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Assessment of Simplified Models for Lateral Load Analysis of Unreinforced Masonry Buildings

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Abstract: The purpose of this paper is to assess the relative accuracy of various models for the analysis of unreinforced masonry buildings, primarily for use by practising engineers. It also seeks to establish whether and under what circumstances a straightforward equivalent frame model can be used for design and/or assessment purposes. First in the elastic range, using both fine and coarse planar meshes, a number of parametric assessments employing finite element FE models of two-dimensional and three-dimensional structures have been carried out. The identical structures were then subjected to evaluations utilising comparable frames and different stiff offset arrangements. To verify the accuracy of the inferences made from the elastic analysis, two-dimensional nonlinear static "pushover" evaluations of the FE and comparable frame models were carried out. The outcomes displayed here This will further clarify if it is feasible to use straightforward and affordable analytical models as a tool for the practical design and/or evaluation of ordinary masonry constructions.

keywords: Masonry; Lateral loads; Buildings; Load analysis; Finite element method; Models.

1.Introduction:

Practical analysis of unreinforced masonry (URM) buildings for design and/or assessment purposes is typically carried out using static analysis involving planar [two-dimensional (2D) models], and isotropic homogeneous linear elastic behavior is assumed. However, currently available analytical tools for URM also include finite element (FE) models based on isotropic–orthotropic homogeneous nonlinear material, or even heterogeneous nonlinear material assumptions. Furthermore, discrete element formulations are available, focusing on the nonlinear behavior of joints between masonry units.

Existing analytical techniques for the nonlinear analysis of masonry buildings are virtually solely employed for research because to their high processing costs and more importantly the high analytical skills necessary for their implementation.

The potential exception of monumental structures for which a more expensive than usual expense of examination is regarded justified. For the practical analysis of typical URM structures, equivalent frame analysis, based on the linear isotropic material assumption, has been widely employed in place of the more sophisticated FE technique. This has occasionally been complemented by limit analysis, which involves simplified models of important structural elements and aims to estimate the maximum load that can be supported for a certain collapse mechanism.

The main objective of this paper is the comparative evaluation of commonly used techniques for practical analysis of realistic URM structures subjected to lateral loading in addition to their gravity loads. The strategy adopted consists in first establishing the conditions under which the commonly used equivalent frame and finite element approaches yield results that can be considered as equivalent from the practical design and/or assessment point of view, in the case of the simplest realistic masonry structure, the 2D perforated wall, assumed to remain in the (quasi-)elastic range. Then, the two approaches are applied in the case of an actual three-dimensional (3D) masonry building with and without floor diaphragms, using again the elastic behavior assumption. Subsequently, the critical issue of inelastic behavior is tackled, by analyzing the response to combined horizontal and vertical loading of the previously studied perforated wall, applying the pushover procedure for both the equivalent frame and the finite element model of the structure. A key component of the latter part of the study is a newly devised procedure for modeling inelastic response in the equivalent frame model.

It is noted that while methodologies like the ones mentioned above have been employed in numerous studies, the majority of them only focus on one approach, which is assumed to be the most effective in the scenario investigated a priori (a nice collection of such works may be found, among others, in ICCROM, 1990!). The studies by Seible and Kingsley (1991), Karantoni and Fardis (1992), and Magenes and La Fontana (1998) are three of the few that compare the equivalent frame and the FE technique in a fair amount of detail.

2.Analytical Models

2.1.Elastic Analysis of Perforated Wall

A typical 2D masonry structure, a perforated wall of regular geometry, as shown in Fig. 1, was analyzed by two different approaches, an equivalent frame model and a finite element model consisting of plane stress elements. This is the same structure previously studied by Seible and Kingsley (1991), which facilitates comparisons.

Ten different models were analyzed for this 2D structure, six of which were equivalent frame models and four involved FE meshes of planar elements. Linear elastic analysis of the different models was carried out using the (SAP2000 -Computers and Structures Inc. 1999) software package.

Regarding the equivalent frame models, three different patterns of rigid offsets were used for simulating the finite width or depth of the piers and spandrels, and the resulting three models were analyzed with and without the diaphragm constraint.

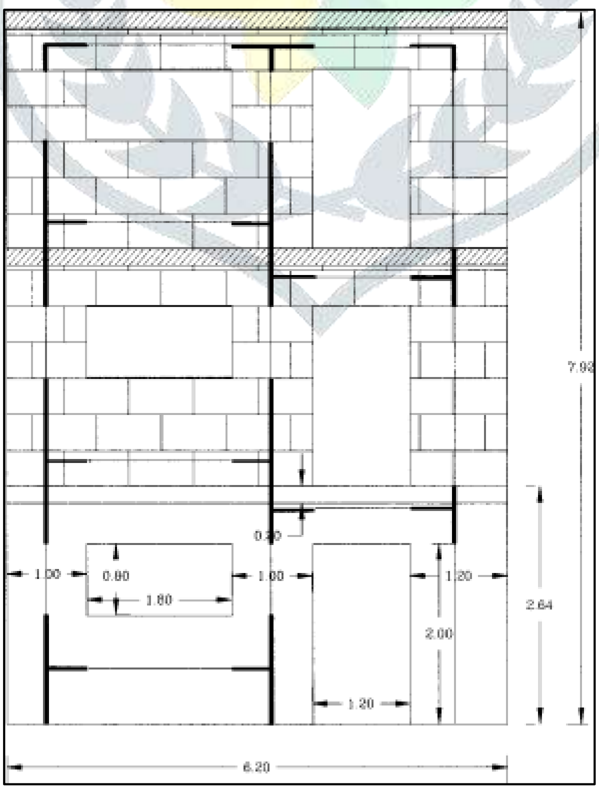


Fig. 1. Geometry of perforated masonry wall studied and equivalent frame model with full horizontal and vertical offsets

Regarding the FE models, two different meshes of varying refinement were used, and each of them was analyzed with and without a diaphragm constraint at the floor levels. As to the mesh refinement, two different approaches were adopted, the coarsest one, essentially defined by the geometry of the wall resulting in a total of 36 elements, and a refined one involving a total of 156 elements.

The magnitude of the stiff offsets to be imposed in the corresponding frame models is a topic that is currently up for debate in the literature. While it might seem logical to simply align them with Because of the actual vertical and horizontal offsets in the width and depth of the piers, the frame is typically stiffer than the actual perforated wall. This is especially true whenever attempts are made to mimic the structure's crack pattern, a problem that is addressed later in this paper. Here, three different instances were taken into account within the context of elastic analysis while taking into account earlier suggestions made in relation to this problem:

- Full horizontal rigid offsets
- Full horizontal and vertical rigid offsets (Fig. 1); and
- Full horizontal and half vertical rigid offsets.

All the models used for the perforated wall analysis are summarized in Table 1.

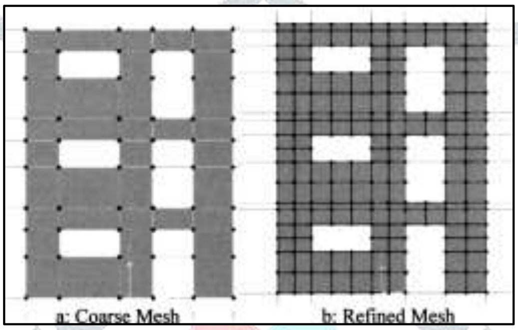


Fig. 2. Finite element models for two-dimensional (2D) wall (a) coarse mesh(b) refined mesh

2.2Elastic Analysis of Three-Dimensional Building

The analysis was then extended to the 3D case, by considering an actual two-story stone masonry building situated in Kalamata, Greece, which was damaged by the 1986 earthquake that hit the city. The geometry of the building is shown in Fig. 3. This building was previously analyzed by Karantoni and Fardis (1992), who have performed several parametric analyses involving elastic models of varying complexity.

Seven different analyses of this building were performed, two involving equivalent frame models, and five involving FE models. The equivalent frame models were analyzed using the full rigid offsets option with different assumptions regarding the diaphragm constraint, while the FE models were analyzed using different meshes and also including or excluding the diaphragm constraint at the floor level. Consideration of the diaphragm constraint is a critical issue, especially in the assessment of existing structures for possible retrofitting, since it is very common in old masonry buildings to have wooden floors, which cannot be considered to impose a rigid diaphragm constraint on the structure, particularly when their connection to the masonry walls is rather loose. Hence, in order to evaluate the effect of different floor systems on the same structure, two different cases were studied, one with a wooden floor and one with a reinforced concrete (R/C) floor, with due account of the different stiffness, as well as the different gravity loads in each structure. The previously described models are summarized in Table 2. Six of them were analyzed under static loading, while one FE model (FE-R) was also analyzed dynamically. The dynamic analysis was performed for the case of the loosely connected wooden floor (practically no diaphragm action), with distributed masses at all nodes of the walls to simulate their mass, and concentrated masses at the floor level to simulate the extra weight carried and transferred by the floor’s wooden beams. This was an analysis deemed necessary in view of the significant discrepancies found in the cases with and without a rigid diaphragm (discussed in the next section), and represents a possible solution to a commonly encountered problem in practical analysis of masonry structures wherein, due to the absence of diaphragms, judgment is required in applying the lateral (seismic) loading.

Table 1. Planar Wall Models

Name	Description
EF1	Only horizontal rigid zones and no diaphragm constraint
EF1D	Only horizontal rigid zones with diaphragm constraint
EF2	Horizontal and vertical rigid zones and no diaphragm constraint
EF2D	Horizontal and vertical rigid zones with diaphragm constraint
EF3	Horizontal and half vertical rigid zones and no diaphragm constraint
EF3D	Horizontal and half vertical rigid zones with diaphragm constraint
FE-R	A refined mesh of finite elements with no diaphragm constraints
FE-RD	A refined mesh of finite elements with diaphragm constraints
FE-C	A coarse mesh of finite elements with no diaphragm constraints
FE-CD	A coarse mesh of finite elements with diaphragm constraints

Table 2. Three Dimensional Building Models

Name	Description
EF	Equivalent frame without any diaphragms (wooden floors)
EF-D	Equivalent frame with diaphragms (concrete floors)
FE-C	A coarse mesh without any diaphragms (wooden floors)
FE-CD	A coarse mesh with diaphragms (concrete floors)
FE-R	A refined mesh without any diaphragms (wooden floors)
FE-RD	A refined mesh with diaphragms (concrete floors)
FE-R (DYN)	A coarse mesh without any diaphragms (wooden floors) used for dynamic analysis

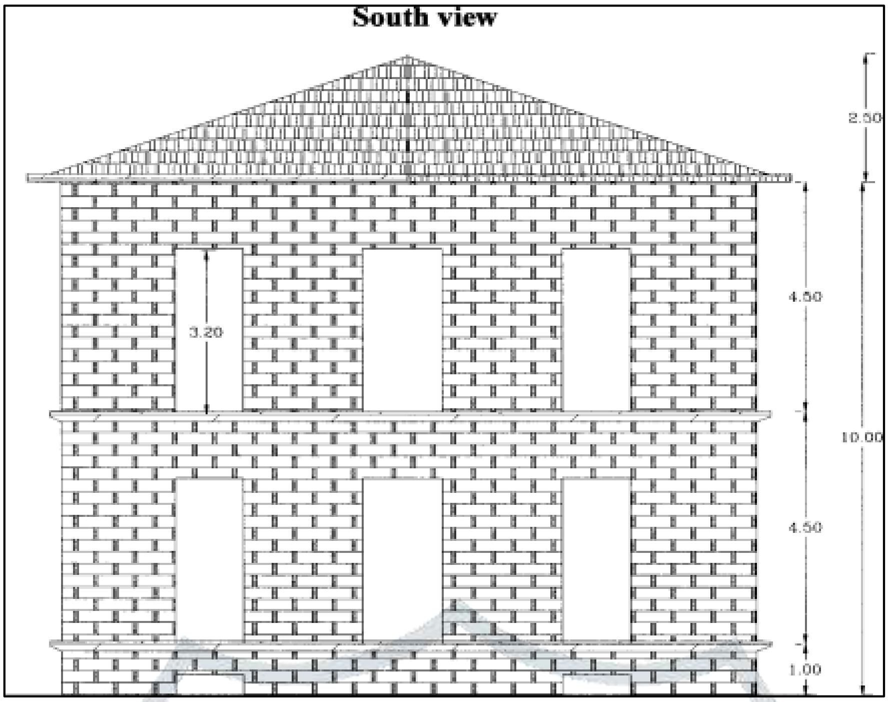


Fig. 3. Side view of three-dimensional (3D) masonry building studied

Nonlinear Static Analysis of Perforated Wall

The planar perforated wall previously analyzed elastically, was subsequently analyzed using the *ANSYS* nonlinear FE code (ANSYS Inc. 1997), based on the standard smeared crack approach and involving the 3D Solid 65 element available primarily for modeling concrete. The material constitutive law (Willam-Warnke model) used for masonry is shown in Fig. 4. The equivalent uniaxial stress–strain relationship was a parabolic one, with an elastic modulus coinciding with the value previously used for elastic analysis, E_{el}=1000 f_c (CEN TC 250 1995), where f_c is the compressive strength of the masonry wall, ultimate compressive strain s_u=0.002, and tensile strength f_t=0.1*f_c.The biaxial strength envelope produced as a projection of the 3D failure surface, both shown in Fig. 4, is in good agreement with the one proposed on the basis of test results (Dhanasekar et al. 1985). Whenever the biaxial tensile strength is exceeded the element is assumed to have cracked, while a compressive strain in excess of s_u is considered as crushing of the element. The element stiffness matrix is updated whenever failure according to either criterion occurs. After cracking, a residual shear stiffness is retained, equal to 60% of the uncracked value.

The use of a tensile strength equal to 10% of the compressive strength for URM, as well as the use of 60% of the uncracked shear stiffness after closing of a crack, were decided on the basis of a sensitivity analysis performed for a half-scale URM building tested at Ismes (Benedetti et al. 1998). The alternative models used are shown in Fig. 5(a), and it is clear that the aforementioned values result in the best prediction of the experimental response.

Further information, especially regarding the tensile strength, can be found in the experimental work of Dhanasekar et al. (1985) work of Ignatakis et al. (1989), where it is shown that, in the tension-compression region of the biaxial and the analytical strength envelope, a high nonlinearity is observed, and for a com- pressive stress of 10% of the uniaxial compressive strength, the corresponding tensile strength is 9% of the compressive strength. This corroborates the results of the sensitivity analysis which shows that the f_t=0.1• f_c case [*ANSYS* A in Fig. 5(a)] best matches the experimental data. Finally, some additional sensitivity analyses were run using a reduced elastic modulus E_{el}=550 f_c , in line with some recent recommendations (FEMA 1997) for existing structures; some improvement was noted in matching the initial slope of the experimental curves [Fig. 5(b)], but strength and the post-elastic branches that were the focus of this part of the study was not affected.

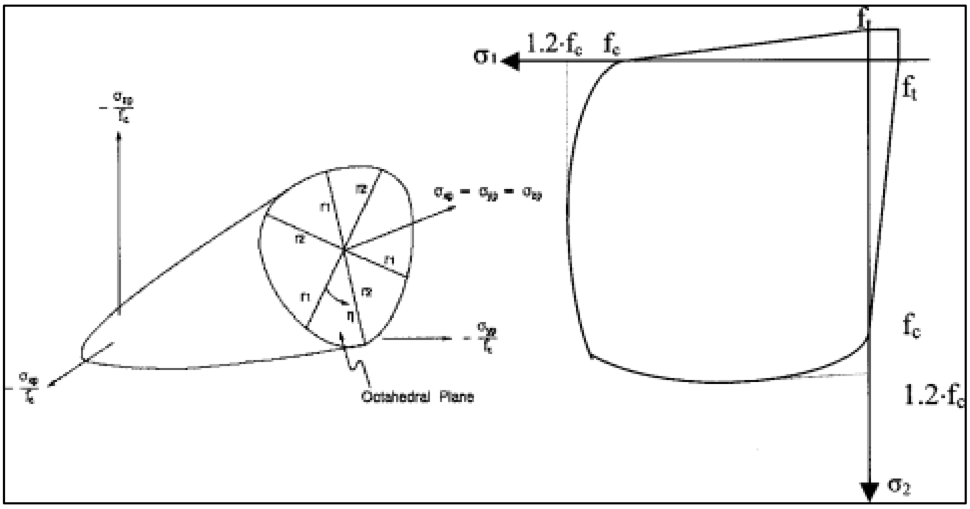


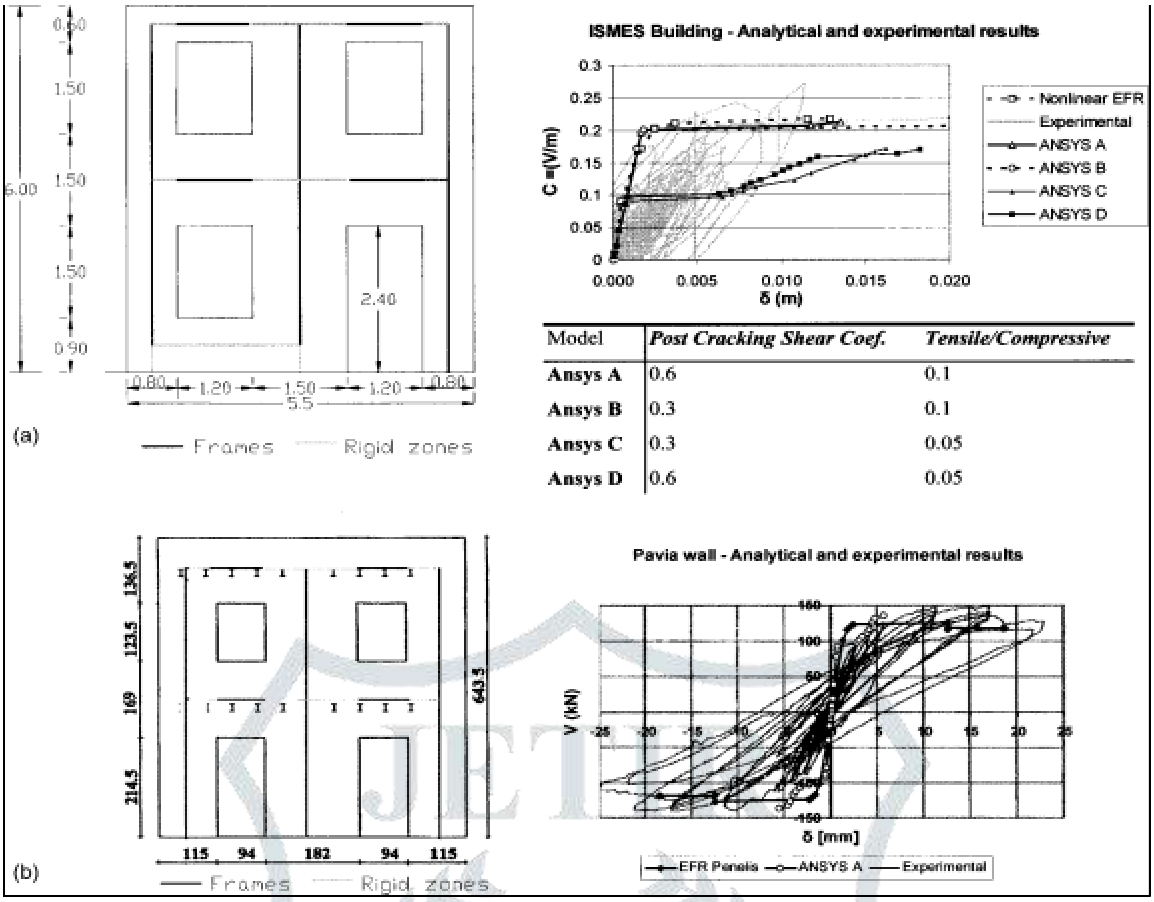
Fig. 4. Three-dimensional failure surface and corresponding biaxial strength envelope for unreinforced masonry (URM)

The nonlinear analysis was performed by applying the gravity loads and the lateral loads in a stepwise fashion. The step-by-step analysis was carried out, using a force-based convergence criterion, until failure of several elements due to cracking or crushing led to deterioration of the stiffness matrix, and the procedure be- came unstable.

The wall was also analysed inelastically (pushover analysis) in addition to the finite element method utilising the equivalent frame method. The modelling of the inelastic behaviour of critical sections is crucial in this regard. For this analysis, the nonlinear SAP2000 programme was employed, and point hinge line elements with rigid offsets at the ends were used to simulate both piers and spandrels. Analytical phenomenological closed-form solutions for flexure and shear were integrated with a hybrid semi analytical semi statistical model to compute nonlinear moment-rotation curves for the point hinges at the element ends. With no brickwork tensile strength, the expected stress-strain behaviour is the same as that in the FE model.

Experimental information from the has been used to validate the suggested pushover analysis approach. From the pseudodynamic test of a full-scale perforated URM wall at Pavia (Magenes and La Fontana 1998) and the shake table test of a half-scale URM building at Ismes (Benedetti et al., 1998), both depicted in Fig. 5. In a recent study by the second author (Penelis 2000), the analogous frame technique for brick walls is discussed in more detail

Since for practical assessment purposes full nonlinear procedures (whenever feasible) are still cumbersome, it was considered worth investigating some simplified analytical procedures that can yield useful, albeit rough, indications of the strength margins in the structure. Hence, a nonlinear analysis of the base cross section of the wall was carried out, using the same material properties as in the pushover analysis. Then, the forces at the base calculated from the elastic frame model with full horizontal and vertical rigid offsets (Fig. 1), were checked against the axial force versus yield moment interaction surfaces calculated from section analysis, in order to evaluate the resulting overstrength factor and compare it with the one calculated from the actual pushover analysis.



Model	Post Cracking Shear Coef.	Tensile/Compressive
Ansys A	0.6	0.1
Ansys B	0.3	0.1
Ansys C	0.3	0.05
Ansys D	0.6	0.05

Fig. 5. Validation of nonlinear models against test data: (a) half-scale URM building tested at Ismes; (b)pseudodynamic test of full scale perforated URM wall tested at Pavia.

Discussion of Results

Elastic Analysis of 2D Perforated Wall

In view of the current trend towards displacement-based design and assessment, the focus of the analysis was on story displacements calculated using the different models; these are summarized in Fig. 6. It is recalled that the parameters under evaluation were the extent of the rigid zones in the equivalent frame model, the mesh refinement in the finite element model, and the diaphragm constraint in all the models.

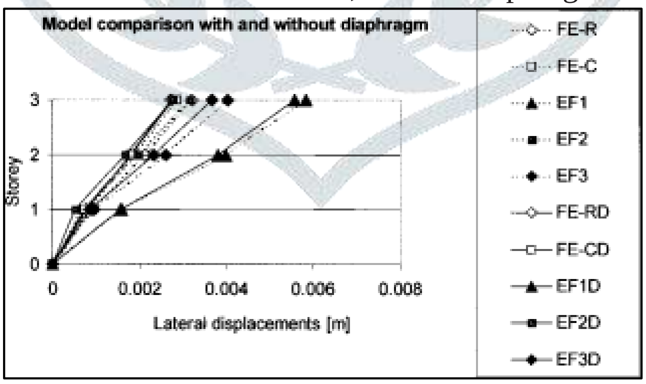


Fig. 6. Story displacements for 2D wall calculated using different equivalent frame (EF) and finite element (FE) models (see model nomenclature in Table 1)

The first conclusion from Fig. 6 is that in the two-dimensional models the effect of the diaphragm constraint is negligible (results from models without diaphragm are shown with dotted lines in the figure, while results from models with diaphragm constraint are shown with full lines), hence there is no need for special modeling effort to account for this effect in the analysis.

It appears from the results in Fig. 6 that the degree of mesh refinement in the finite element models does not produce results that are significantly different from those produced by the coarse mesh, so it is difficult to justify the higher computational cost of the more refined mesh. Contrary to the diaphragm effect, it is believed that the finding about mesh refinement should not be applied universally because perforated walls with more complex (irregular) geometries may be more sensitive to the FE mesh employed.

Table 3. Comparison of Internal Forces Calculated by Equivalent Frame (EF) and Finite Element (FE) Models

	Axial force (KN)			Shear force (KN)		
a/a	EF2	FE-R	Diff. (%)	EF2	FE-R	Diff. (%)
Left pier	−85	−65.4	23.0	15.6	19.0	16.0
Middle pier	−147	−172.0	14.5	28.6	23.5	17.7
Right pier	−171	−166.6	2.5	15.9	18.0	11.4

The most significant finding from this series of analyses is the observation that the rigid offset pattern assumption in equivalent frame models is a crucial one, as demonstrated by the significantly different displacement values in Fig. 6, and that the model (EF2) with full horizontal and vertical rigidity is the most important one. Offsets (shown in Fig. 1) produces outcomes that are extremely similar to those from the revised FE model, making it an appealing alternative. a realistic analysis tools. However, it should be noted that all of the displacements depicted in Fig. 6 were calculated using the comparable frame and FE models, which all assume elastic behaviour. Despite how prevalent this assumption may be (in current design practise), it is still very different from the actual behaviour. condition, especially when the final limit state under seismic loading is considered. Later in this work, it is determined whether the simple equivalent frame is suitable for nonlinear analysis.

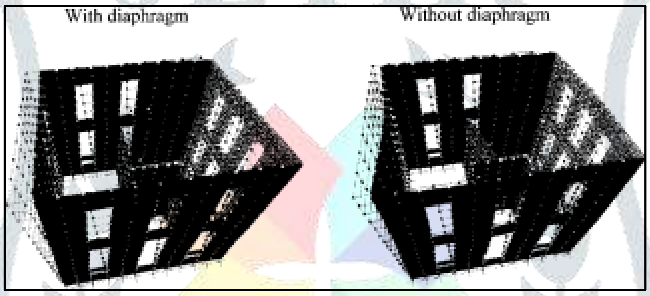


Fig. 7. Deformation modes of 3D masonry building with and with rigid diaphragms, subjected to seismic loading

acceptable in terms of design. However, FE modelling and appropriate mesh refinement become necessary if determining the structure's stress state is a problem (which isn't always the case in a design setting). This study discovered that the stress distributions derived from the analysis with the refined mesh could not be accurately represented by the coarser mesh.

Elastic Analysis of 3D Building

In the different 3D models of the building (Fig. 3), the key parameters under consideration were the diaphragm constraint and
the mesh refinement that has a significant impact on the cost of three-dimensional models.

The displacements of the building, as predicted by the refined finite element model, are shown in Fig. 7 for the cases of a R/C floor (left) that is stiff enough to provide a diaphragm, and a wooden floor (right). The building is subjected to normal gravity
loads and 100% of the design seismic load in one direction plus 30% of it in the other direction (typical code loading). In the building with a R/C diaphragm the seismic forces are mainly resisted by the walls parallel to the excitation, i.e., the structural elements resist the seismic action in their strong direction, while the out-of-plane displacements of the walls are negligible, as indicated by the displacement pattern on the left of Fig. 7. In the case of a relatively light and flexible floor, such as the wooden one, which does not impose a diaphragm constraint on the structure, out-of-plane deformations of the walls are rather significant, as shown on the right of Fig. 7. It is noted that the present analysis was based on the assumption that intersecting walls of the building always remain tied together, which is not necessarily the case in old buildings with poor detailing of corners and without ring beams or tie rods; in fact, separation

at the corners is quite common in these structures, and drastically changes their collapse mechanism.

The displacements of the building calculated using the different models (see Table 2) are summarized in Fig. 8. It is clear that, depending on the floor system, displacements can vary drastically, a somewhat surprising result that needs some explanation. The first reason for the discrepancy is that the heavier R/C floor significantly increases the total mass of the structure (by a factor of about 2 in the building studied, considering both the floors and
the roof as either wooden or concrete); this leads to a corresponding doubling of the inertial (seismic) forces, calculated from the Eurocode 8 (CEN TC 250 1994) design spectrum for a design acceleration of 0.24 g and a behavior factor (response modifiction factor) q=1.5. A further reason refers to the distribution of seismic forces along the structure, an issue that becomes crucial

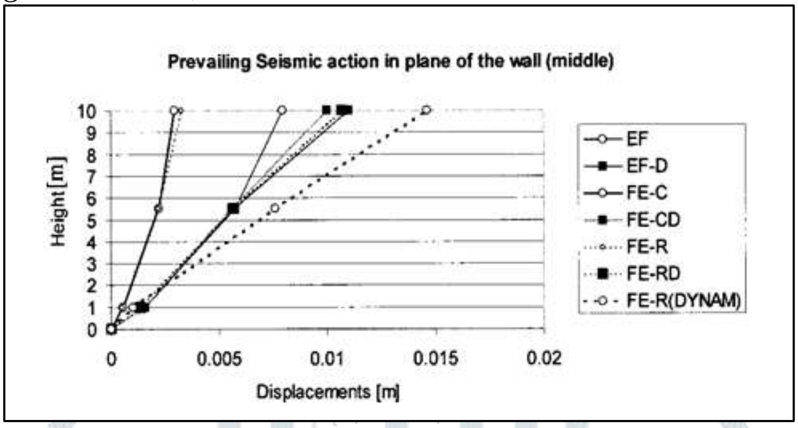


Fig. 8. Displacements at story centers for 3D structure, calculated using different equivalent frame (EF)and finite element (FE) models see model nomenclature in Table 2

As a result, each element must have an inertial load given to it (the product of the element mass and seismic acceleration), which results in an essentially uniform load pattern rather than the "triangular" one required by code procedures. The top displacement of the cantilever subjected to a triangular pattern of concentrated (at floor levels) forces is found to be 1.9 times the corresponding displacement calculated using a uniformly distributed loading that gives the same base shear in the current analysis. This problem was addressed by calculating the equivalent uniform load that yields the same base shear as the desired triangular load pattern in a simple cantilever. This combined result of the significantly higher mass and the various load distribution, which causes the dramatically variable displacement values depicted in Figure 8 (Fig.).

Although it is frequently used, using a mass-proportional ("uniform") load distribution to describe the real seismic loading in a structure without floor diaphragms may not be precise enough. Since the seismic forces in the static situations were calculated using the design spectrum, a modal dynamic analysis of the FE model with the wooden floor (last row in Table 2) was also performed. The findings in Fig. 8 demonstrate how unreliable and unconservative the aforementioned assumption is, making a modal dynamic analysis with a realistic mass distribution preferable.
the impact of model type is concerned

According to calculated displacements, as depicted in Fig. 8, the equivalent 3D frame model with the full rigid offsets produces displacements that are very similar to the refined FE model in the case with a diaphragm constraint at each floor level. However, the difference is significant in the case without a diaphragm, for the reasons previously discussed.

The observation made in the previous section (2D analysis) regarding the degree of refinement in FE modelling, that reasonable estimates of displacements can be obtained with relatively coarse meshes, is even more valid in the 3D case, as the computational cost involved in refining the mesh is not justified by the marginally improved accuracy in the results (see Fig. 8). Actually, keeping in mind

Any attempt to over-refine the finite element model appears to be a fruitless exercise given the uncertainties related to the seismic input and the fact that the properties of masonry, which is a laddered, nonlinear, heterogeneous material, are expressed in terms of a (constant) Young's modulus and a shear modulus.

Nonlinear Static Analysis of Perforated Wall

The pushover curves for the perforated masonry wall calculated from two different methods, one with a finite element model and

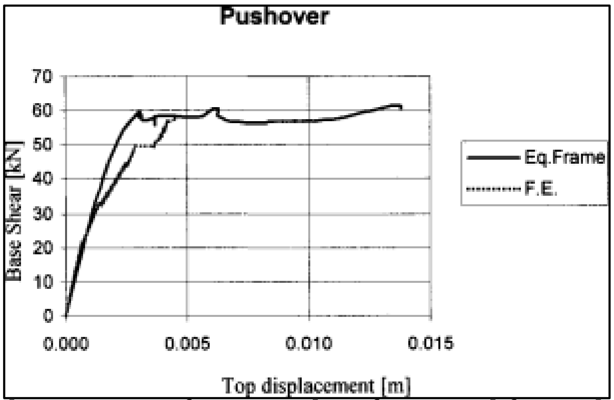


Fig. 9. Pushover curves for equivalent frame and finite element models

Fig. 9 displays one equivalent frame model and the other. Both the initial stiffness and the ultimate strength are in good agreement, but since the FE analysis with ANSYS was force controlled and the equivalent frame analysis with SAP2000 was displacement controlled, there is no meaningful way to compare the final displacement.

The failure mode of the structure can also be predicted using inelastic analysis, either by graphing the broken or crushed parts in the FE analysis or by displaying the state of each plastic hinge in the equivalent frame analysis. Fig. 10(a) displays a "snapshot" of the structure at the analysis's final stage (base shear 57.7 kN). Typical of a wall of this size design, high tensile stresses mostly form at the spandrel edges (where there is a low shear span ratio), which causes cracking and ultimately results in solution instability. Fig. 10(b) displays the distribution of "plastic hinges" throughout the height of the wall. The damage patterns from the two assessments are generally comparable, with the major failure occurring at the spandrels in both cases.

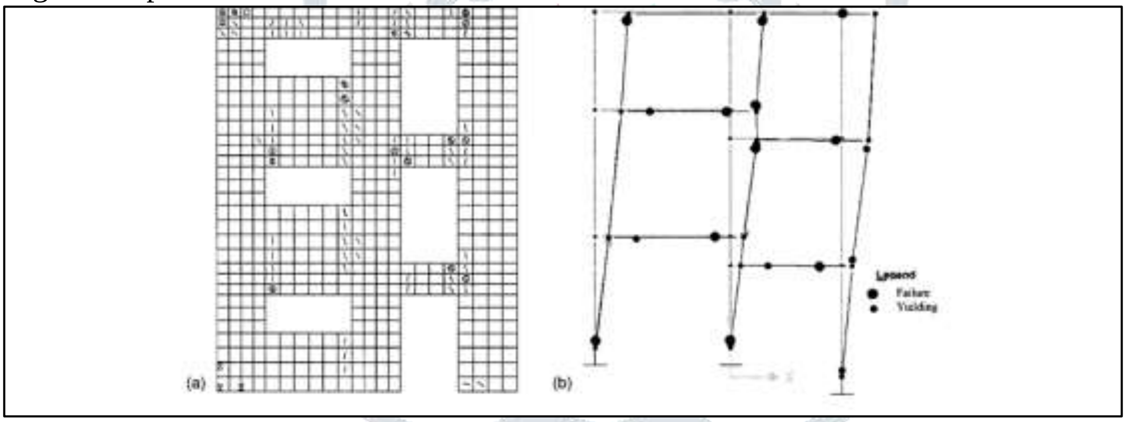


Fig. 10. Distribution of damage in wall; (a) cracked pattern (only tensile cracks) from FE pushover analysis, (b) failure mode at last step of equivalent frame pushover analysis

As a further comparison between simplified and rigorous procedures, the available overstrength factor, defined as the ratio of the ultimate to design load, was considered, using the procedure described in the previous section. The ultimate moment versus axial force interaction diagrams at the base of the wall were calculated considering all the cross sections of the base and the most critical one was found to be the left pier of the ground floor. The resulting axial force (N) –bending moment (M) interaction diagram along with the M ,N pair calculated from elastic analysis ofthe equivalent frame is shown in Fig. 11. On the basis of this diagram, the over strength factor was estimated to be 1.3. Theover strength factor calculated on the basis of the nonlinear FE

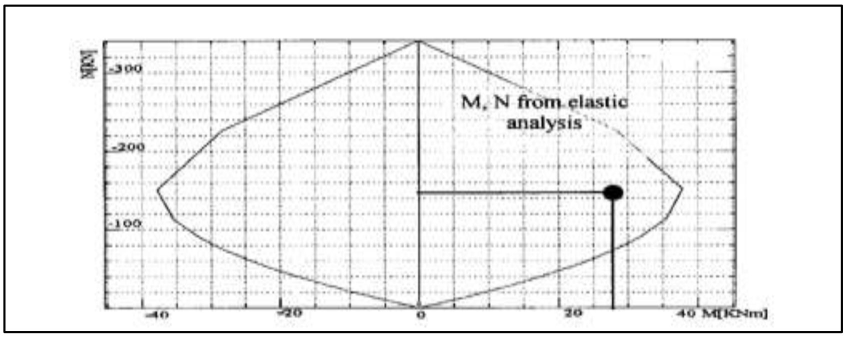


Fig. 11. Bending moment versus axial force interaction diagram at most critical cross section of wall

analysis was 0.97 (57.7 kN) and the over strength factor calculated from the equivalent frame nonlinear analysis was 1.00 (60 kN) as seen from the pushover curves of Fig. 9. The difference between section analysis and equivalent frame analysis is due to the fact that the frame model fails in the spandrels, whereas section analysis focused on the ultimate resistance of the piers. The reason for not performing the same type of section analysis for the spandrels is that these are composed of two different members from different materials (masonry supported by a wooden or R/C beam) having no firm interconnection. In general, the masonry part of the spandrel fails in a brittle fashion whenever masonry strength is exceeded but retains a substantial residual strength due to the supporting beam. Subject to the previous limitations, the correlation between the nonlinear FE analysis and the elastic analysis using an equivalent frame (combined with a strength calculation at the critical section) may be deemed satisfactory for practical purposes. In either case special attention should be paid to the forces and deformations imposed on the spandrels.

It should be noted that the various nonlinear methods are presented in a decreasing cost of analysis order,i.e.,starting from the most time consuming (FE analysis with *ANSYS*, requiring 30 min or more on a PC) to the least time consuming (equiv. frame analysis with *SAP2000*, requiring about 3 min).

Conclusions

The finding that an elastic frame model with full vertical and horizontal stiff offsets was found to be precise enough to be employed for the analysis of unreinforced masonry buildings is perhaps the most significant conclusion from the perspective of practical application.

Diaphragms should only be taken into account in 3D models since they act in the plane perpendicular to the walls, negligibly affecting 2D models while having a significant impact on 3D models' ability to simulate the behaviour of a structure as a whole. According to the results of the current study, particular caution must be used when loading patterns for models with and without a diaphragm.

It does not seem that the finite element modelling mesh refinement is vital aspect for the overall behaviour, however it's important to pinpoint significant areas where the meshing needs to be locally modified to capture stress or strain concentrations (such short spandrels and piers), primarily based on engineering judgement.

Finally, this study has demonstrated that it is possible to do an efficient and reasonably accurate nonlinear analysis of masonry buildings using software that is readily accessible on the market. This finding is crucial for evaluating URM buildings that are already in existence.

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