\mu}\beta - \langle x \rangle$. Hence the result.

4. Sober $\mathcal{M}_X\, \hat{\mu}\beta\, R_0$ spaces

Definition 4.1 A m-space $(X, \mathcal{M}_X)$ is said to be Sober $\mathcal{M}_X\, \hat{\mu}\beta\, R_0$ if $\bigcap_{x \in X} \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x\}) = \emptyset$.

Theorem 4.2 A m -space $(X, \mathcal{M}_X)$ is Sober $\mathcal{M}_X\, \hat{\mu}\beta\, R_0$ if and only if $\mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\}) \neq X$, for every $x \in X$.

Proof. Suppose that the space $(X, \mathcal{M}_X)$ be Sober $\mathcal{M}_X\, \hat{\mu}\beta\, R_0$. Assume that there is a point y in X such that $\mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{y\}) = X$. Then $y \notin O$ which O is some proper $\mathcal{M}_X\, \hat{\mu}\beta$ -open subset of X. This implies that $y \in \bigcap_{x \in X} \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x\})$. But this is a contradiction.

Now assume that $\mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\}) \neq X$ for every $x \in X$. If there exists a point y in X such that $y \in \bigcap_{x \in X} \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x\})$, then every $\hat{\mu}\beta$-open set containing y must contain every point of X. This implies that the space X is the unique $\mathcal{M}_X\, \hat{\mu}\beta$-open set containing y. Hence $\mathcal{M}_X\, \hat{\mu}\beta_{ker}(\{x\}) = X$ which is a contradiction. Therefore is $(X, \mathcal{M}_X)$ is Sober $\mathcal{M}_X\, \hat{\mu}\beta\, R_0$.

Theorem 4.3 If the m-space X is Sober $\mathcal{M}_X\, \hat{\mu}\beta\, R_0$ and Y is any m-space, then the product X x Y is Sober $\mathcal{M}_X\, \hat{\mu}\beta\, R_0$.

Proof. By showing that $\bigcap_{(x,y) \in X \times Y} \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x, y\}) = \emptyset$, we are done. We have: $\bigcap_{(x,y) \in X \times Y} \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x, y\}) \subset \bigcap_{(x,y) \in X \times Y} \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x\}) \times \bigcap_{(x,y) \in X \times Y} \hat{\mu}\beta_{\mathcal{M}_X} cl(\{y\}) = \bigcap_{x \in X} \hat{\mu}\beta_{\mathcal{M}_X} cl(\{x\}) \times \bigcap_{y \in Y} \hat{\mu}\beta_{\mathcal{M}_X} cl(\{y\}) \subset \emptyset \times Y = \emptyset$.

REFERENCES

[1] E. Ott, C. Grebogi, J.A. Jorke. Controlling Chaos, *Phys. Rev. Lett.*, 64, pp. 1196–1199, 1990.

[2] J. Ruan, Z. Huang. An improved estimation of the fixed point's neighborhood in controlling discrete chaotic systems, *Commun. Nonlinear Sci. Numer. Simul.*, 3,pp. 193 197, 1998.

[3] H. Maki, J. Umehara, T. Noiri. Every topological space is pre T1/2, *Mem. Fac. Sci.* Kochi Univ. Ser. Math., pp. 33–42.

[4] M. Alimohammady, M. Roohi. Linear minimal spaces, to appear.

[5] M. Alimohammady, M. Roohi. Fixed Point in Minimal Spaces, Nonlinear Analysis: Modelling and Control, 2005, Vol. 10, No. 4, 305–314

[6] A. Csaszar. Generalized topology: generalized continuity, *Acta. Math. Hunger.*, 96 pp. 351–357, 2002.6. S. Lugojan. Generalized Topology, *Stud. Cerc. Math.*, 34, pp. 348 360, 1982.

[7] V. Popa, T. Noiri. On M-continuous functions, *Anal. Univ. "Dunarea Jos"-Galati, Ser*, Mat. Fiz, Mec. Teor. Fasc. II, 18(23), pp. 31–41, 2000.

[8] E.Rosas, N.Rajesh and C.Carpintero, Some new types of open sets and closed sets in minimal structure-I, Int. Mat. Forum 4(44)(2009), 2169-2184.

[9] H. Maki. On generalizing semi-open sets and preopen sets, in: *Meeting on Topolgical* Spaces Theory and its Application, August 1996, pp. 13–18.

[10] H.Maki, K.C.Rao and A.Nagoor Gani, On generalizing semi-open and preopen sets,

[11] T. Noiri. On -sets and related spaces, in: *Proceedings of the* 8*th Meetings on* Topolgical Spaces Theory and its Application, August 2003, pp. 31–41.

[12] M. Caldas and D.N. Georgiou, More on -semiopen sets, Note di Matematica 22, n. 2, 2003, 113-126.

[13] J.Subashini, Separation Axioms on $\hat{\mu}$B Closed Sets in Minimal Structure. (Submitted).

[14] Young Key Kim and Devi R, the -closure and the -kernel via -open sets, Journalof the Chungcheong mathematical society,Volume 23, No. 1, March 2010