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Priority Biserial Queues In Fuzzy Environment

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Abstract: **The present research investigates the membership function value of performance indicators for a parallel biserial queue network setup connected to a single server in series when the inter-arrival time and service time are uncertain. Basically, α- cut approach, triangular fuzzy numbers and operations are used to determine fuzzy queue parameters. The methodology used to convert input data into fuzzy numbers and discussed various queue characteristics in fuzzy. Both Yager's formula and Robust's Ranking methods have been applied to convert queues' fuzziness into precise values. The model finds its applications in registration counter, hospitals, assembling lines, educational institutions, manufacturing industries and many fields. To check the efficiency of model a numerical illustration is used.**

***Keywords*: Bi-serial, fuzzy number, priority, crisp values, ranking method**

I. Introduction

Queuing theory have wide range of applications in real world. Waiting in queues are always undesirable at the cost of time and resources. We use different methods and models to analyze waiting lines and to reduce waiting time. Priority in queuing system is an important aspect to ease the complicated situations. Many researchers gave their contribution to analyze priority queuing models. Priority with fuzziness increase their applications. The researchers Zadeh [1978], Negi and Lee [1992], Kao [1999], Chen [2005,2006], Ritha and Robert [2009,2010], Singh & Kusum [2011], Seema & Gupta [2013], Mittal Meenu et al [2015,2016] used Zadeh principal to analyzed fuzzy queue models. W. Ritha & S. Josephine [2017] discussed the fuzziness of priority queues with L-R method. B. Kalpana and N. Anusheela [2018], B. Kalpana [2021] used the L-R technique or different type of fuzzy numbers to assess, non-preemptive priority queue performance indicators. K. Selvakumari and S. Revathi [2021] provides a new ranking method to check the effectiveness of model of non-preemptive priority queue with unequal service charges in fuzzy environment. Saini A, Gupta D and Tripathi A.K. [2022] analyzed biserial queue network with the concept of priority.

In the present paper, we discuss queue characteristics of priority biserial queues system in fuzzy environment to make the model more realistic and relevant in real world situations. Here, we work on the model given by Seema et al [2013] in extended form by applying the concept of priority.

II. Fuzzy Set

A set $\tilde{F}: X \to \mu_{\tilde{F}}(X)$ is said to be fuzzy, if it is defined as $\tilde{F} = \{(x, \mu_{\tilde{F}}(x)) : x \in X,\ \mu_{\tilde{F}}(x) \in [0,1]\}$ where the universal set is X and $\mu_{\tilde{F}}(x)$ represent the function of membership degree and the membership function's value, $\mu_{\tilde{F}}(x) = \begin{cases} 0, if\ x \notin X \\ 1, if\ \ x\ \in X \end{cases}$

III. Fuzzy Triangular Number

If the following criteria are met, a number F = (f, g, h) represents a fuzzy triangular number with $\mu_{\tilde{F}}(X)\ as$ membership function value.

$$\mu_{\tilde{F}}(X) = \begin{cases} \dfrac{x-f}{g-f}, & f \le x \le g \\ \dfrac{h-x}{h-g}, & g \le x \le h \\ 0, otherwise \end{cases}$$

IV. Fuzzy Arithmetic Operations

The fundamental arithmetic operations on two triangular fuzzy numbers, F = (f_1, g_1, h_1) and G = (f_2, g_2, h_2), are the following:

i. Addition of two fuzzy numbers = $\tilde{F} + \tilde{G}$ = $(f_1+ f_2, g_1+ g_2, h_1+ h_2)$
ii. Difference of two fuzzy numbers = $\tilde{F} - \tilde{G}$ = $(f_1- f_2, g_1- g_2, h_1- h_2)$ if DP($\tilde{F}$)≥ DP($\tilde{G}$), where DP($\tilde{F}$)= $\frac{h_1-f_1}{2}$ & DP($\tilde{G}$) = $\frac{h_2-f_2}{2}$, DP denotes different point of triangular fuzzy numbers, otherwise $\tilde{F} - \tilde{G}$ = $(f_1- h_2, g_1- g_2, h_1- f_2)$
iii. Multiplication = $\tilde{F} \times \tilde{G}$= $(f_1g_2+ g_1f_2 - g_1g_2, g_1g_2, h_1g_2+g_1h_2 - g_1g_2)$
iv. Division = $\tilde{A}/\tilde{B}$ = $(\frac{2f_1}{f_2+h_2}, \frac{g_1}{g_2}, \frac{2h1}{f_2+h_2})$

V. Robust Ranking Formula

Robust Ranking formula for a triangular fuzzy number F = (f_1, f_2, f_3) is defined as

R(F) $=\int_0^1 (0.5)(f_\alpha^L + f_\alpha^U)d\alpha$,   where $(f_\alpha^L, f_\alpha^U) = \{(f_2 - f_1)\alpha + f_1, f_3 - (f_3 - f_2)\alpha\}$ is $\alpha - cut$.

VI. Notations

Table 1: symbols used in the paper

m = incoming items	
$\widetilde{\lambda_{i,j}}$ = fuzzy arrival of jobs , *where* i = 1,2 & j = L, H	$\lambda_{i,j}$ = arrival of jobs, i = 1,2 & j = L, H
$\widetilde{\mu_{i,j}}$ = fuzzy service rate, i = 1,2 & j = L, H	$\mu_{i,j}$ = *service rate*, i = 1,2 & j = L, H
$\widetilde{\alpha_{i\,j}}$ = fuzzy probabilities of high priority items from i'th server to j'th server	$\alpha_{i\,j}$ = moving probability of high priority customers
$\widetilde{\alpha'_{ij}}$ = fuzzy probabilities of low priority items from i'th server to j'th server	α'_{ij} = *moving probability* of low priority items
$\tilde{L}$ = System fuzzy average queue length	L = average queue length
$E(\widetilde{w})$ = fuzzy expected time spent in the system	E(w) = expected waiting time

VII. Mathematical Model

The purposed model consists three server C_{11}, C_{12} & C_2. The server C_2 is common server linked with bi-serial server C_{11} & C_{12}. Two types of customers with pre-emptive priority, one is high priority and second is low priority with Poisson arrival rates λ_{1H} & λ_{1L} *and* λ_{2H} & λ_{2L}respectively entered into the system at C_{11} and C_{12} from outside. Both type low and high priority Customers who arrived at C_{11} after getting the service join either server C_2 or C_{12} with conditions $\alpha_{12} + \alpha_{13} = 1, \alpha'_{12} + \alpha'_{13} = 1$. Those reached at C_{12}, after taking service move either server C_{11} or C_2 for service. After receiving successful service, customer leave the system with possible conditions $\alpha_{21} + \alpha_{23} = 1, \alpha'_{21} + \alpha'_{23} = 1$. The service time at each server is exponentially distributed with service rate $\mu_{1H}, \mu_{1L}, \mu_{2H}, \mu_{2L}$& μ_2 respectively.

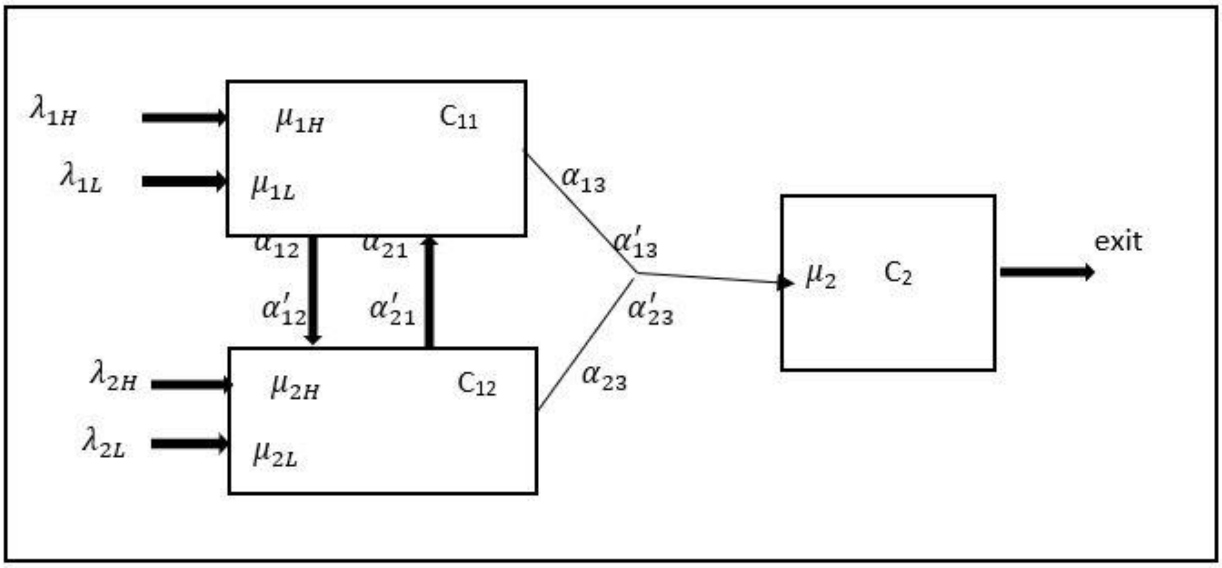

Figure 1 : Priority Biserial Queue Model

Let us define probability distribution function $P_{m_{1L},m_{1H},m_{2L},m_{2H},m_2}(t)$ of $m_{1L}, m_{1H}, m_{2L}, m_{2H}, m_2$ customers at any time t, where *arrival is always* $m_{1L}, m_{1H}, m_{2L}, m_{2H}, m_2 \geq 0$.

Now, differential equations in steady – state form are

$$(\lambda_{1H} + \lambda_{1L} + \lambda_{2H} + \lambda_{2L} + \mu_{1H} + \mu_{1L} + \mu_2)P_{m_{1L},m_{1H},m_{2L},m_{2H},m_2} = \lambda_{1H}P_{m_{1L},m_{1H-1},m_{2L},m_{2H},m_2} + \lambda_{1L}P_{m_{1L-1},m_{1H},m_{2L},m_{2H},m_2} + \lambda_{2H}P_{m_{1L},m_{1H},m_{2L},m_{2H-1},m_2} + \lambda_{2L}P_{m_{1L},m_{1H},m_{2L-1},m_{2H},m_2} + \mu_{1H}\alpha_{12}P_{m_{1L},m_{1H+1},m_{2L},m_{2H-1},m_2} + \mu_{1H}\alpha_{13}P_{m_{1L},m_{1H+1},m_{2L},m_{2H},m_{2-1}} + \mu_{2H}\alpha_{21}P_{m_{1L},m_{1H-1},m_{2L},m_{2H+1},m_2} + \mu_{2H}\alpha_{23}P_{m_{1L},m_{1H},m_{2L},m_{2H+1},m_2-1} + \mu_3P_{m_{1L},m_{1H},m_{2L},m_{2H},m_{2+1}} \quad (1)$$

Like (1) we have 31 more differential equations and solve them by using generating function technique. Then find the solution in the form

$$G(Y_1,Y_2,Y_3,Y_4,Y_5)=\frac{\begin{array}{c}\left[\mu_{1H}\left(1-\frac{\alpha_{12}}{Y_1}Y_3-\frac{\alpha_{13}}{Y_1}Y_5\right)-\mu_{1L}\left(1-\frac{\alpha'_{12}}{Y_2}Y_4-\frac{\alpha'_{13}}{Y_2}Y_5\right)\right]G_1+\\ \left[\mu_{2H}\left(1-\frac{\alpha_{21}}{Y_3}Y_1-\frac{\alpha_{23}}{Y_3}Y_5\right)-\mu_{2L}\left(1-\frac{\alpha'_{21}}{Y_4}Y_2-\frac{\alpha'_{23}}{Y_4}Y_5\right)\right]G_2+\\ \mu_3\left(1-\frac{1}{Y_5}\right)G_3+\mu_{1L}\left(1-\frac{\alpha'_{12}}{Y_2}Y_4-\frac{\alpha'_{13}}{Y_2}Y_5\right)G_4+\mu_{2L}\left(1-\frac{\alpha'_{21}}{Y_4}Y_2-\frac{\alpha'_{23}}{Y_4}Y_5\right)G_5\end{array}}{\begin{array}{c}\lambda_{1H}(1-Y_1)+\lambda_{1L}(1-Y_2)+\lambda_{2H}(1-Y_3)+\lambda_{2L}(1-Y_4)+\mu_{1H}\left(1-\frac{\alpha_{12}}{Y_1}Y_3-\frac{\alpha_{13}}{Y_1}Y_5\right)+\\ \mu_{2H}\left(1-\frac{\alpha_{21}}{Y_3}Y_1-\frac{\alpha_{23}}{Y_3}Y_5\right)+\mu_3\left(1-\frac{1}{Y_5}\right)\end{array}}$$

Solution of the model in stochastic environment is

$$P_{m_{1L},m_{1H},m_{2L},m_{2H},m_2}=(1-G_1)^{m_{1L}}(1-G_2)^{m_{1H}}(1-G_3)^{m_{2L}}(1-G_4)^{m_{2H}}(1-G_5)^{m_2}G_1G_2G_3G_4G_5$$

$$=\gamma_1^{m_{1L}}\gamma_2^{m_{1H}}\gamma_3^{m_{2L}}\gamma_4^{m_{2H}}\gamma_5^{m_2}(1-\gamma_1)(1-\gamma_2)(1-\gamma_3)(1-\gamma_4)(1-\gamma_5)$$

Here, utilization of servers is represented by

$$\gamma_1=\frac{\lambda_{1H}+\alpha_{21}\lambda_{2H}}{\mu_{1H}(1-\alpha_{12}\alpha_{21})}$$

$$\gamma_2=\frac{\lambda_{2H}+\alpha_{12}\,\lambda_{1H}}{\mu_{2H}(1-\alpha_{12}\alpha_{21})}$$

$$\gamma_3=\frac{\begin{array}{c}\alpha_{13}(1-\alpha'_{12}\alpha'_{21})(\lambda_{1H}+\alpha_{21}\lambda_{2H})+\alpha'_{13}(1-\alpha_{12}\alpha_{21})(\lambda_{1L}+\alpha'_{21}\lambda_{2L})\\ +\alpha_{23}(1-\alpha'_{12}\alpha'_{21})(\lambda_{2H}+\alpha_{12}\,\lambda_{1H})+\alpha'_{23}(1-\alpha_{12}\alpha_{21})(\lambda_{2L}+\alpha'_{12}\lambda_{1L})\end{array}}{\mu_2(1-\alpha_{12}\alpha_{21})(1-\alpha'_{12}\alpha'_{21})}$$

$$\gamma_4=\frac{\mu_{1L}(1-\alpha'_{12}\alpha'_{21})(\lambda_{1H}+\alpha_{21}\lambda_{2H})+\mu_{1H}(1-\alpha_{12}\alpha_{21})(\lambda_{1L}+\alpha'_{21}\lambda_{2L})}{\mu_{1L}\mu_{1H}(1-\alpha_{12}\alpha_{21})(1-\alpha'_{12}\alpha'_{21})}$$

$$\gamma_5=\frac{\mu_{2L}(1-\alpha'_{12}\alpha'_{21})(\lambda_{2H}+\alpha_{12}\lambda_{1H})+\mu_{2H}(1-\alpha_{12}\alpha_{21})(\lambda_{2L}+\alpha'_{12}\lambda_{1L})}{\mu_{2L}\mu_{2H}(1-\alpha_{12}\alpha_{21})(1-\alpha'_{12}\alpha'_{21})}$$

And solution of the model exist if $\gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5\le 1$

Mean queue length

$$L=\frac{\gamma_1}{(1-\gamma_1)}+\frac{\gamma_2}{(1-\gamma_2)}+\frac{\gamma_3}{(1-\gamma_3)}+\frac{\gamma_4}{(1-\gamma_4)}+\frac{\gamma_5}{(1-\gamma_5)}$$

Expected waiting time

E(w) $=\frac{L}{\lambda}$, $\lambda=\lambda_{1H}+\lambda_{1L}+\lambda_{2H}+\lambda_{2L}$

VIII. Solution Process

In order to transform the input data into the fuzzy numbers and estimate the parameters in fuzzy to estimate a parameter as a fuzzy number, we use fuzzy triangular numbers and adopt the same methodology as by Sameer & Gupta [2013], Buckley and Qu [1990].

Therefore, $\widetilde{\lambda_{ij}}=(\lambda_{ij}^1,\lambda_{ij}^2,\lambda_{ij}^3)$, $\widetilde{\mu_{ij}}=(\mu_{ij}^1,\mu_{ij}^2,\mu_{ij}^3)$, $\widetilde{\alpha_{ij}}=(\alpha_{ij}^1,\alpha_{ij}^2,\alpha_{ij}^3)$, $\widetilde{\alpha'_{ij}}=(\alpha_{ij}'^1,\alpha_{ij}'^2,\alpha_{ij}'^3)$

For various values of i & j

Therefore, $\alpha_{\lambda_{ij}}=[\alpha(\lambda_{ij}^2-\lambda_{ij}^1)+\lambda_{ij}^1,\lambda_{ij}^3-\alpha(\lambda_{ij}^3-\lambda_{ij}^2)]$

$\alpha_{\mu_{ij}}=[\alpha(\mu_{ij}^2-\mu_{ij}^1)+\mu_{ij}^1,\mu_{ij}^3-\alpha(\mu_{ij}^3-\mu_{ij}^2)]$

$\alpha_{\alpha_{ij}}=[\alpha(\alpha_{ij}^2-\alpha_{ij}^1)+\alpha_{ij}^1,\alpha_{ij}^3-\alpha(\alpha_{ij}^3-\alpha_{ij}^2)]$

$\alpha_{\alpha'_{ij}}=[\alpha(\alpha_{ij}'^2-\alpha_{ij}'^1)+\alpha_{ij}'^1,\alpha_{ij}'^3-\alpha(\alpha_{ij}'^3-\alpha_{ij}'^2)]$

Now,

$$\alpha_{\gamma_1}=\frac{\alpha_{\lambda_{1H}}+\alpha_{\alpha_{21}}\alpha_{\lambda_{2H}}}{\alpha_{\mu_{1H}}(1-\alpha_{\alpha_{21}}\alpha_{\alpha_{12}})}$$

$$\alpha_{\lambda_{1H}}+\alpha_{\alpha_{21}}\alpha_{\lambda_{2H}}=\begin{Bmatrix}(\alpha(\lambda_{1H}^2-\lambda_{1H}^1)+\lambda_{1H}^1)+[(\alpha(\alpha_{21}^2-\alpha_{21}^1)+\alpha_{21}^1)(\alpha(\lambda_{2H}^2-\lambda_{2H}^1)+\lambda_{2H}^1)],\\ (\lambda_{1H}^3-\alpha(\lambda_{1H}^3-\lambda_{1H}^2)+(\alpha_{21}^3-\alpha(\alpha_{21}^3-\alpha_{21}^2))(\lambda_{2H}^3-\alpha(\lambda_{2H}^3-\lambda_{2H}^2)))\end{Bmatrix}$$

$$\alpha_{\mu_{1H}}(1-\alpha_{\alpha_{21}}\alpha_{\alpha_{12}})=\begin{Bmatrix}(\alpha(\mu_{1H}^2-\mu_{1H}^1)+\mu_{1H}^1)[1-(\alpha(\alpha_{21}^2-\alpha_{21}^1)+\alpha_{21}^1)(\alpha(\alpha_{12}^2-\alpha_{12}^1)+\alpha_{12}^1)],\\ (\mu_{1H}^3-\alpha(\mu_{1H}^3-\mu_{1H}^2))[1-(\alpha_{21}^3-\alpha(\alpha_{21}^3-\alpha_{21}^2))(\alpha_{12}^3-\alpha(\alpha_{12}^3-\alpha_{12}^2))]\end{Bmatrix}$$

$$\alpha_{\gamma_1}=\begin{Bmatrix}\dfrac{(\alpha(\lambda_{1H}^2-\lambda_{1H}^1)+\lambda_{1H}^1)+[(\alpha(\alpha_{21}^2-\alpha_{21}^1)+\alpha_{21}^1)(\alpha(\lambda_{2H}^2-\lambda_{2H}^1)+\lambda_{2H}^1)]}{(\mu_{1H}^3-\alpha(\mu_{1H}^3-\mu_{1H}^2))[1-(\alpha_{21}^3-\alpha(\alpha_{21}^3-\alpha_{21}^2))(\alpha_{12}^3-\alpha(\alpha_{12}^3-\alpha_{12}^2))]},\\ \dfrac{(\lambda_{1H}^3-\alpha(\lambda_{1H}^3-\lambda_{1H}^2)+(\alpha_{21}^3-\alpha(\alpha_{21}^3-\alpha_{21}^2))(\lambda_{2H}^3-\alpha(\lambda_{2H}^3-\lambda_{2H}^2)))}{(\alpha(\mu_{1H}^2-\mu_{1H}^1)+\mu_{1H}^1)[1-(\alpha(\alpha_{21}^2-\alpha_{21}^1)+\alpha_{21}^1)(\alpha(\alpha_{12}^2-\alpha_{12}^1)+\alpha_{12}^1)]}\end{Bmatrix}$$

Take $\alpha=0\ \&\ \alpha=1, then$ utilization factor reduces in approximate triangular fuzzy number as

$$\widetilde{\gamma_1}=\left\{\frac{\lambda_{1H}^1+\alpha_{21}^1\lambda_{2H}^1}{\mu_{1H}^3(1-\alpha_{12}^3\alpha_{21}^3)},\frac{\lambda_{1H}^2+\alpha_{21}^2\lambda_{2H}^2}{\mu_{1H}^2(1-\alpha_{21}^2\alpha_{12}^2)},\frac{\lambda_{1H}^3+\alpha_{21}^3\lambda_{2H}^3}{\mu_{1H}^1(1-\alpha_{21}^1\alpha_{12}^1)}\right\}=(\gamma_1^1,\gamma_1^2,\gamma_1^3)$$

$$\widetilde{\gamma_2}=\left\{\frac{\lambda_{2H}^1+\alpha_{12}^1\lambda_{1H}^1}{\mu_{2H}^3(1-\alpha_{12}^3\alpha_{21}^3)},\frac{\lambda_{2H}^2+\alpha_{12}^2\lambda_{1H}^2}{\mu_{2H}^2(1-\alpha_{21}^2\alpha_{12}^2)},\frac{\lambda_{2H}^3+\alpha_{12}^3\lambda_{1H}^3}{\mu_{2H}^1(1-\alpha_{21}^1\alpha_{12}^1)}\right\}=(\gamma_2^1,\gamma_2^2,\gamma_2^3)$$

$$\widetilde{\gamma_3}=\begin{Bmatrix}\frac{\alpha_{13}^1(\lambda_{1H}^1+\alpha_{21}^1\lambda_{2H}^1)}{\mu_3^3(1-\alpha_{12}^3\alpha_{21}^3)}+\frac{\alpha_{13}'^1(\lambda_{1L}^1+\alpha_{21}'^1\lambda_{2L}^1)}{\mu_3^3(1-\alpha_{12}'^3\alpha_{21}'^3)}+\frac{\alpha_{23}^1(\lambda_{2H}^1+\alpha_{12}^1\lambda_{1H}^1)}{\mu_3^3(1-\alpha_{12}^3\alpha_{21}^3)}+\frac{\alpha_{23}'^1(\lambda_{2L}^1+\alpha_{12}'^1\lambda_{1L}^1)}{\mu_3^3(1-\alpha_{12}'^3\alpha_{21}'^3)},\\ \frac{\alpha_{13}^2(\lambda_{1H}^2+\alpha_{21}^2\lambda_{2H}^2)}{\mu_3^2(1-\alpha_{21}^2\alpha_{12}^2)}+\frac{\alpha_{13}'^2(\lambda_{1L}^2+\alpha_{21}'^2\lambda_{2L}^2)}{\mu_3^2(1-\alpha_{21}'^2\alpha_{12}'^2)}+\frac{\alpha_{23}^2(\lambda_{2H}^2+\alpha_{12}^2\lambda_{1H}^2)}{\mu_3^2(1-\alpha_{21}^2\alpha_{12}^2)}+\frac{\alpha_{23}'^2(\lambda_{2L}^2+\alpha_{12}'^2\lambda_{1L}^2)}{\mu_3^2(1-\alpha_{21}'^2\alpha_{12}'^2)},\\ \frac{\alpha_{13}^3(\lambda_{1H}^3+\alpha_{21}^3\lambda_{2H}^3)}{\mu_3^1(1-\alpha_{21}^1\alpha_{12}^1)}+\frac{\alpha_{13}'^3(\lambda_{1L}^3+\alpha_{21}'^3\lambda_{2L}^3)}{\mu_3^1(1-\alpha_{21}'^1\alpha_{12}'^1)}+\frac{\alpha_{23}^3(\lambda_{2H}^3+\alpha_{12}^3\lambda_{1H}^3)}{\mu_3^1(1-\alpha_{21}^1\alpha_{12}^1)}+\frac{\alpha_{23}'^3(\lambda_{2L}^3+\alpha_{12}'^3\lambda_{1L}^3)}{\mu_3^1(1-\alpha_{21}'^1\alpha_{12}'^1)}\end{Bmatrix}=(\gamma_3^1,\gamma_3^2,\gamma_3^3)$$

$$\widetilde{\gamma_4}=\begin{Bmatrix}\frac{(\lambda_{1H}^1+\alpha_{21}^1\lambda_{2H}^1)}{\mu_{1H}^3(1-\alpha_{12}^3\alpha_{21}^3)}+\frac{(\lambda_{1L}^1+\alpha_{21}'^1\lambda_{2L}^1)}{\mu_{1L}^3(1-\alpha_{12}'^3\alpha_{21}'^3)},\\ \frac{(\lambda_{1H}^2+\alpha_{21}^2\lambda_{2H}^2)}{\mu_{1H}^2(1-\alpha_{21}^2\alpha_{12}^2)}+\frac{(\lambda_{1L}^2+\alpha_{21}'^2\lambda_{2L}^2)}{\mu_{1L}^2(1-\alpha_{21}'^2\alpha_{12}'^2)},\\ \frac{(\lambda_{1H}^3+\alpha_{21}^3\lambda_{2H}^3)}{\mu_{1H}^1(1-\alpha_{21}^1\alpha_{12}^1)}+\frac{(\lambda_{1L}^3+\alpha_{21}'^3\lambda_{2L}^3)}{\mu_{1H}^1(1-\alpha_{21}'^1\alpha_{12}'^1)}\end{Bmatrix}=(\gamma_4^1,\gamma_4^2,\gamma_4^3)$$

$$\widetilde{\gamma_5}=\begin{Bmatrix}\frac{(\lambda_{2H}^1+\alpha_{12}^1\lambda_{1H}^1)}{\mu_{2H}^3(1-\alpha_{12}^3\alpha_{21}^3)}+\frac{(\lambda_{2L}^1+\alpha_{12}'^1\lambda_{1L}^1)}{\mu_{2L}^3(1-\alpha_{12}'^3\alpha_{21}'^3)},\\ \frac{(\lambda_{2H}^2+\alpha_{12}^2\lambda_{1H}^2)}{\mu_{2H}^2(1-\alpha_{21}^2\alpha_{12}^2)}+\frac{(\lambda_{2L}^2+\alpha_{12}'^2\lambda_{1L}^2)}{\mu_{2L}^2(1-\alpha_{21}'^2\alpha_{12}'^2)},\\ \frac{(\lambda_{2H}^3+\alpha_{12}^3\lambda_{1H}^3)}{\mu_{2H}^1(1-\alpha_{21}^1\alpha_{12}^1)}+\frac{(\lambda_{2L}^3+\alpha_{12}'^3\lambda_{1L}^3)}{\mu_{2L}^1(1-\alpha_{21}'^1\alpha_{12}'^1)}\end{Bmatrix}=(\gamma_5^1,\gamma_5^2,\gamma_5^3)$$

Fuzzy length of queues

$$L_1=\frac{\widetilde{\gamma_1}}{1-\widetilde{\gamma_1}},\quad L_2=\frac{\widetilde{\gamma_2}}{1-\widetilde{\gamma_2}},\quad L_3=\frac{\widetilde{\gamma_3}}{1-\widetilde{\gamma_3}},\quad L_4=\frac{\widetilde{\gamma_4}}{1-\widetilde{\gamma_4}},\quad L_5=\frac{\widetilde{\gamma_5}}{1-\widetilde{\gamma_5}}$$

$$\tilde{L}=\widetilde{L_1}+\widetilde{L_2}+\widetilde{L_3}+\widetilde{L_4}+\widetilde{L_5}$$

Expected fuzzy waiting time

$$E(\widetilde{w})=\frac{\tilde{L}}{\tilde{\lambda}},\quad \tilde{\lambda}=\widetilde{\lambda_{1L}}+\widetilde{\lambda_{1H}}+\widetilde{\lambda_{2L}}+\widetilde{\lambda_{2H}}$$

To preserve the fuzziness of input data, membership function used to describe the performance measures of the system. To defuzzify the characteristics of system from fuzzy values to crisp values Yager's Formula $B=\frac{b_1+2b_2+b_3}{4}$ & $\tilde{B}=(b_1,b_2,b_3)$ and Robust Ranking Formula have been used.

IX. Numerical Illustration

Table 2: Input Fuzzy Dimensions

Customers In Number	Arrival Times	Service Costs	Probabilities	
$m_{1L}=4$	$\widetilde{\lambda_{1L}}=(1,2,3)$	$\widetilde{\mu_{1L}}=(14,16,18)$	$\widetilde{\alpha_{12}}=(.2,.4,.6)$	$\widetilde{\alpha_{12}'}=(.4,.5,.6)$
$m_{1H}=6$	$\widetilde{\lambda_{1H}}=(2,3,4)$	$\widetilde{\mu_{1H}}=(17,18,19)$	$\widetilde{\alpha_{13}}=(.8,.6,.4)$	$\widetilde{\alpha_{13}'}=(.6,.5,.4)$
$m_{2L}=3$	$\widetilde{\lambda_{2L}}=(1,2,1)$	$\widetilde{\mu_{2L}}=(14,15,16)$	$\widetilde{\alpha_{21}}=(.5,.6,.7)$	$\widetilde{\alpha_{23}'}=(.9,.7,.5)$
$m_{2H}=5$	$\widetilde{\lambda_{2H}}=(3,4,5)$	$\widetilde{\mu_{2H}}=(16,18,20)$	$\widetilde{\alpha_{23}}=(.5,.4,.3)$	
		$\widetilde{\mu_2}=(19,17,15)$	$\widetilde{\alpha_{21}'}=(.1,.3,.5)$	

Table 3: Fuzzy Parameters

Server Utilizations	Partial Queue Lengths
$\widetilde{\gamma_1} = (.3176, .3947, .4902)$	$\widetilde{L_1} = (.5328, .6521, .8223)$
$\widetilde{\gamma_2} = (.2931, .3801, .5139)$	$\widetilde{L_2} = (.4914, .6132, .8615)$
$\widetilde{\gamma_3} = (.7001, .6469, .4588)$	$\widetilde{L_3} = (1.6647, 1.8321, 1.0909)$
$\widetilde{\gamma_4} = (.4049, .5859, .7506)$	$\widetilde{L_4} = (.9589, 1.4149, 1.7776)$
$\widetilde{\gamma_5} = (.4181, .6154, .7222)$	$\widetilde{L_5} = (.9727, 1.6001, 1.6801)$

Fuzzy Mean Queue Length

$\tilde{L} = (4.6205, 6.1124, 6.2324)$

Fuzzy Waiting Time

$E(\widetilde{w}) = \frac{\tilde{L}}{\tilde{\lambda}} = (.4621, .5557, .6232)$

$\tilde{\lambda} = (7, 11, 13)$

To defuzzied the expected waiting time, we have used Yager's formula

$E(\tilde{L})$ = 5.7694

$E(\tilde{\lambda})$ =10.5

$E(\widetilde{w})$ = .5492

Now, the precise value for the mean length of customer queues using Robust's Ranking Techniques [2011] is R ($\tilde{L}$) = 5.42645 and the crisp value for the expected waiting time is $R(\widetilde{w})$ = .54265. Same as from Yager's formula the crisp values of mean length of queue and waiting time are 5.7694 & $.5492$ respectively. That shows the length of system queues $\tilde{L} = (4.6205, 6.1124, 6.2324)$ never falls below 4.6205 & *above* 6.2324. The exact mean queue length is 6.1124. average time spent by customer in the system $E(\widetilde{w}) = (.4621, .5557, .6232)$ lies between $.4621$ & $.6232$. The most possible value of it is $.5557$. From above calculation, it is clear that utilization of servers by both type of customers low & high is 39%, 38%, 64%, 58%, 61%.

X. Conclusion

The queue characteristics of pre-emptive priority biserial queues with unequal service rates have been studied in fuzzy environment. Priority queuing system is helpful in designing and analyzing various queue network system such as communication network, manufacturing industries, hospitals etc. In literature, various methods have been used by researchers to calculate fuzzy queue characteristics. But here, we are using $\alpha - cut$ method that use Zadeh extension principle to calculate membership function value of average length of queue, waiting time and server utilizations by both type of pre-emptive priority customers. The present work is more suitable as the study of priority queues in fuzzy environment related to real- life and numerical illustration support the validity of work. The future work can be done in examining the efficiency of the model with more servers and considering the following - batch arrival, feedback and impatient behavior of customers.

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