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An Approach To Solve Integral Equation Using Second and Third Order B-Spline Wavelets

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Abstract : It was proven that semi-orthogonal wavelets approximate the solution of integral equation very finely over the orthogonal wavelets. Here we used the compactly supported semi-orthogonal B-spline wavelets generated in our paper " Compactly Supported B-spline Wavelets with Orthonormal Scaling Functions " satisfying the Daubechies conditions, to solve the Fredholm integral equation . The generated wavelets satisfies all the properties on the bounded interval. The method is computationally easy, which is illustrated with two examples whose solution closely resembles the exact solution as the order of wavelet increases.

***IndexTerms* - B-spline wavelets, dual wavelets, integral equation.**

I. INTRODUCTION

Integral equations are find very vast usage in many areas of engineering, physics, applied mathematics and many more. Here we seek to resolve a class of integral equation called Fredholm integral equation. There are various methods like variational method, callocation type method and integrated callocation method are known to estimate the solution of integral[1]. Some of the methods convert the integral equation into non linear equation while in some others method it transform to a set of algebraic equations.

Wavelets due to its outstanding properties like vanishing moment, compact support, are good candidates for providing fast algorithm in numerical aspects in approximating[3,4,5,6]. In the present paper , we apply compactly supported semi orthogonal B-Spline wavelet generated in our paper[7] for bounded interval to solve the linear Fredholm integral equation of form

$$y(x) = f(x) + \int_0^1 k(x,t)y(t)dt \qquad 0 \le x \le 1$$

where f and k are given continuous function. Due to the interesting features like smoothness which increases with order of vanishing moment and closed form expression of compactly supported spline wavelets, it was widely used in solving numerical problems. The wavelet formed satisfy all the properties on a bounded interval. In [8] it was shown that semi-orthogonal wavelets are better than orthogonal for integral equation application.

II. SECOND AND THIRD ORDER B-SPLINE WAVELETS ON [0,1]

The wavelets are generally defined as

$$\psi_{j,k}(x) = \psi(2^j x - k) \qquad 0 \le k \le 2^j - 1$$

that is, the translation and dilation of mother wavelets. Here j is called the octave level and $j = j_0$ is lowest octave level, first obtained in[2] for B-spline semi-orthogonal wavelets of order m as

$$2^{j_0} \ge 2m - 1$$

to give a complete wavelet in interval [0,1]. Here the actual coordinate position x is related to x_j as $x = 2^j x$. The second order B-spline scaling function are given by

$$B_{j,k}(x) = \begin{cases} x_j - k & k \le x_j \le k+1 \\ 2 - (x_j - k) & k+1 \le x_j \le k+2 \\ 0 & otherwise \end{cases} \qquad k = 0, \dots, x_j - 2 \qquad (2.1)$$

For j=2 the inner scaling functions are obtained by

$$B_{2,0}(x) = \begin{cases} 4x & 0 \le x \le \frac{1}{4} \\ 2 - 4x & \frac{1}{4} \le x \le \frac{1}{2} \\ 0 & otherwise \end{cases} \qquad B_{2,1}(x) = \begin{cases} 4x - 1 & 0 \le x \le \frac{1}{2} \\ 3 - 4x & \frac{1}{2} \le x \le \frac{3}{4} \\ 0 & otherwise \end{cases}$$

$$B_{2,2}(x) = \begin{cases} 4x - 2 & \frac{1}{2} \le x \le \frac{3}{4} \\ 4 - 4x & \frac{3}{4} \le x \le 1 \\ 0 & otherwise \end{cases}$$

The L.H.S and R.H.S boundary scaling functions are

$$B_{2,-1}(x) = \begin{cases} 1-4x & 0 \le x \le \frac{1}{4} \\ 0 & otherwise \end{cases}$$

$$B_{2,3}(x) = \begin{cases} 4x-3 & \frac{3}{4} \le x \le 1 \\ 0 & otherwise \end{cases}$$

The second order B-spline wavelet obtained in [7] are given by

$$\psi_{j,k}(x) = \frac{1}{2}\begin{cases} \frac{-2}{\sqrt{2}}(x_j-k) & k \le x_j \le k+\frac{1}{2} \\ 2\left(\frac{3}{\sqrt{2}}-1\right)(x_j-k)-\frac{4}{\sqrt{2}}+1 & k+\frac{1}{2} \le x_j \le k+1 \\ 2(3-\frac{2}{\sqrt{2}})(x_j-k)+\frac{6}{\sqrt{2}}-7 & k+1 \le x_j \le k+\frac{3}{2} \\ -2\left(3+\frac{2}{\sqrt{2}}\right)(x_j-k)+\frac{6}{\sqrt{2}}+11 & k+\frac{3}{2} \le x_j \le k+2 \\ 2\left(\frac{3}{\sqrt{2}}+1\right)(x_j-k)-\frac{14}{\sqrt{2}}-5 & k+2 \le x_j \le k+\frac{5}{2} \\ -\frac{2}{\sqrt{2}}(x_j-k)+\frac{6}{\sqrt{2}} & k+\frac{5}{2} \le x_j \le k+3 \\ 0 & otherwise \end{cases} \tag{2.2}$$

The inner wavelet functions are obtained as

$$\psi_{2,0}(x) = \frac{1}{2}\begin{cases} \frac{-2}{\sqrt{2}}(4x) & 0 \le x_j \le \frac{1}{8} \\ 2\left(\frac{3}{\sqrt{2}}-1\right)(4x)-\frac{4}{\sqrt{2}}+1 & \frac{1}{8} \le x_j \le \frac{1}{4} \\ 2\left(3-\frac{2}{\sqrt{2}}\right)(4x)+\frac{6}{\sqrt{2}}-7 & \frac{1}{4} \le x_j \le \frac{3}{8} \\ -2\left(3+\frac{2}{\sqrt{2}}\right)(4x)+\frac{6}{\sqrt{2}}+11 & \frac{3}{8} \le x_j \le \frac{1}{2} \\ 2\left(\frac{3}{\sqrt{2}}+1\right)(4x)-\frac{14}{\sqrt{2}}-5 & \frac{1}{2} \le x_j \le \frac{5}{8} \\ -\frac{2}{\sqrt{2}}(4x)+\frac{6}{\sqrt{2}} & \frac{5}{8} \le x_j \le \frac{3}{4} \\ 0 & otherwise \end{cases}$$

$$\psi_{2,1}(x) = \frac{1}{2}\begin{cases} \frac{-2}{\sqrt{2}}(4x-1) & \frac{1}{4} \le x_j \le \frac{3}{8} \\ 2\left(\frac{3}{\sqrt{2}}-1\right)(4x-1)-\frac{4}{\sqrt{2}}+1 & \frac{3}{8} \le x_j \le \frac{1}{2} \\ 2\left(3-\frac{2}{\sqrt{2}}\right)(4x-1)+\frac{6}{\sqrt{2}}-7 & \frac{1}{2} \le x_j \le \frac{5}{8} \\ -2\left(3+\frac{2}{\sqrt{2}}\right)(4x-1)+\frac{6}{\sqrt{2}}+11 & \frac{5}{8} \le x_j \le \frac{3}{4} \\ 2\left(\frac{3}{\sqrt{2}}+1\right)(4x-1)-\frac{14}{\sqrt{2}}-5 & \frac{3}{4} \le x_j \le \frac{7}{8} \\ -\frac{2}{\sqrt{2}}(4x-1)+\frac{6}{\sqrt{2}} & \frac{7}{8} \le x_j \le 1 \\ 0 & otherwise \end{cases}$$

The L.H.S and R.H.S boundary wavelet function are given as

$$\psi_{2,-1}(x) = \frac{1}{2}\begin{cases} 2\left(4-\frac{5}{\sqrt{x}}\right)(4x+1)+\frac{14}{\sqrt{2}}-10 & 0 \le x \le \frac{1}{8} \\ -2\left(3+\frac{1}{\sqrt{2}}\right)(4x+1)+\frac{2}{\sqrt{x}}+11 & \frac{1}{8} \le x \le \frac{1}{4} \\ 2\left(\frac{3}{\sqrt{2}}+1\right)(4x+1)-\frac{14}{\sqrt{2}}-5 & \frac{1}{4} \le x \le \frac{3}{8} \\ -\frac{2}{\sqrt{2}}(4x+1)+\frac{6}{\sqrt{2}} & \frac{3}{8} \le x \le \frac{1}{2} \\ 0 & otherwise \end{cases}$$

$$\psi_{2,2}(x) = \frac{1}{2}\begin{cases} \frac{-2}{\sqrt{2}}(4x-2) & 0 \le x \le \frac{1}{8} \\ 2\left(\frac{3}{\sqrt{2}}-1\right)(4x-2)-\frac{4}{\sqrt{2}}+1 & \frac{1}{8} \le x \le \frac{1}{4} \\ 2\left(3-\frac{1}{\sqrt{2}}\right)(4x-2)+\frac{4}{\sqrt{2}}-7 & \frac{1}{4} \le x \le \frac{3}{8} \\ -2\left(4+\frac{5}{\sqrt{2}}\right)(4x-2)+\frac{16}{\sqrt{2}}+14 & \frac{3}{8} \le x \le \frac{1}{2} \\ 0 & otherwise \end{cases}$$

The third order B-spline scaling function and B-spline wavelet function are given by

$$B_{j,k}(x) = \begin{cases} \frac{1}{2}(x_j-k)^2 & k \le x_j \le k+1 \\ -(x_j-k)^2+(x_j-k)-\frac{3}{2} & k+1 \le x_j \le k+2 \\ \frac{1}{2}(x_j-k)^2-3(x_j-k)+\frac{9}{2} & k+2 \le x_j \le k+3 \\ 0 & otherwise \end{cases} \tag{2.3}$$

and $$\psi_{j,k}(x) = \frac{1}{2}\begin{cases} I & k \le x_j \le k+\frac{1}{2} \\ II & k+\frac{1}{2} \le x_j \le k+1 \\ III & k+1 \le x_j \le k+\frac{3}{2} \\ IV & k+\frac{3}{2} \le x_j \le k+2 \\ V & k+2 \le x_j \le k+\frac{5}{2} \\ VI & k+\frac{5}{2} \le x_j \le k+3 \\ VII & k+3 \le x_j \le k+\frac{7}{2} \\ VIII & k+\frac{7}{2} \le x_j \le k+4 \\ 0 & otherwise \end{cases} \qquad k = 0, \dots, 2^j - 4 \qquad (2.4)$$

where

$$I = -3(x_j - k)^2$$

$$II = -\frac{3}{2}\left[-(2x_j - 2k)^2 + 3(2x_j - 2k) - \frac{3}{2}\right] + \frac{7}{4}(2x_j - 2k - 1)^2$$

$$III = \frac{7}{2}\left[-(2x_j - 2k - 1)^2 + 3(2x_j - 2k - 1) - \frac{3}{2}\right] - \frac{3}{2}\left[\frac{1}{2}(2x_j - 2k)^2 - 3(2x_j - 2k) + \frac{9}{2}\right]$$

$$IV = \frac{7}{2}\left[\frac{1}{2}(2x_j - 2k - 1)^2 - 3(2x_j - 2k - 1) + \frac{9}{2}\right] - 3(2x_j - 2k - 3)^2$$

$$V = -6\left[-(2x_j - 2k - 3)^2 + 3(2x_j - 2k - 3) - \frac{3}{2}\right] + \frac{11}{4}(2x_j - 2k - 4)^2$$

$$VI = -6\left[\frac{1}{2}(2x_j - 2k - 3)^2 - 3(2x_j - 2k - 3) + \frac{9}{2}\right] + \frac{11}{2}\left[-(2x_j - 2k - 4)^2 + 3(2x_j - 2k - 4) - \frac{3}{2}\right] - \frac{3}{4}(2x_j - 2k - 5)^2$$

$$VII = \frac{11}{2}\left[\frac{1}{2}(2x_j - 2k - 4)^2 - 3(2x_j - 2k - 4) + \frac{9}{2}\right] - \frac{3}{2}\left[-(2x_j - 2k - 4)^2 + 3(2x_j - 2k - 4) - \frac{3}{2}\right] \qquad VIII = -\frac{3}{2}\left[\frac{1}{2}(2x_j - 2k - 5)^2 - 3(2x_j - 2k - 5) + \frac{9}{2}\right]$$

The L.H.S and R.H.S baundary scaling function and wavelet function are given by

$$B_{3,-1}(x) = \begin{cases} -(8x+1)^2 + (1-8x)^2 + 3(8x+1) - \frac{3}{2} & 0 \le x \le \frac{1}{8} \\ \frac{1}{2}(8x+1)^2 - 3(8x+1) + \frac{9}{2} & \frac{1}{8} \le x \le \frac{1}{4} \\ 0 & otherwise \end{cases}$$

$$B_{3,6}(x) = \begin{cases} \frac{1}{2}(8x-6)^2 & \frac{3}{4} \le x \le \frac{7}{8} \\ -(8x-6)^2 + 3(16x-16) + \frac{1}{2}(10-8x)^2 + \frac{6}{2} & \frac{7}{8} \le x \le 1 \\ 0 & otherwise \end{cases}$$

$$\psi_{3,-1}(x) = \frac{1}{2}\begin{cases} I' & 0 \le x \le \frac{1}{16} \\ II' & \frac{1}{16} \le x \le \frac{1}{8} \\ V & \frac{1}{8} \le x \le \frac{3}{16} \\ VI & \frac{3}{16} \le x \le \frac{1}{4} \\ VII & \frac{1}{4} \le x \le \frac{5}{16} \\ VIII & \frac{5}{16} \le x \le \frac{3}{8} \\ 0 & otherwise \end{cases}$$

and $$\psi_{3,5}(x) = \frac{1}{2}\begin{cases} I & \frac{5}{8} \le x \le \frac{11}{16} \\ II & \frac{11}{16} \le x \le \frac{3}{4} \\ III & \frac{3}{4} \le x \le \frac{13}{16} \\ IV & \frac{13}{16} \le x \le \frac{7}{8} \\ V' & \frac{7}{8} \le x \le \frac{15}{16} \\ VI' & \frac{15}{16} \le x \le 1 \\ 0 & otherwise \end{cases}$$

where $$I' = \frac{7}{2}\left\{-(16x+1)^2 + 3(16x+1) - \frac{3}{2}\right\} - \frac{3}{2}\left\{\frac{1}{2}(16x+2)^2 - 3(16x+2) - (2-16x)^2 + 3(2-16x) + \frac{6}{2}\right\} + \frac{7}{4}(-16x+1)^2$$

$$II' = \frac{7}{2}\left\{\frac{1}{2}(16x+1)^2 - 3(16x+1) + \frac{9}{2}\right\} - 3(16x-1)^2 - 3(-8x+1)^2$$

$$V' = -6\left\{-(16x-13)^2 + 3(16x-13) - \frac{3}{2}\right\} + \frac{11}{4}(16x-14)^2 - \frac{3}{2}\left\{\frac{1}{2}(16x-17)^2 + 3(16x-17) + \frac{9}{2}\right\}$$

$$VI' = -6\left\{\frac{1}{2}(16x-13)^2 - 3(16x-13) + \frac{9}{2}\right\} - \frac{3}{4}(16x-15)^2 - \frac{3}{2}\left\{-(16x-17)^2 - 3(16x-17) - \frac{3}{2}\right\} + \frac{11}{2}\{-(16x-14)^2 + 3(16x-14) + \frac{1}{2}(16x-18)^2 + 3(16x-18) + 3\}$$

The inner third order B-spline scaling function are obtained by substituting $j = 3$ and $k = 0,1,2,3,4,5$ in eqn(2.3). And the inner wavelet functions are obtained by putting $j = 3$ and $k = 0,1,2,3,4$ in equation (2.4). Fig(2.1) shows the B-spline wavelets for $m = 2$ & $m = 3$.

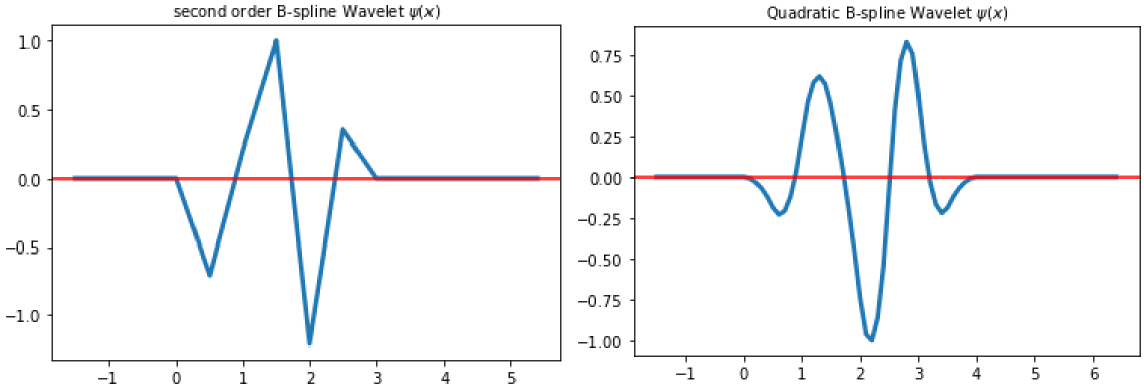

Fig.(2.1): Second and third B-spline wavelet i,e for m=2 and m=3 resp.

III. Function Approximation

A function g(x) defined over [0,1]may be represented by B-spline wavelets as

$$g(x) = \sum_{k=-1}^{2^j-(m-1)} a_{j,k} B_{j,k}(x) + \sum_{k=-1}^{2^j-m} \alpha_{j,k}\psi_{j,k}(x), \qquad j = 2,3,\dots\dots M$$

where $B_{j,k}$ and $\psi_{j,k}$ are the scaling and wavelet functions and m is the order of B-spline wavelets. Also,

$$g(x) = A_g^T \Psi(x)$$

where

$$A_g = [a_{j,-1}, a_{j,0}, \dots, a_{j,2^j-(m-1)}]^T$$

$$\Psi = [B_{j,-1}, B_{j,0}, \dots, B_{j,2^j-(j-1)}, \psi_{j,-1}, \psi_{j,2}, \dots, \psi_{j,2^j-m}]^T$$

where $a_{j,k} = \int_0^1 g(x)\,\tilde{B}_{j,k}(x)dx$ and $\alpha_{j,k} = \int_0^1 g(x)\,\tilde{\psi}_{j,k}(x)dx$.

$\tilde{B}_{j,k}$ and $\tilde{\psi}_{j,k}$ are duals of $B_{j,k}$ and $\psi_{j,k}$ resp. The duals can be obtained as follows:

Let $\phi = [B_{j,-1}, B_{j,0}, \dots, B_{j,2^j-(m-1)}]^T$

$\psi = [\psi_{j,-1}, \psi_{j,0}, \dots, \psi_{j,2^j-m}]^T$

then, $\int_0^1 \phi\,\phi^T dx = P_1$ and $\int_0^1 \psi\,\psi^T dx = P_2$ (3.1)

Let $\tilde{\phi}$ and $\tilde{\psi}$ are the dual function of ϕ and ψ resp. given by

$$\tilde{\phi} = [\tilde{B}_{j,-1}, \tilde{B}_{j,0}, \dots, \tilde{B}_{j,2^j-(m-1)}]^T$$

$$\tilde{\psi} = [\tilde{\psi}_{j,-1}, \tilde{\psi}_{j,0}, \dots, \tilde{\psi}_{j,2^j-m}]^T \qquad (3.2)$$

such that, $\int_0^1 \tilde{\phi}\,\phi^T dx = I_1$ and $\int_0^1 \tilde{\psi}\,\psi^T dx = I_2$ (3.3)

where I_1 and I_2 are identity matrices.

From Eq. (3.1) and Eq. (3.2),

$$\tilde{\phi} = P_1^{-1}\phi \quad \text{and} \quad \tilde{\psi} = P_2^{-1}\psi$$

IV. Fredholm Integral Equations

Here we consider Fredholm Integral Equations of type

$$y(x) = f(x) + \int_0^1 k(x,t)\,y(t)dt \qquad 0 \le x \le 1 \qquad (4.1)$$

and solve this equation by second order B-spline wavelets for $j = 2$. Let first approximate y(x) as

$$y(x) = A_y^T \Psi(x) \qquad (4.2)$$

where $\Psi(x) = \left[B_{2,-1}, B_{2,0}, \dots, B_{2,3}, \psi_{2,-1}, \dots, \psi_{2,2}\right]^T$

Also denoted as $\Psi(x) = [\psi_1, \psi_2, \dots, \psi_6, \dots. \psi_9]^T$ (4.3)

we expand $f(x)$ and $k(x,t)$ by B-spline dual wavelets defined as in Eq. (3.2) as

$$f(x) = A_f^T \tilde{\Psi}(x) \qquad (4.4)$$

and $k(x,t) = \tilde{\Psi}^T(t)\Theta\tilde{\Psi}(x)$ (4.5)

where Θ is a 9×9 matrix for second order B-spline wavelet with $j = 2$ with its elements

$$\Theta_{ij} = \int_0^1 [\int_0^1 k(x,t)\,\psi_i(t)dt]\,\psi_j(x)dx$$

From Eq.(4.2) and Eq.(4.5),

$$\int_0^1 k(x,t)\,y(t)dt = \int_0^1 \tilde{\Psi}^T(t)\Theta\tilde{\Psi}(x)\,A_y^T\Psi(t)dt = \Theta\tilde{\Psi}(x)A_y^T = \Theta A_y^T\tilde{\Psi}(x) \qquad (4.6)$$

Using Eq.(4.2),Eq.(4.4) and Eq.(4.6) in Eq.(4.1), we get

$$A_y^T\Psi(x) = A_f^T\tilde{\Psi}(x) + \Theta A_y^T\tilde{\Psi}(x)$$

Multiplying the above equation by $\tilde{\Psi}^T(x)$ and integrating from 0 to 1, we get

$$A_y^T \int_0^1 \tilde{\Psi}^T(x)\,\Psi(x)dx = A_f^T + \Theta A_y^T \qquad (4.7)$$

Putting $\int_0^1 \tilde{\Psi}^T(x)\,\Psi(x)dx = P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$.

For 2nd order B-spline scaling function and wavelet function, for j=2,

$$P_1 = \begin{bmatrix} 0.0833 & 0.0417 & 0.0000 & 0.0000 & 0.0000 \\ 0.0417 & 0.1667 & 0.0417 & 0.0000 & 0.0000 \\ 0.0000 & 0.0417 & 0.1667 & 0.0417 & 0.0000 \\ 0.0000 & 0.0000 & 0.0417 & 0.1667 & 0.0417 \\ 0.0000 & 0.0000 & 0.0000 & 0.0417 & 0.0833 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0.1347 & -0.034 & 0.0000 & 0.0000 \\ -0.034 & 0.1667 & -0.0417 & 0.000 \\ 0.0000 & -0.0417 & 0.1667 & -0.049 \\ 0.0000 & 0.0000 & -0.049 & 0.2819 \end{bmatrix}$$

thus,

$$A_y^T \mathrm{P} = A_f^T + \Theta A_y^T$$

$$A_y^T(P - \Theta) = A_f^T$$

$$or, \quad A_y^T = A_f^T(P - \Theta)^{-1}$$

In this way, from Eq.(4.2) , we get the numerical solution of the integral Eq.(4.1).Similar process is applied for higher order wavelets and higher octave levels.

V. Illustrative Examples

1. Consider the equation

$$y(x) = e^x - \frac{e^{x+1} - 1}{x + 1} + \int_0^1 e^{xt}\, y(t)dt, \qquad 0 \le x \le 1$$

with exact solution $y(x) = e^x$.

The solution for $y(x)$ is obtained by the method explain in section 4 with second(m=2) and third(m=3) order B-spline wavelet for different values of j. The numerical solution obtained with exact solution $y(x) = e^x$ are given in Table 5.1

Table 5.1. Exact and obtained solution

x	m = 2		m = 3	Exact Sol.
	j = 2	j = 3	j = 3	
0	0.954713	0.981062	1	1
0.1	1.06087	1.12446	1.09688	1.10517
0.2	1.26066	1.20488	1.21838	1.2214
0.3	1.38671	1.34623	1.35443	1.34986
0.4	1.45637	1.49318	1.49702	1.49182
0.5	1.67171	1.6511	1.6467	1.64872
0.6	1.80969	1.82285	1.82085	1.82212
0.7	2.01761	2.01214	2.01787	2.01375
0.8	2.20405	2.22317	2.21904	2.22554
0.9	2.41732	2.44861	2.45811	2.4596

2. Consider the equation

$$y(x) = e^{2x+\frac{1}{3}} - \frac{1}{3}\int_0^1 e^{2x-\frac{5t}{3}}\, y(t)dt, \qquad 0 \le x \le 1$$

with exact solution $y(x) = e^{2x}$.

The numerical solution obtained for m=2 and m=3 for different values of j together with exact solution $y(x) = e^{2x}$ are given in Table 5.2

Table 5.2. Exact and approximate solution

x	m = 2		m = 3	Exact sol^n
	j = 2	j = 3	j = 3	
0	0.892668	0.957694	1	1
0.1	1.14382	1.26296	1.20363	1.2214
0.2	1.58615	1.45818	1.48601	1.49182
0.3	1.90841	1.81305	1.82929	1.82212
0.4	2.16427	2.2306	2.23898	2.22554
0.5	2.77233	2.72857	2.7078	2.71828
0.6	3.2909	3.32686	3.32117	3.32012
0.7	4.0913	4.04803	4.07837	4.0552
0.8	4.84134	4.93798	4.91508	4.95303
0.9	5.85308	5.99318	6.03669	6.04965

VI. CONCLUSION

In this paper, we present a method to solve linear Fredholm integral equations. Our approximation is based on compactly supported semi-orthogonal B-spline wavelet generated in our previous paper. Two examples are illustrated to check the validity and significance of the proposed technique. The solution of the examples reveals that the exactness of solution increases as the order of B-spline wavelet and octave level increases.

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