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EFFECT OF MHD FLOWS DUE TO SECOND GRADE FLUID OVER A ROATING POROUS DISK

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Abstract

The aim of the present paper is to study the effect of MHD flows due to second grade fluid over a rotating porous disk .The nonlinear problem is solved analytically using Laplace transform technique and inverse Laplace transform techniques. The effects of all the parameters on the flow are carefully examined. The graphical result are found using MATLAB. It is observed that increase of second grade parameter causes an increase in the boundary layer thickness and it is clearly examined that an increase of the magnetic parameter results the decrease in the boundary layer thickness. It is also found that real and imaginary part of velocity profiles first decreased and then increased by increasing the second grade parameter.

Keywords: Second grade parameter, Magnetic parameter, Porosity parameter.

INTRODUCTION

The theoretical study of the flow near a rotating disk of infinite extent can be traced back to Von Karma's similarity analysis. That is why the flow is widely known as Von karman's flow. He assumed that the flow possessed axial symmetry, and introduced a similarity transformation which reduced the Navier – Stokes equation into a system of coupled nonlinear ordinary differential equations. These equations have been used as a test problem for numerical methods and in the study of matched asymptotic expansions. This problem has received considerable attention over the years and different extensions of Von Karman's swirling flow problem have been made to address various applications, for instance Benton (1966); Kuiken (1971); Riley (1964); Sahoo (2009); Ariel (2003);

The viscous laminar flow between porous disks has recently been studied by several authors. Elkouh (1969) have obtained the solutions of laminar flow between non-rotating and rotating porous disks with equal suction/injection through porous discs. Gaur (1972) has discussed the viscous incompressible fluid flow between two infinite porous rotating discs. Narayana (1972) has considered the steady flow of a Newtonian fluid between two infinite parallel discs when one disc (upper) is rotating and other disc (lower) is at rest with uniform suction at the stationary disc. Rudraiah et. al. (1974) has studied a singular perturbation problem of non – Newtonian fluid flow between porous discs. Rudraiah et. al. (1974) obtained the solutions for both small and large values of cross-flow Reynolds number by regular perturbation and matched asymptotic expansions technique, respectively. Sacheti and Bhatt (1975) have discussed the steady laminar flow of a non Newtonian fluid with suction / injection through disks and heat transfer through parallel disks.

However, the possibility of an exact solution for the flow due to a rotating disk in a fluid which is at infinity and its rotating disk in a fluid which is at infinity and is rotating rigidly has been implied by Berker (1982). Parter and Rajagopal (1984) have established the existence solutions which do not possess axial symmetry, to the Navier─Stokes equations for the problem governing the flow of infinite disks rotating about a common axis. Based on that work, Huilgol & Rajagopal (1987) have shown that in the case of certain non-Newtonian fluid models, solutions that lack axisymmetry are possible. Recently, Turkyilmazoglu (2009) has obtained exact solutions to the Navier ─ Stokes for the swirling flow problem in such a way that the physical quantities are allowed to develop non-axisymmetrically over a rotating disk.

It is a well-known fact that the Navier – Stokes equations seem to be a weak model for a class of real fluids, called non- Newtonian fluid. During the last few decades, considerable efforts have been developed to the study of flow on non – Newtonian fluids because of their technological applications. A vast amount of literature is now available for the flow problems associated with non Newtonian fluids in a variety of situations. One important and simple model of non-Newtonian fluids for which one can reasonably hope to obtain analytical solutions is the second grade fluid. The study of fluid flowing between parallel porous / non – porous disks is of practical importance in the design of thrust bearings, radial diffusers, etc.

Magnetohydrodynamics (MHD) is an academic discipline, which studies the dynamic behaviours of the interaction between magnetic fields and electrically conducting fluids. Examples of such fluids are numerous including plasmas, liquid metals, and salt water or electrolytes. The MHD flow is encountered in a variety of applications such as MHD power generators, MHD pumps, MHD accelerators, and MHD flow meters and it can also be expanded into various industrial uses.

During the past decades, a great deal of papers in literatures used a combination of Navier – Stokes equations and Maxwell's equations to describe the MHD flow of the Newtonian and electrically conducting fluid. Sayed-Ahmed and Attia (1998) studied the unsteady Couette flow and heat transfer of a dusty conducting fluid between two parallel plates with variable viscosity and electrical conductivity. Osalusi et. al. (2007) solved unsteady MHD and slip over a porous rotating disk in the presence of Hall and ion- slip currents a shooting method.

However, the Newtonian fluid is the simplest to be solved and its application is very limited. In practice, many complex fluids such as blood, suspension fluids, certain oils, greases, and polymer solutions, elastomers, and many emulsions have been treated as non –Newtonian fluids.

From the literature, the non – Newtonian fluids principally classified on the basis of their behaviour in shear. A fluid with a linear relationship between the shear stress and the shear rate, giving rise a constant viscosity, is always characterized to be a Newtonian fluid. Based on the knowledge of solutions to Newtonian fluid, the different fluids can be extended, such as Maxwell fluids, Voigt fluids, Oldroyd − B fluid, Rivlin − Ericsen fluids, and power-law fluids.

Based upon the previous studies, this chapter is extended for the flow characteristics of the MHD flow of second grade fluid. MHD flow due to eccentric rotations of a porous disk and an oscillating second grade fluid at infinity is studied when the disk and the fluid at infinity rotate with same angular velocity. The effects of material parameter of second grade fluid on the velocity profiles are discussed. The effects of all the parameters on the flow are carefully examined. The results which have found are reported for conclusion.

MATHEMATICAL FORMULATION

The flow of an incompressible second grade fluid, neglecting thermal effects and body forces is given by

$$\mathrm{div}\, \mathrm{v} = 0$$

$$\rho \frac{\mathrm{dv}}{\mathrm{dt}} = \mathrm{divT}$$

Where the Cauchy stress tensor T in an incompressible and Rivlin – Ericsen fluid of second grade is related to the fluid motion in the following manner (Rivlin and Ericksen

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2$$

$$A_1 = (\mathrm{grad}V) + (\mathrm{grad}V)^T$$

$$A_2 = \frac{d}{dt}A_1 - A_1(\mathrm{grad}V) + (\mathrm{grad}V)^T A_1$$

Here V is the velocity vector field, p is the fluid pressure, ρ the constant fluid density, μ the constant coefficient of viscosity, $\frac{d}{dt}$ the material time derivative and α_1 and α_2 the normal stress moduli.

According to Dunn and Fosdick the second grade fluid model is compatible with thermodynamics when the Helmholtz free energy of the fluid is minimum for the fluid in equilibrium. The fluid model then has general and pleasant roundedness and stability properties. The Clausius – Duhem inequality and the assumption that the Helmholtz free energy is minimum in equilibrium provided the following restrictions

$$\mu \geq 0, \qquad \alpha_1 \geq 0, \qquad \alpha_1 + \alpha_2 = 0$$

Here the flow is dealt with second grade fluid flow, the strict inequality holds true.

Fosdick and Rajagopal (1979) have shown that when $\alpha_1 < 0$, the fluid exhibits anomalous behaviour that is incompatible with any fluid of rheological interest, and so results in a fluid that is unstable.

Here, an electrically conducting second grade fluid occupying a space $z > 0$ in contact with an infinite porous disk at z = 0 is considered. The axis of rotation of the disk and that of the fluid at infinity are assumed to be in the plane x = 0. The distance between the axes is being considered as l. The porous disk and the fluid at infinity are initially rotating about the z^1 – axis with the same angular velocity Ω. At time t = 0, suddenly the disk and the fluid at infinity starts rotation about the z – axis with angular velocity Ω. Additionally it is assumed that the fluid at infinity oscillates with frequency k. The fluid is electrically conducting by a magnetic field B_0 applied transversely to the flow.

Since the derivation of equation for the flow of an incompressible fluid when porous boundary and fluid exhibit a state of non-coaxial rotation, the velocity field for such a motion is defined by Erdogan (1997) as

$$u = -\Omega y + f(z,t), \; v = \Omega x + g(z,t)$$

along with the initial and boundary conditions are of the form

$$u = -\Omega y, \qquad v = \Omega x, \qquad \text{as } z = 0, t > 0,$$

$$u = -\Omega y + \Omega l \cos kt, \qquad v = \Omega x, \text{as } z \to \infty, t > 0$$

$$u = -\Omega y + \Omega l, \qquad v = \Omega x, \quad \text{at} \quad z > 0, \; t = 0 \qquad (5.5)$$

which along with equation $\nabla . V = 0$ yields for uniform porosity that

$w = -w_0$ where $w_0 > 0$ *corresponds to the uniform suction velocity and* $w_0 < 0$ *indicates the blowing.*

Making use of equation (5.4) in (5.2) and eliminating the pressure from the resulting equations give

$$\frac{\alpha_1}{\rho}\frac{\partial^3 F}{\partial z^2\,\partial t}+\left(\nu-i\Omega\,\frac{\alpha_1}{\rho}\right)\frac{\partial^2 F}{\partial z^2}-\frac{\alpha_1}{\rho}w_0\frac{\partial^3 F}{\partial z^3}-\left(N_1+i\Omega\right)F-\frac{\partial F}{\partial t}+w_0\frac{\partial F}{\partial z}=\left(i+\frac{N_1}{\Omega}\right)\text{coskt}+\text{ksinkt}$$

With the following boundary and initial conditions

$f(0,t) \ = 0, g(0,t) = 0$

$f(\infty,t) = \ \Omega l\ \text{coskt}, g(\infty,t) = 0$

$f(z,0) \ = \Omega l, g(z,0) = 0$

$F(z,t) = \frac{f}{\Omega l} + i\frac{g}{\Omega l}$.

introducing

$$\xi=\sqrt{\frac{\Omega}{2\nu}}z,\quad \tau=\Omega t,\quad c=\frac{k}{\Omega},\ n=\frac{N_1}{\Omega},\quad \epsilon=\frac{w_0}{\sqrt{2\nu\Omega}},$$

$\alpha = \frac{\alpha_1\Omega}{\nu}$, $v = \frac{\mu}{\rho}$,

Equation (5.7) and conditions (5.8) become

$$\alpha\frac{\partial^3 F}{\partial \xi^2\,\partial \tau}-\alpha\epsilon\frac{\partial^3 F}{\partial \xi^3}+(1-i\alpha)\frac{\partial^2 F}{\partial \xi^2}-2\frac{\partial F}{\partial \tau}+2\epsilon\frac{\partial F}{\partial \xi}-(n+i)F=-2(i+n)\text{cosc}\tau+2\text{csinc}\tau$$

(5.10)

$F(0,\tau) = 0, F(\infty,t) = \text{cosc}\tau, F(\xi,0) = 1$

writing

$F(0,\tau) = H(\xi,\tau)e^{-i\tau}$ (5.12)

The problem containing equations (5.10) and (5.11) become

$$\alpha\frac{\partial^3 H}{\partial \xi^2\,\partial \tau}-\alpha\epsilon\frac{\partial^3 H}{\partial \xi^3}+(1-2i\alpha)\frac{\partial^2 H}{\partial \xi^2}-2\frac{\partial H}{\partial \tau}+2\epsilon\frac{\partial H}{\partial \xi}-2nH=-(i+n)\ X\left[e^{i(1+c)\tau}+e^{i(1-c)\tau}\right]$$

$$-(i+n)\ X\left[e^{i(1+c)\tau}+e^{i(1-c)\tau}\right]$$

$$-ic\left[e^{i(1+c)\tau}-e^{i(1-c)\tau}\right] \quad (5.13)$$

$H(0,\tau) = 0,$

$H(\infty,\tau) = \frac{1}{2}\left[e^{i(1+c)\tau}+e^{i(1-c)\tau}\right],$ (5.14)

$$H(\xi,0) = 1,$$

In order to find the solution of equations (5.13) and (5.14), the Laplace transform pair can be written as

$$\overline{H} = \int_0^\infty H(z,t)\, e^{-st} dt, \quad s > 0,$$

$$H = \frac{1}{2\pi i}\int_0^\infty \overline{H}\,(z,s) e^{st} ds \qquad (5.15)$$

In transformed s- plane, the problem becomes

$$\alpha \in \frac{d^3\overline{H}}{d\xi^3} - (1 - 2i\alpha + \alpha s)\frac{d^2\overline{H}}{\partial\xi^2} - 2\epsilon\frac{dH}{d} - 2(s+n)\overline{H}$$

$$= 2 + 2\,(i+n)$$

$$X\left[\frac{1}{s-i(1+c)} + \frac{1}{s-i(1-c)}\right] + ic\left[\frac{1}{s-i(1+c)} - \frac{1}{s-i(1-c)}\right] \qquad (5.16)$$

$$H(0,s) = 0$$

$$\overline{H}(\infty,s) = \frac{1}{2}\left[\frac{1}{s-i(1+c)} + \frac{1}{s-i(1-c)}\right] \qquad (5.17)$$

Since the equation of (5.16) is of third order, perturbation solution can be obtained by assuming the non – Newtonian fluid parameter to be very small by Beard and walters ( )

Therefore, $\overline{H}$ can be written as

$$\overline{H} = \overline{H}_1 + \alpha\overline{H}_2 + O(\alpha^2) \qquad (5.18)$$

using equation (5.18) into equations (5.16) and (5.17) and then equating the terms of like powers of α, the following systems can be obtained.

System of order zero

$$\frac{d^2\overline{H}_1}{d\xi^2} + 2 \in \frac{d\overline{H}_1}{d\xi} - 2(s+n)\overline{H}_1 = -2 - (i+n)\left[\frac{1}{s-i(1+c)} + \frac{1}{s-i(1-c)}\right]$$

$$-ic\left[\frac{1}{s-i(1+c)} - \frac{1}{s-i(1-c)}\right] \qquad (5.19)$$

$$\overline{H}_1(0,s) = 0$$

$$\overline{H}_1(\infty,s) = \left[\frac{1}{s-i(1+c)} + \frac{1}{s-i(1-c)}\right]$$

(5.20)

System of order one

$$\in \frac{d^3\overline{H}_1}{d\xi^3} - \frac{d^2\,\overline{H}_2}{d\xi^2} - (s-2i)\frac{d^2\overline{H}_1}{\partial\xi^2} - 2\epsilon\frac{d\overline{H}_2}{d\xi} + 2(s+n)\overline{H}_2 = 0, \qquad (5.21)$$

$$\overline{H}_2(0,s) = 0,$$

$$\overline{H}_1(\infty,s) = 0 \qquad (5.22)$$

Zeroth order solution

The solution of Equations (5.42) that satisfies the boundary conditions (5.45) is

$$\overline{H}_1 = \frac{1}{2}\left[\frac{1}{s-i\,(1+c)}+\frac{1}{s-i\,(1-c)}\right] + e^{-\left[\epsilon+\sqrt{\epsilon^2+2(s+n)}\right]\xi} + \frac{1}{2}\left[\frac{1}{s-i\,(1+c)}+\frac{1}{s-i\,(1-c)}\right] \qquad (5.23)$$

Taking Laplace inversion for the above solution $\overline{H}_1$ becomes

$$H_1(\xi,t) = -\frac{1}{4}\begin{pmatrix} e^{-(\epsilon+x_1^*)\xi+i(1+c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_1^*\sqrt{\frac{\tau}{2}}\right] \\ +e^{-(\epsilon-x_1^*)\xi+i(1+c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_1^*\sqrt{\frac{\tau}{2}}\right] \\ +e^{-(\epsilon+x_1^*)\xi+i(1-c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_1^*\sqrt{\frac{\tau}{2}}\right] \\ +e^{-(\epsilon+x_1^*)\xi+i(1-c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_1^*\sqrt{\frac{\tau}{2}}\right] \end{pmatrix}$$

$$+\frac{1}{2}\left(e^{i(1+c)\tau}+e^{i(1-c)\tau}\right) \qquad (5.24)$$

First order solution

Using the zeroth order solution (5.46) into equations (5.44), the solution can be written as

$$\frac{d^2\overline{H}_1}{d\xi^2} + 2\epsilon\frac{d\overline{H}_2}{d\xi} - 2(s+n)\overline{H}_2 = -[\epsilon G^3 + (s-2i)G^2]Ae^{-G\xi}, \qquad (5.25)$$

where

$$A = \frac{1}{2}\left[\frac{1}{s-i(1+c)}+\frac{1}{s-i(1+c)}-\frac{1}{s-i(1-c)}-\frac{1}{s-i(1-c)}\right] \qquad (5.26)$$

$$G = \epsilon + \sqrt{\epsilon^2 + 2(s+n)} \qquad (5.27)$$

The solution of the first order system is

$$\overline{H}_2 = A(\epsilon G^3 + (s-i)G^2)\frac{\xi e^{-\left(\epsilon+\sqrt{\epsilon^2+2(s+n)}\right)\xi}}{\sqrt{\epsilon^2+2(s+n)}} \qquad (5.28)$$

Taking inverse Laplace transform of above equation,

$$H_2(\xi,t)= \frac{\xi}{4}\begin{pmatrix} \left\{\frac{1}{X_1^*}(A_1+A_2 i(1+c)+(1+c)^2)-A_3-A_4 i(1+c)\right\} \\ X\, e^{-(\in+X_1^*)\xi+ic\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_1^*\sqrt{\frac{\tau}{2}}\right] \\ -\left\{\frac{1}{X_1^*}(A_1+A_2 i(1+c)+(1+c)^2)-A_3-A_4 i(1+c)\right\} \\ X\, e^{-(\in-X_1^*)\xi+ic\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_1^*\sqrt{\frac{\tau}{2}}\right] \\ -\left\{\frac{1}{X_2^*}(A_1+A_2 i(1-c)+(1-c)^2)+A_3+A_4 i(1-c)\right\} \\ X\, e^{-(\in+X_1^*)\xi+ic\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_1^*\sqrt{\frac{\tau}{2}}\right] \\ \left\{\frac{1}{X_2^*}(A_1+A_2 i(1-c)+(1-c)^2)-A_3-A_4 i(1-c)\right\} \\ X\, e^{-(\in-X_2^*)\xi+ic\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_2^*\sqrt{\frac{\tau}{2}}\right] \end{pmatrix}$$

$$-\frac{\xi}{2}\left[\frac{2A_2+\in^2+2n}{\sqrt{2\pi\tau}}+\frac{2\xi A_4-\xi^2}{\sqrt{2\pi\tau^3}}+\frac{1-2i}{\sqrt{2\pi\tau^5}}\right]e^{-\in\xi-(\in^2+2n)\frac{\tau}{2}-\frac{\xi^2}{2\tau}} \qquad (5.29)$$

The analytical solution of the problem up to the order α can be written as

$$H(\xi,t)= -\frac{1}{4}\begin{pmatrix} \left\{1+\alpha\xi\left(\frac{1}{X_1^*}(A_1+A_2 i(1+c)+(1+c)^2)-(A_3+iA_4 i(1+c))\right)\right\} \\ X\, e^{-(\in+X_1^*)\xi+i(1+c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_1^*\sqrt{\frac{\tau}{2}}\right] \\ +\left\{1-\alpha\xi\frac{1}{X_1^*}\left((A_1+A_2 i(1+c)+(1+c)^2)-A_3-A_4 i(1+c)\right)\right\} \\ X\, e^{-(\in-X_1^*)\xi+i(1+c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_1^*\sqrt{\frac{\tau}{2}}\right] \\ +\left\{1+\alpha\xi\left(\frac{1}{X_1^*}(A_1+A_2 i(1-c)+(1-c)^2)+A_3+A_4 i(1+c)\right)\right\} \\ X\, e^{-(\in+X_1^*)\xi+i(1-c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_2^*\sqrt{\frac{\tau}{2}}\right] \\ +\left\{1-\alpha\xi\left(\frac{1}{X_1^*}(A_1+A_2 i(1-c)+(1-c)^2)-A_3-A_4 i(1-c)\right)\right\} \\ X\, e^{-(\in-X_1^*)\xi+i(1+b)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_2^*\sqrt{\frac{\tau}{2}}\right] \end{pmatrix}$$

$$-\alpha\xi\left[\frac{2A_2+\in^2+2n}{\sqrt{2\pi\tau}}+\frac{2\xi A_4-\xi^2}{\sqrt{2\pi\tau^3}}+\frac{1-2i}{\sqrt{2\pi\tau^5}}\right]e^{-\in\xi-(\in^2+2n)\frac{\tau}{2}-\frac{\xi^2}{2\tau}}+$$

$$\frac{1}{2}\left(e^{i(1+c)\tau}+e^{i(1-c)\tau}\right) \qquad (5.30)$$

where

$$A_1=2\in^4+(3n-2i)\in^2-2in,\quad A_2=4\in^2+n-2i, \qquad (5.31)$$

$$A_3=4\in^3+\in n-2i\in,\qquad A_4=2\in,$$

Using the equation (5.18) and (5.9) into equation (5.30), the following suction solution is obtained.

for $0 < c < 1$

$$\frac{f}{\Omega l}+i\frac{g}{\Omega l}=-\frac{1}{4}\left(\begin{array}{c}\left\{1+\alpha\xi\left(\frac{1}{X_1^*}(A_1+A_2 i(1+c)+(1+c)^2)-\left(A_3+iA_4 i(1+c)\right)\right)\right\}\\ X\, e^{-(\epsilon+X_1^*)\xi+i(1+c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_1^*\sqrt{\frac{\tau}{2}}\right]\\ +\left\{1-\alpha\xi\frac{1}{X_1^*}\left((A_1+A_2 i(1+c)+(1+c)^2)-A_3-A_4 i(1+c)\right)\right\}\\ X\, e^{-(\epsilon-X_1^*)\xi+i(1+c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_1^*\sqrt{\frac{\tau}{2}}\right]\\ +\left\{1+\alpha\xi\left(\frac{1}{X_1^*}(A_1+A_2 i(1-c)+(1-c)^2)+A_3+A_4 i(1+c)\right)\right\}\\ X\, e^{-(\epsilon+X_1^*)\xi+i(1-c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_2^*\sqrt{\frac{\tau}{2}}\right]\\ +\left\{1-\alpha\xi\left(\frac{1}{X_1^*}(A_1+A_2 i(1-c)+(1-c)^2)-A_3-A_4 i(1-c)\right)\right\}\\ X\, e^{-(\epsilon-X_1^*)\xi+i(1+b)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_2^*\sqrt{\frac{\tau}{2}}\right]\end{array}\right)$$

$$-\alpha\xi\left[\frac{2A_2+\epsilon^2+2n}{\sqrt{2\pi\tau}}+\frac{2\xi A_4-\xi^2}{\sqrt{2\pi\tau^3}}+\frac{1-2i}{\sqrt{2\pi\tau^5}}\right]e^{-\epsilon\xi-(\epsilon^2+2n)\frac{\tau}{2}}-\frac{\xi^2}{2\tau}-i\tau$$

$$+\frac{1}{2}\left(e^{ic\tau}+e^{-ic\tau}\right), \qquad (5.32)$$

for $c > 1$; the suction solution is of the following form

$$\frac{f}{\Omega l}+i\frac{g}{\Omega l}=-\frac{1}{4}$$

$$\left(\begin{array}{c}\left\{1+\beta\xi\left(\frac{1}{X_1^*}(A_1+A_2 i(1+c)+(1+c)^2)-\left(A_3+iA_4 i(1+c)\right)\right)\right\}\\ X\, e^{-(\epsilon+X_1^*)\xi+i(1+c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_1^*\sqrt{\frac{\tau}{2}}\right]\\ +\left\{1-\beta\xi\frac{1}{X_1^*}\left((A_1+A_2 i(1+c)+(1+c)^2)-A_3-A_4 i(1+c)\right)\right\}\\ X\, e^{-(\epsilon-X_1^*)\xi+i(1+c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_1^*\sqrt{\frac{\tau}{2}}\right]\\ +\left\{1+\beta\xi\left(\frac{1}{Y_1^*}(A_1-A_2 i(1-c)+(1-c)^2)+A_3+A_4 i(1+c)\right)\right\}\\ X\, e^{-(\epsilon+Y_1^*)\xi+i(1-c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-Y_1^*\sqrt{\frac{\tau}{2}}\right]\end{array}\right)$$

$$-\beta\xi\left[\frac{2A_2+\epsilon^2+2n}{\sqrt{2\pi\tau}}+\frac{2\xi A_4-\xi^2}{\sqrt{2\pi\tau^3}}+\frac{1-2i}{\sqrt{2\pi\tau^5}}\right]e^{-\epsilon\xi-(\epsilon^2+2n)\frac{\tau}{2}}-\frac{\xi^2}{2\tau}-i\tau$$

$$+\frac{1}{2}\left(e^{ic\tau}+e^{-ic\tau}\right) \qquad (5.33)$$

For the resonant case i.e c = 1, we have

$$\frac{f}{\Omega l}+\frac{g}{\Omega l}=\begin{pmatrix}\left\{1+\alpha\xi\left(\frac{1}{Z_1}\left(A_1+2iA_2+4\quad\right)-A_3-2iA_4\right)\right\}\\ X\,e^{-(\in+Z_1)\xi+i\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-Z_1\sqrt{\frac{\tau}{2}}\right]\\ +\left\{1-\alpha\xi\frac{1}{Z_1}\left((A_1+A_2 2i+4)-A_3-A_4 i2\right)\right\}\\ X\,e^{-(\in+Z_1)\xi+i\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+Z_1\sqrt{\frac{\tau}{2}}\right]\\ +\left\{1+\alpha\xi\left(\frac{A_1}{Z_2}+A_3\right)\right\}\\ X\,e^{-(\in-Z_2)\xi-i(1+c)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-Z_2\sqrt{\frac{\tau}{2}}\right]\\ +\left\{1-\alpha\xi\left(\frac{A_2}{Z_2}+A_3\right)\right\}\\ X\,e^{-(\in+Z_2)\xi-i\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+Z_2\sqrt{\frac{\tau}{2}}\right]\end{pmatrix}$$

$$-\beta\xi\left[\frac{2A_2+\in^2+2n}{\sqrt{2\pi\tau}}+\frac{2\xi A_4-\xi^2}{\sqrt{2\pi\tau^3}}+\frac{1-2i}{\sqrt{2\pi\tau^5}}\right]e^{-\in\xi-\left(\in^2+2n\right)\frac{\tau}{2}-\frac{\xi^2}{2\tau}-i\tau}$$

$$+\frac{1}{2}\left(e^{ic\tau}+e^{-ic\tau}\right), \qquad (5.34)$$

The blowing solution can be obtained by replacing $\in$ by $-\in$ in equations (5.32), (5.33) and (5.34).

5.3 RESULTS AND DISCUSSION

The effects of various parameters on the velocity profiles are drawn through graphs.

The variation of second grade parameter α, magnetic parameter n, porosity parameter $\in$, frequency a and time τ are shown by graphs.

- The real and imaginary parts of the velocity for various values of second grade parameter $\alpha = 0.0, 1.0, 2.0$ are presented in figures.5.3.1 to 5.3.6.
- Figures 5.3.1 and 5.3.2 indicates that $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ first decrease and then increases by increasing α. However in figures 5.3.3 and 5.3.4, $\frac{f}{\Omega l}$ first decreases then increases and $\frac{g}{\Omega l}$ increases.
- In Figures 5.3.5 and 5.3.6 $\frac{f}{\Omega l}$ decreases and $\frac{g}{\Omega l}$ first decreases and then increases by increasing α causes an increase in the boundary layer thickness.
- The effects of magnetic parameter n = 0, 1, 2 are shown in figures 5.3.7 to 5.3.12. It is noted from the graphs that $\frac{f}{\Omega l}$ increase and $\frac{g}{\Omega l}$ decreases by increasing n. Further it is evident from these figures that increase in n, shows the decrease in boundary layer thickness.

- The variation of porosity parameter $\in = -1,0,1$ can be observed from figures 5.3.13 to 5.3.18. These Figures show that large values of $\in$ causes an increase in $\frac{f}{\Omega l}$ and decrease in $\frac{g}{\Omega l}$. The boundary layer thickness decreases by increasing suction and increases for large values of blowing velocity.
- Figures 5.3.19 to 5.3.24 indicates the influence of $\tau = 0.25, 0.5, 0.75$ on the velocity profiles. Obviously $\frac{f}{\Omega l}$ decreases and $\frac{g}{\Omega l}$ increases when large values of τ are taken into account.

The variation of the velocity field with distance from the disk for various values of second grade parameter $\boldsymbol{\alpha}$ when c = 1.5, $\in = \mathbf{0}$, n=0, and $\tau = 1$

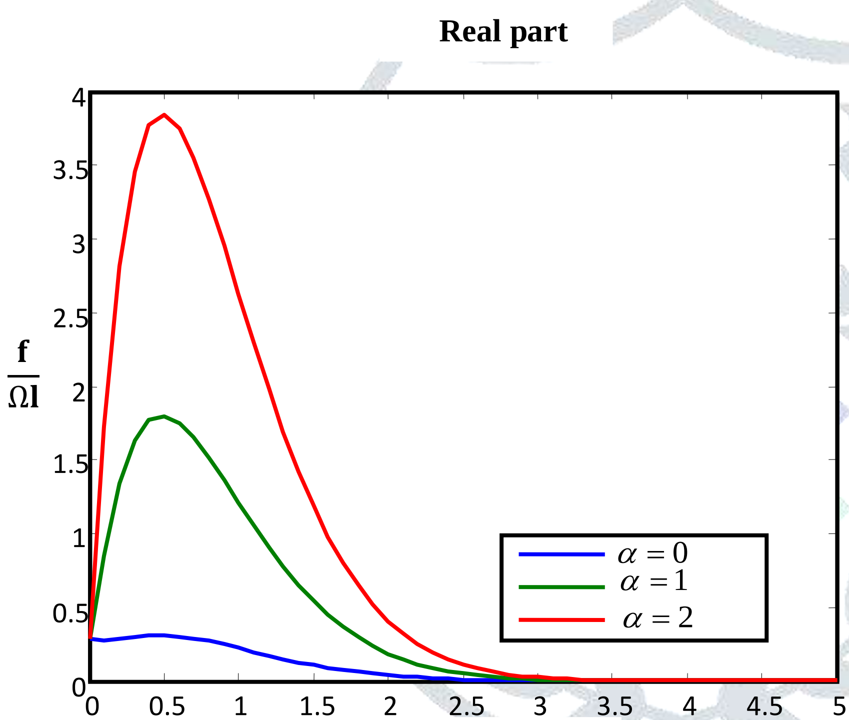

Figure 5.3.1

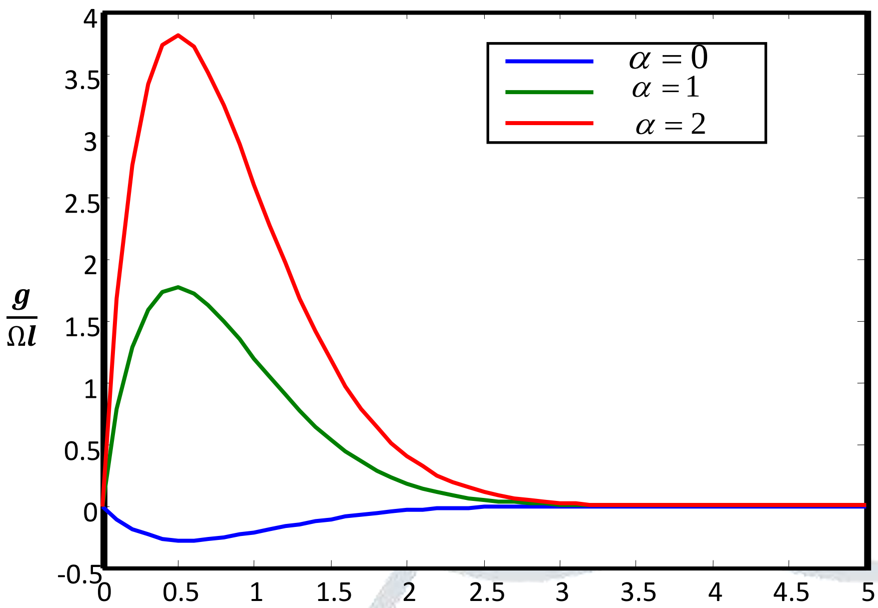

Figure 5.3.2

The variation of the velocity field with distance from the disk for various values of second grade parameter α when c = 0.4, $\in = 0$, n=0, and $\tau = 1$

Real part

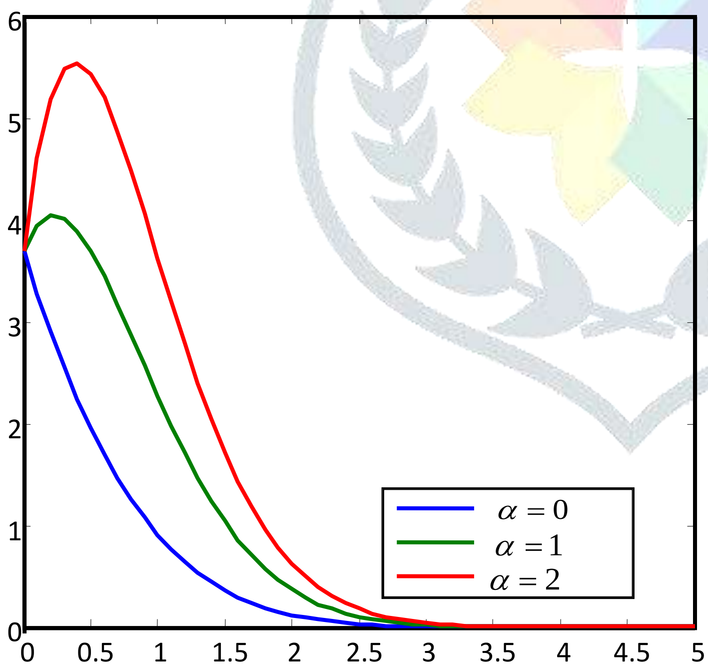

Figure 5.3.3

Imaginary part

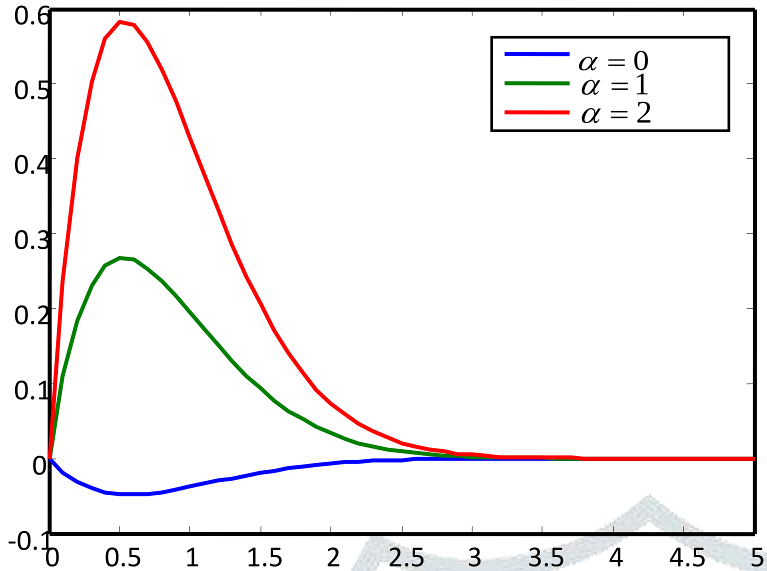

Figure 5.3.4

The variation of the velocity field with distance from the disk for various values of second grade parameter α when c = 1, $\in = 0.4$, n=0, and $\tau = 1$

Real part

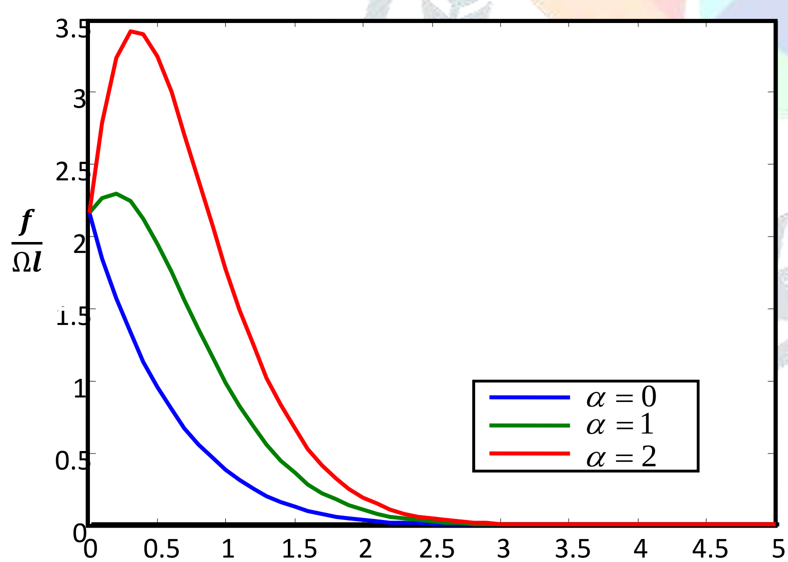

Figure 5.3.5

Imaginary Part

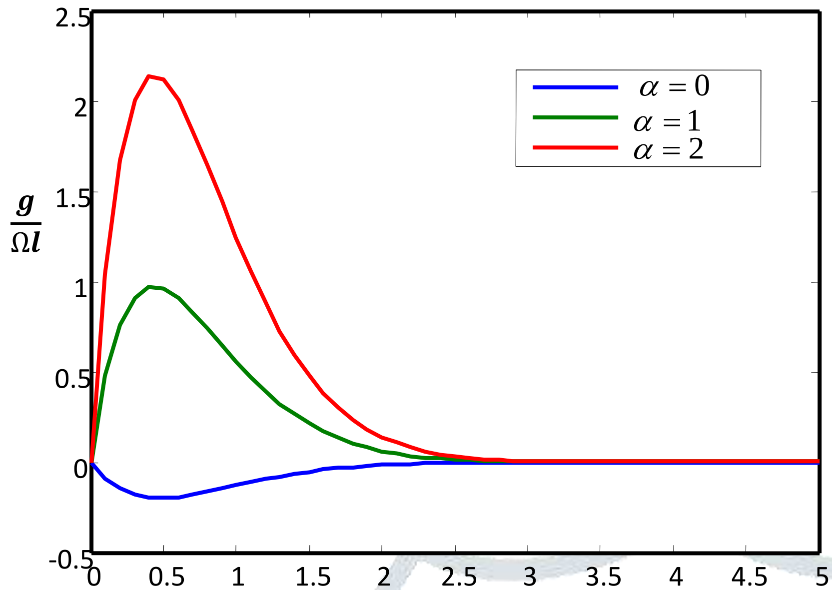

Figure 5.3.6

The variation of the velocity field with distance from the disk for various values of magnetic parameter n when c = 1.4, $\in = 0$, $\alpha = 0.08$ and τ=1

Real part

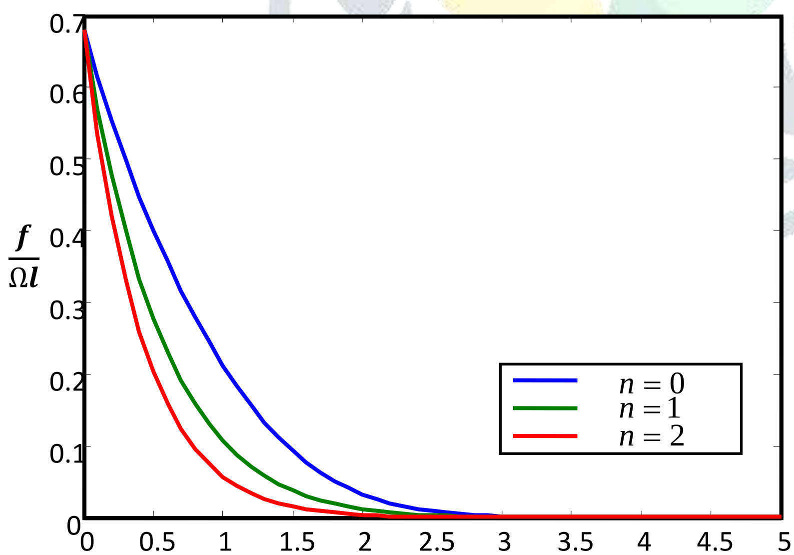

Figure 5.3.7

Imaginary Part

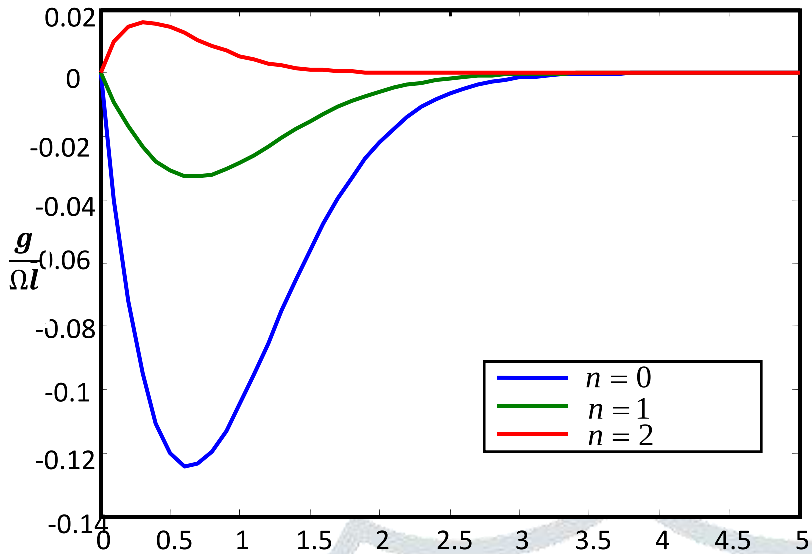

Figure 5.3.8

The variation of the velocity field with distance from the disk for various values of magnetic parameter n when c = 0.4, $\in = 0$**,** $\alpha = 0.08$ **and** $\tau = 1$

Real part

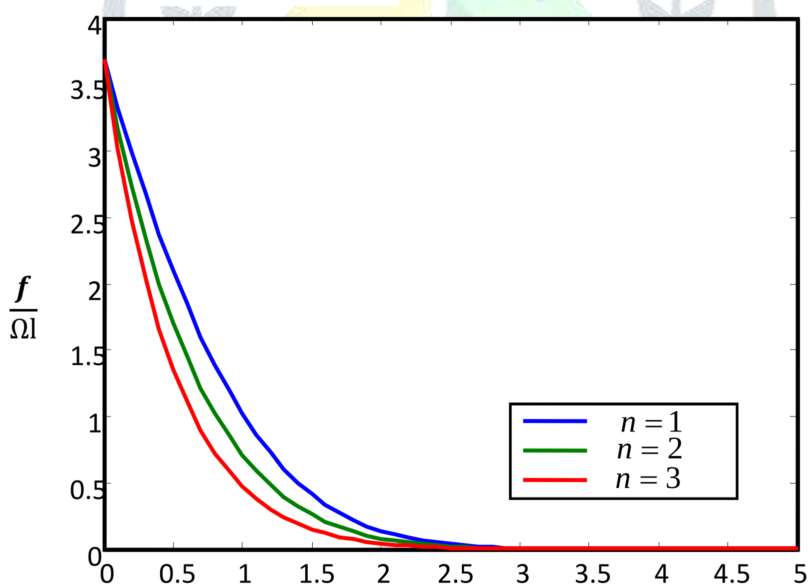

Figure 5.3.9

Imaginary Part

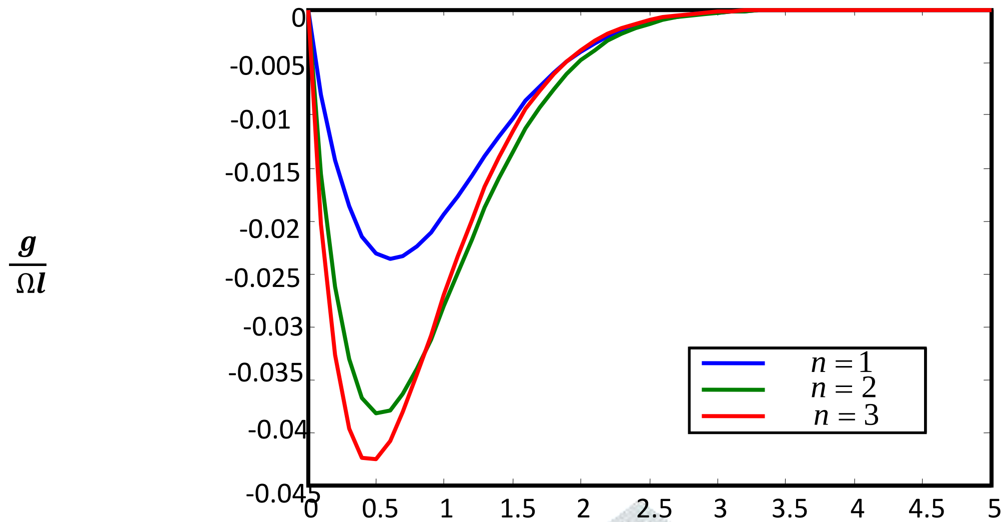

Figure 5.3.10

The variation of the velocity field with distance from the disk for various values of magnetic parameter n when c =1, $\in = 0$, $\boldsymbol{\alpha = 0.08}$ **and** $\boldsymbol{\tau}$**=1**

Real part

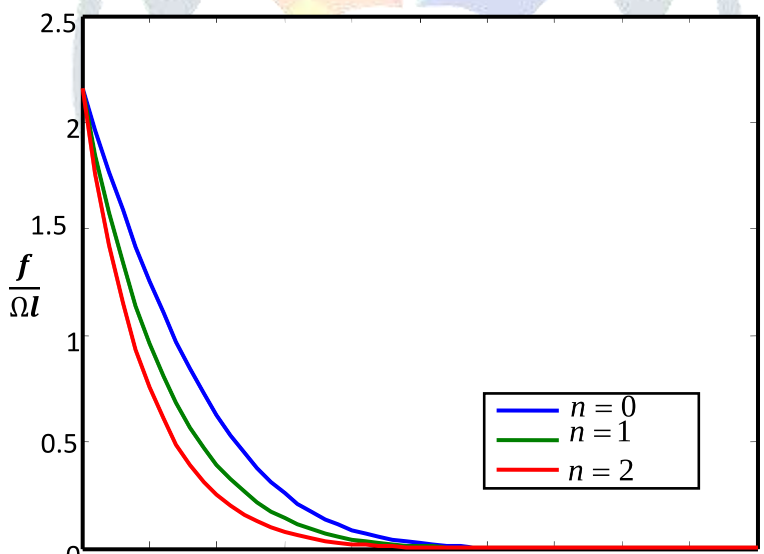

Figure 5.3.11

Imaginary Part

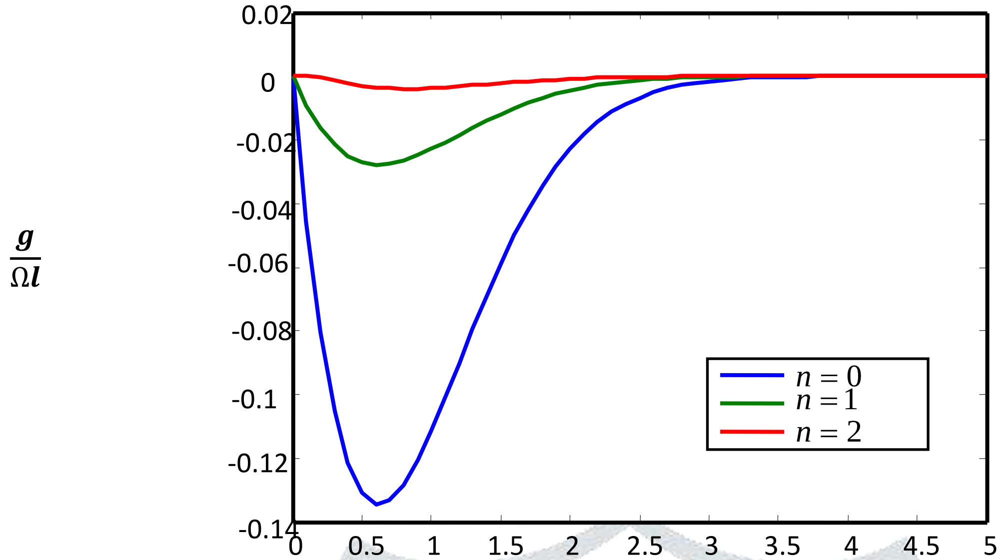

Figure 5.3.12

The variation of the velocity field with distance from the disk for various values of blowing/suction parameter ∈ when c = 1.4, $\in = 0$**,** $\alpha = 0.08$ **and** $\tau = 1$

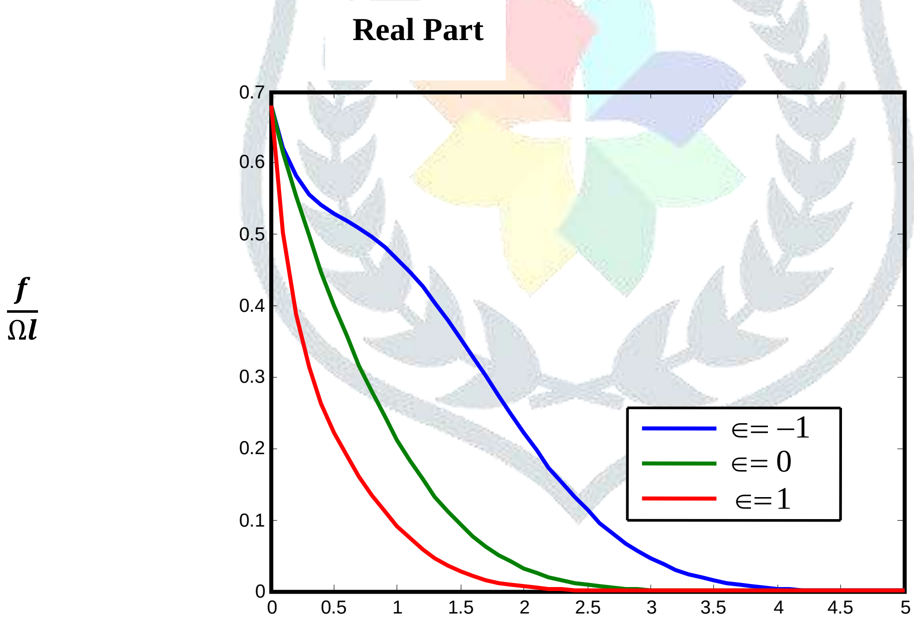

Figure 5.3.14

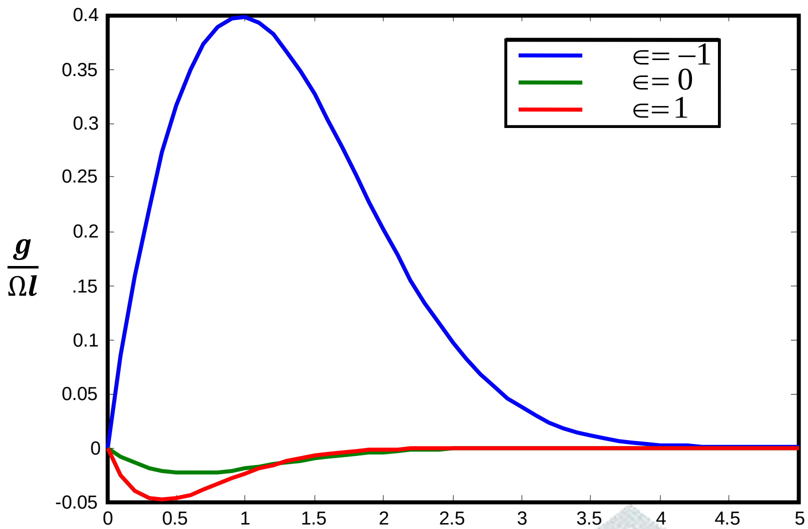

Figure 5.3.15

The variation of the velocity field with distance from the disk for various values of blowing/suction parameter ∈ when c = 0.4, $\in = 0$, $\alpha = 0.08$ and τ=1

Real part

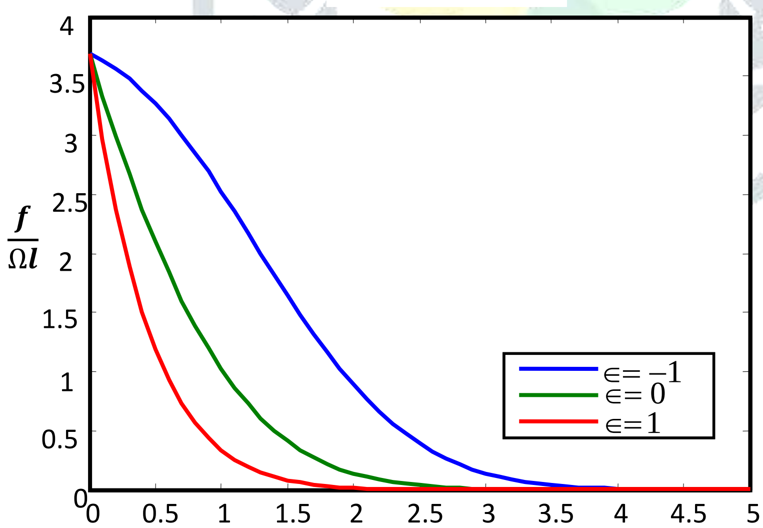

Figure 5.3.15

Imaginary Part

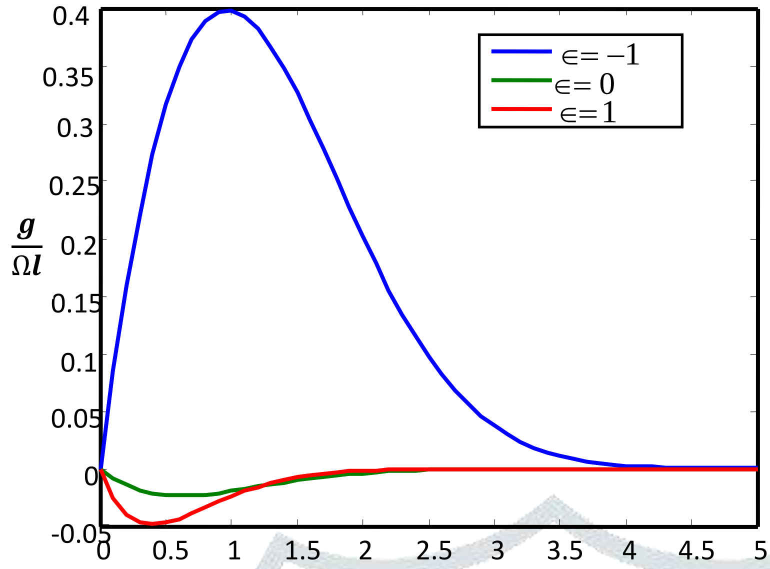

Figure 5.3.16

The variation of the velocity field with distance from the disk for various values of blowing/suction parameter ∈ when c = 1, ∈= 0, α = 0.08 and τ=1

Real part

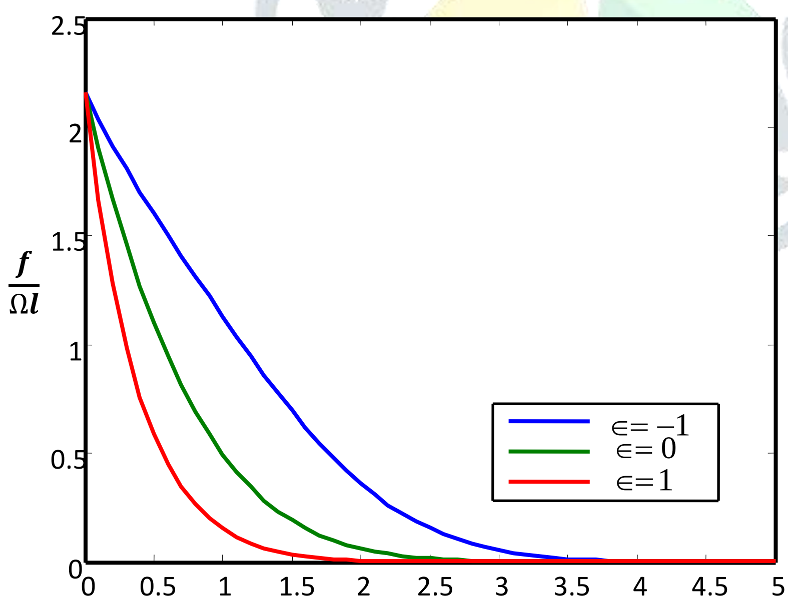

Figure 5.3.17

Imaginary Part

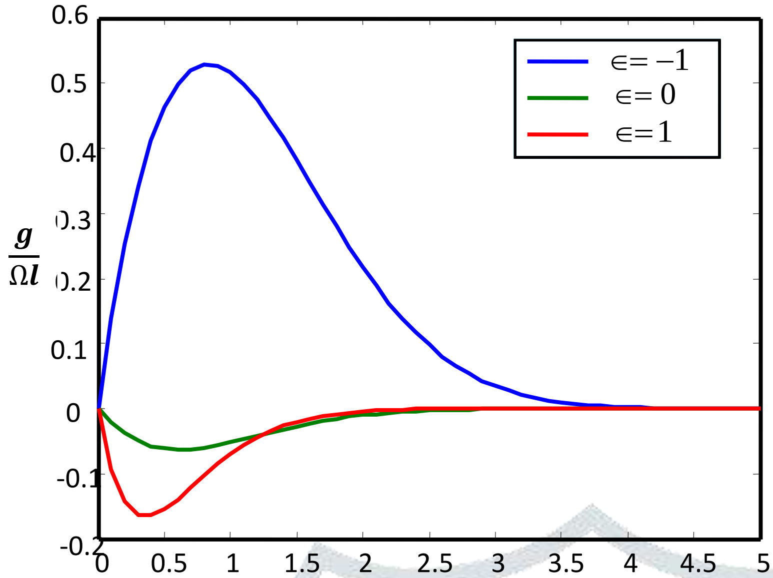

Figure 5.3.18

The variation of the velocity field with distance from the disk for various values of time τ when c = 1.5, n=0, $\alpha = 0.08$, $\in = 0$

Real part

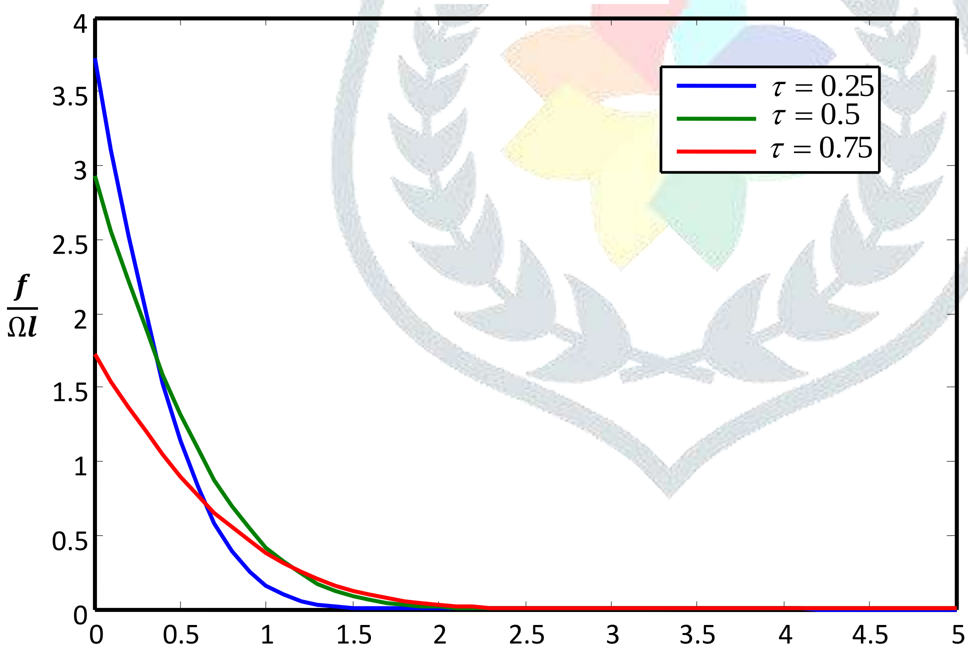

Figure 5.3.19

Imaginary Part

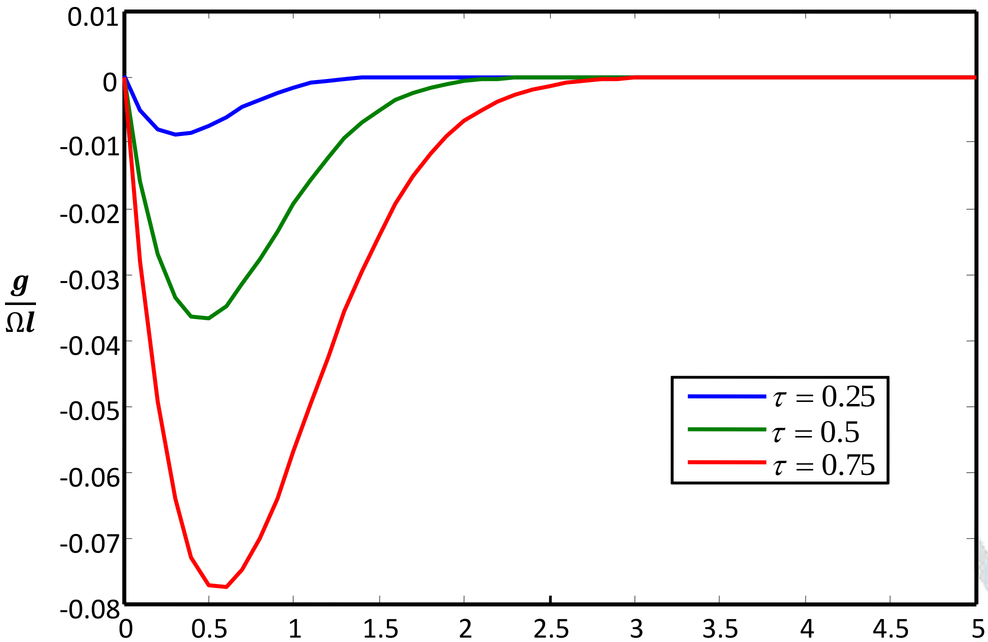

Figure 5.3.20

The variation of the velocity field with distance from the disk for various values of time τ when c = 0.4, n = 0, $\alpha = 0.08$, $\in = 0$

Real part

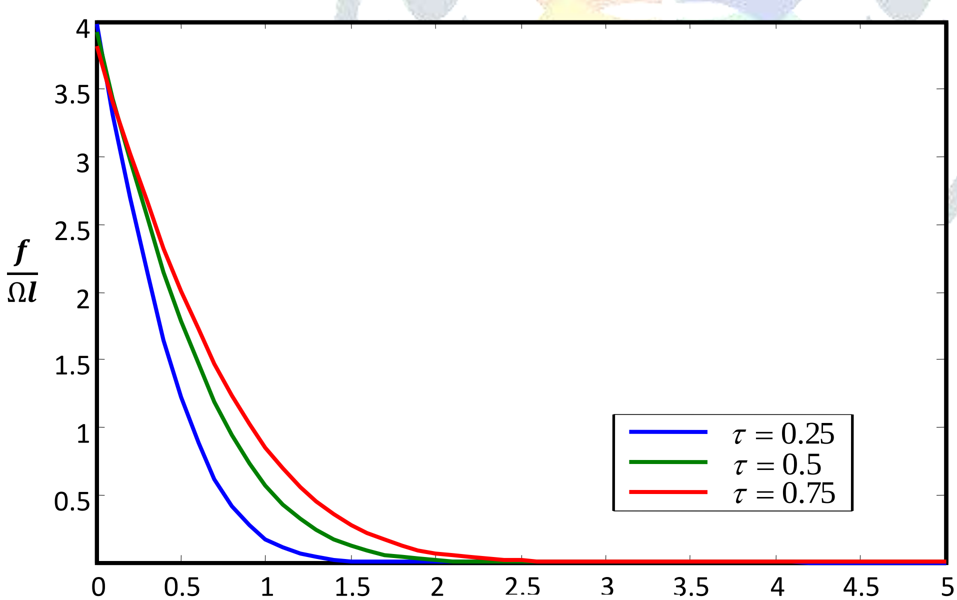

Figure 5.3.21

Imaginary Part

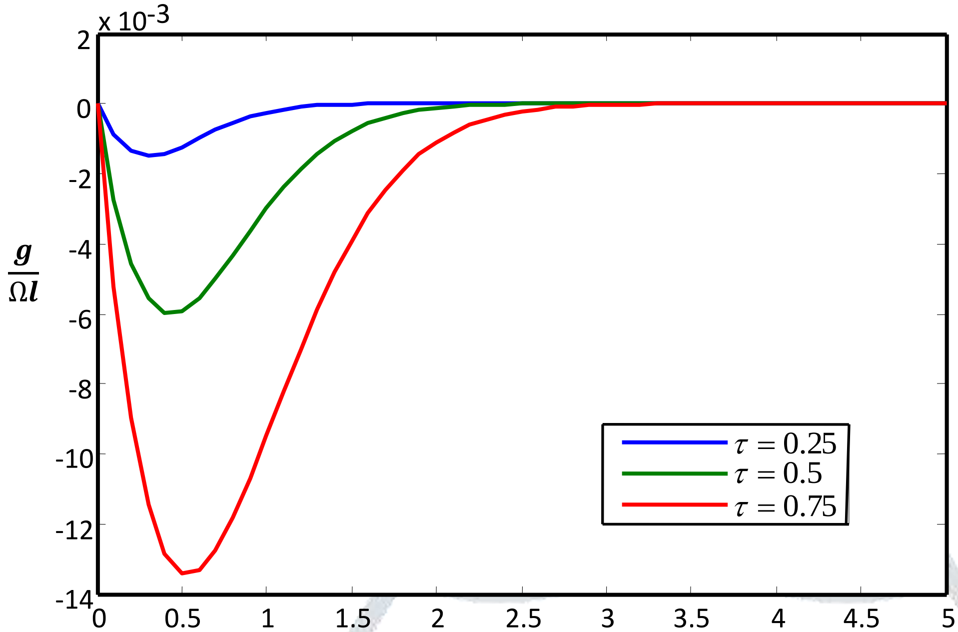

Figure 5.3.22

The variation of the velocity field with distance from the disk for various values of time τ when c = 1, n = 0, $\alpha = 0.08, \in = 0.4$

Real part

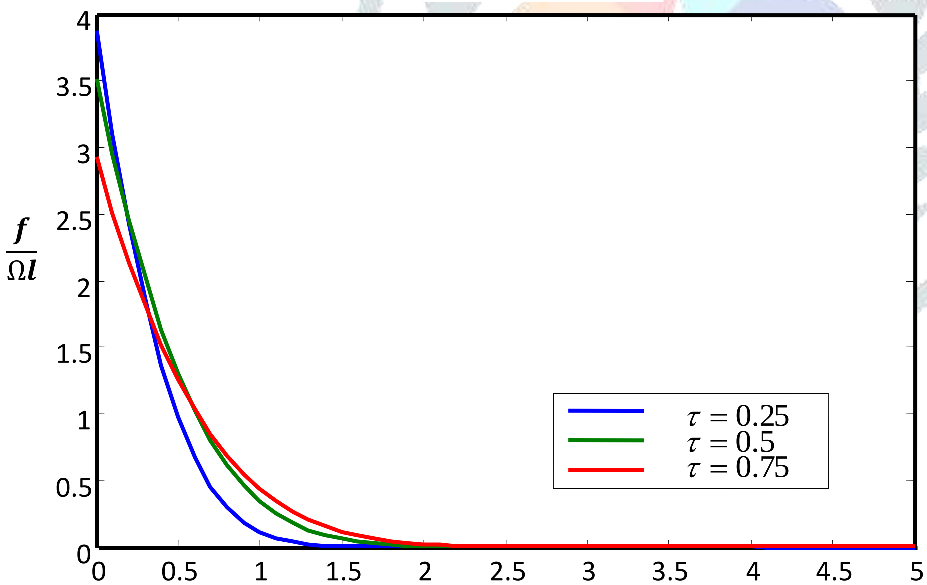

Figure 5.3.23

Imaginary Part

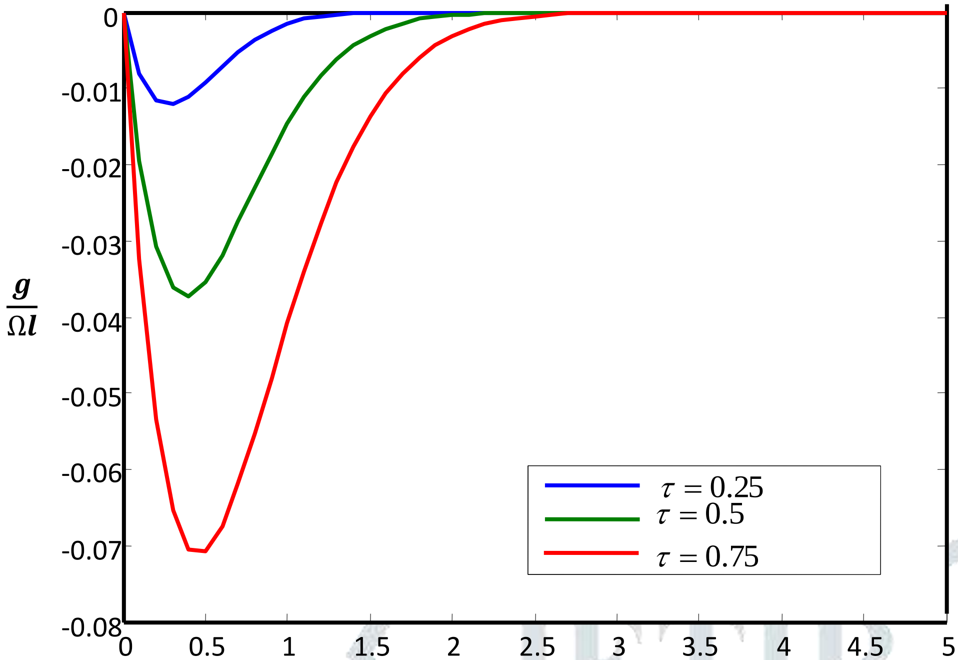

Figure 5.3.24

5.4 EFFECT OF MHD FLOW DUE TO ECCENTRIC ROTATIONS OF A POROUS DISK AND AN OSCILLATING SECOND GRADE FLUID AT INFINITY WITH DIFFERENT FREQUENCIES

The flow description is the same as that of the description given in section 5.3 except the viscous fluid is replaced by the second grade fluid.

Therefore, the resulting second grade fluid problem can be written as

$$\frac{\alpha_1}{\rho}\frac{\partial^3 F}{\partial z^2 \partial t} + \left(\nu - i\Omega\frac{\alpha_1}{\rho}\right)\frac{\partial^2 F}{\partial z^2} - \frac{\alpha_1}{\rho} w_0 \frac{\partial^3 F}{\partial z^3} - \left(N_1 + i\Omega\right)F - \frac{\partial F}{\partial t} + w_0 \frac{\partial F}{\partial z}$$

$$= -\Omega\left(i + \frac{N_1}{\Omega}\right)\cos k_1 t + k\sin k_2 t, \qquad (5.35)$$

with the boundary and initial condition

$$F(0,t) = \cos k_1 t, \quad F(\infty,t) = \cos k_2 t, \qquad F(Z,0) = 1, \qquad (5.36)$$

The above problem in non-dimensional variables can be written as

$$\alpha\frac{\partial^3 F}{\partial \xi^2 \partial \tau} - \alpha\epsilon\frac{\partial^3 F}{\partial \xi^3} + (1 - i\alpha)\frac{\partial^2 F}{\partial \xi^2} - 2\frac{\partial F}{\partial \tau} + 2\in\frac{\partial F}{\partial \xi} - 2(i+n)F =$$

$$-2(i+n)\cos b\tau + 2b\sin b\tau \qquad (5.37)$$

With the boundary and initial conditions

$$F(0,\tau) = \cos at, \quad F(\infty,t) = \cos b\tau, \quad F(\xi,0) = 1, \qquad (5.38)$$

using the equations $F(\xi,t) = H(\xi,\tau)e^{-i\tau}$ into (5.36) to (5.37) the following equations can be obtained

$$\alpha\frac{\partial^3 H}{\partial\xi^2\,\partial\tau} - \alpha\epsilon\frac{\partial^3 H}{\partial\xi^3} + (1-2i\alpha)\frac{\partial^2 H}{\partial\xi^2} - 2\frac{\partial H}{\partial\tau} + 2\in\frac{\partial H}{\partial\xi} - 2nH =$$

$$-2(i+n)\,X\left[e^{i(1+b)\tau} + e^{i(1-b)\tau}\right]$$

$$-ib\left[e^{i(1+b)\tau} - e^{i(1-b)\tau}\right], \qquad (5.39)$$

$$H(0,\tau) = \frac{1}{2}\left[e^{i(1+a)\tau} + e^{i(1-a)\tau}\right],$$

$$H(\infty,\tau) = \frac{1}{2}\left[e^{i(1+b)\tau} + e^{i(1-b)\tau}\right],$$

$$H(\xi,0) = 1, \qquad (5.40)$$

Taking Laplace transform into (5.39) and (5.40) the following equation is obtained

$$\alpha\in\frac{d^3\overline{\overline{H}}}{d\xi^3} - (1-2i\alpha+\alpha s)\frac{d^2\overline{\overline{H}}}{\partial\xi^2} - 2\epsilon\frac{dH}{d} - 2(s+n)\overline{H}$$

$$= 2 + 2\,(i+n)$$

$$\left[\frac{1}{s-i(1+b)} + \frac{1}{s-i(1-b)}\right] + ib\left[\frac{1}{s-i(1+b)} - \frac{1}{s-i(1-b)}\right] \qquad (5.41)$$

$$\overline{H}\;(0,s) = \frac{1}{2}\left[\frac{1}{s-i(1+a)} + \frac{1}{s-i(1-a)}\right]$$

$$\overline{H}(\infty,s) = \frac{1}{2}\left[\frac{1}{s-i(1+b)} + \frac{1}{s-i(1-b)}\right] \qquad (5.42)$$

Using perturbation method in equation (5.75), the following systems can be obtained

System of order zero

$$\frac{d^2\overline{H}_1}{d\xi^2} + 2\in\frac{\overline{H}_1}{d\xi} - 2(s+n)\overline{H}_1 =$$

$$-2 - (i+n)\left[\frac{1}{s-i(1+b)} + \frac{1}{s-i(1-b)}\right] - ib\left[\frac{1}{s-i(1+b)} - \frac{1}{s-1(1-b)}\right] \qquad (5.43)$$

$$\overline{H}\;(0,s) = \frac{1}{2}\left[\frac{1}{s-i(1+a)} + \frac{1}{s-i(1-a)}\right]$$

$$\overline{H}(\infty,s) = \frac{1}{2}\left[\frac{1}{s-i(1+b)} + \frac{1}{s-i(1-b)}\right] \qquad (5.44)$$

System of order one

$$\frac{d^2\overline{H}_1}{d\xi^3} - 2\in\frac{d\overline{H}_2}{d\xi^2} - (s-2i)\frac{d^2\overline{H}_1}{d\xi^2} - 2\in\frac{d\overline{H}_2}{d\xi} + 2(s+n)\overline{H}_2 = 0, \qquad (5.45)$$

$$\overline{H}_2(0,s) = 0,$$

$$\overline{H}_2(\infty, s) = 0 \tag{5.46}$$

Taking inverse Laplace transform into (5.45) and (5.46) and then solving the resulting integrals, the system become

$$H_1(\xi, t) = \frac{1}{4}\left(\begin{array}{c} e^{-(\epsilon+X_1)\xi+i(1+a)\tau}\operatorname{Erfc}\left[\frac{\xi}{\sqrt{2\tau}} - X_1\sqrt{\frac{\tau}{2}}\right] \\ +e^{-(\epsilon-X_1)\xi+i(1+a)\tau}\operatorname{Erfc}\left[\frac{\xi}{\sqrt{2\tau}} + X_1\sqrt{\frac{\tau}{2}}\right] \\ +e^{-(\epsilon+X_2)\xi+i(1-a)\tau}\operatorname{Erfc}\left[\frac{\xi}{\sqrt{2\tau}} - X_2\sqrt{\frac{\tau}{2}}\right] \\ +e^{-(\epsilon-X_2)\xi+i(1-a)\tau}\operatorname{Erfc}\left[\frac{\xi}{\sqrt{2\tau}} + X_2\sqrt{\frac{\tau}{2}}\right] \\ -e^{-(\epsilon+X_3)\xi+i(1+b)\tau}\operatorname{Erfc}\left[\frac{\xi}{\sqrt{2\tau}} - X_3\sqrt{\frac{\tau}{2}}\right] \\ -e^{-(\epsilon-X_3)\xi+i(1+b)\tau}\operatorname{Erfc}\left[\frac{\xi}{\sqrt{2\tau}} + X_3\sqrt{\frac{\tau}{2}}\right] \\ -e^{-(\epsilon+X_4)\xi+i(1-b)\tau}\operatorname{Erfc}\left[\frac{\xi}{\sqrt{2\tau}} - X_4\sqrt{\frac{\tau}{2}}\right] \\ -e^{-(\epsilon-X_4)\xi+i(1-b)\tau}\operatorname{Erfc}\left[\frac{\xi}{\sqrt{2\tau}} + X_4\sqrt{\frac{\tau}{2}}\right] \end{array}\right)$$

$$+\frac{1}{2}\left(e^{i(1+b)\tau} + e^{i(1-b)\tau}\right), \tag{5.47}$$

$$H_2(\xi,t)=\frac{1}{4}\left[\begin{array}{c}
\left\{\frac{1}{X_1}(A_1+A_2 i\,(1+\alpha)+(1+\alpha)^2)+A_3+A_4 i(1+\alpha)\right\}\\
Xe^{-(\in+X_1)\xi+ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_1\sqrt{\frac{\tau}{2}}\right]\\
+\left\{\frac{1}{X_1}\left(-A_1-A_2 i(1+\alpha)\quad-(1+\alpha)^2\right)+A_3+A_4 i(1+\alpha)\right\}\\
Xe^{-(\in-X_1)\xi+ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_1\sqrt{\frac{\tau}{2}}\right]\\
+\left\{\frac{1}{X_2}\left(A_1+A_2 i(1-\alpha)\quad+(1-\alpha)^2\right)+A_3+A_4 i(1-\alpha)\right\}\\
e^{-(\in+X_2)\xi+ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_2\sqrt{\frac{\tau}{2}}\right]\\
\left\{\frac{1}{X_2}\left(-A_1-A_2 i(1-\alpha)\quad-(1-\alpha)^2\right)+A_3+A_4 i(1-\alpha)\right\}\\
Xe^{-(\in-X_1)\xi+i(1+a)\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_2\sqrt{\frac{\tau}{2}}\right]\\
+\left\{\frac{1}{X_3}\left(-A_1-A_2 i(1+b)\quad-(1+b)^2\right)-A_3-A_4 i(1+b)\right\}\\
Xe^{-(\in+X_3)\xi+ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_3\sqrt{\frac{\tau}{2}}\right]\\
+\left\{\frac{1}{X_3}\left(A_1+A_2 i(1+b)\quad\right)-(1+b)^2-A_3-A_4 i(1+b)\right\}\\
+e^{-(\in-X_3)\xi+ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_3\sqrt{\frac{\tau}{2}}\right]\\
+\left\{\frac{1}{X_3}\left(-A_1-A_2 i(1-b)\quad-(1-b)^2\right)-A_3-A_4 i(1-b)\right\}\\
Xe^{-(\in+X_3)\xi+ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_4\sqrt{\frac{\tau}{2}}\right]\\
+\left\{\frac{1}{X_4}\left(A_1+A_2 i(1-b)\quad+(1-b)^2\right)-A_3-A_4 i(1-b)\right\}\\
Xe^{-(\in-X_4)\xi+ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_4\sqrt{\frac{\tau}{2}}\right]
\end{array}\right] \qquad (5.48)$$

From equations (5.18), (5.47) and (5.48), the suction solution for $a < 1$, $b < 1$ can be written as

$$\frac{f}{\Omega l}+i\frac{g}{\Omega l}=$$

$$\frac{1}{4}\left[\begin{array}{c} \left\{1+\alpha\xi\left(\frac{1}{X_1}\left(\left(A_1+A_2 i(1+\alpha)\quad\right)+(1+\alpha)^2\right)+A_3+A_4 i(1+\alpha)\right)\right\} \\ Xe^{-(\epsilon+X_1)\xi+ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_1\sqrt{\frac{\tau}{2}}\right] \\ +\left\{1-\alpha\xi\left(\frac{1}{X_1}\left(\left(A_1-A_2 i(1+\alpha)\quad\right)+(1+\alpha)^2\right)-A_3-A_4 i(1+\alpha)\right)\right\} \\ Xe^{-(\epsilon-X_1)\xi+ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_1\sqrt{\frac{\tau}{2}}\right] \\ +\left\{1+\alpha\xi\left(\frac{1}{X_2}\left(A_1+A_2 i(1-\alpha)\quad\right)+(1-\alpha)^2\right)+A_3+A_4 i(1-\alpha)\right\} \\ Xe^{-(\epsilon+X_2)\xi-ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_2\sqrt{\frac{\tau}{2}}\right] \\ +\left\{1-\alpha\xi\left(\left(\frac{1}{X_2}\left(A_1+A_2 i(1-\alpha)\quad\right)+(1-\alpha)^2\right)-A_3-A_4 i(1+\alpha)\right)\right\} \\ Xe^{-(\epsilon-X_1)\xi-ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_2\sqrt{\frac{\tau}{2}}\right] \\ -\left\{1+\alpha\xi\left(\left(\frac{1}{X_3}\left(\left(A_1+A_2 i(1+b)\quad\right)+(1+b)^2\right)\right)+A_3+A_4 i(1+b)\right)\right\} \\ Xe^{-(\epsilon+X_3)\xi+ib\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_3\sqrt{\frac{\tau}{2}}\right] \\ -\left\{1-\alpha\xi\left(\frac{1}{X_3}\left(A_1+A_2 i(1+b)\quad\right)+(1+b)^2-A_3-A_4 i(1+b)\right)\right\} \\ +e^{-(\epsilon-X_3)\xi+ib\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_3\sqrt{\frac{\tau}{2}}\right] \\ -\left\{1+\alpha\xi\left(\frac{1}{X_4}\left(A_1-A_2 i(1-b)\quad-(1-b)^2\right)+A_3+A_4 i(1-b)\right)\right\} \\ Xe^{-(\epsilon+X_4)\xi+ib\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_4\sqrt{\frac{\tau}{2}}\right] \\ -\left\{1-\alpha\xi\left(\frac{1}{X_4}\left(A_1+A_2 i(1-b)\quad+(1-b)^2\right)-A_3-A_4 i(1-b)\right)\right\} \\ Xe^{-(\epsilon-X_4)\xi+ib\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_4\sqrt{\frac{\tau}{2}}\right] \end{array}\right]$$

$$+\ \frac{1}{2}\left(e^{ib\tau}+e^{-ib\tau}\right) \qquad (5.49)$$

For a > 1, b > 1 , the system becomes

$$\frac{f}{\Omega l}+i\frac{g}{\Omega l}=$$

$$\frac{1}{4}\left[\begin{array}{c}
\left\{1+\alpha\xi\left(\frac{1}{X_1}\left((A_1+iA_2(1+a)+)+(1+\alpha)^2\right)+A_3+iA_4(1+a)\right)\right\} \\
Xe^{-(\epsilon+X_1)\xi+ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_1\sqrt{\frac{\tau}{2}}\right] \\
+\left\{1-\alpha\xi\left(\frac{1}{X_1}\left(A_1+A_2i(1+a)+(1+\alpha)^2\right)-A_3-A_4i(1+a)\right)\right\} \\
Xe^{-(\epsilon-X_1)\xi+ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_1\sqrt{\frac{\tau}{2}}\right] \\
+\left\{1+\alpha\xi\left(\frac{1}{Y_1}(A_1-A_2(a-1)+(1-a)^2)+A_3-iA_4(a-1)\right)\right\} \\
Xe^{-(\epsilon+Y_1)\xi-i\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-Y_1\sqrt{\frac{\tau}{2}}\right] \\
+\left\{1-\alpha\xi\left(\frac{1}{Y_1}(A_1-A_2 2i(a-1)+(1-a)^2)-A_3+A_4i(a-1)\right)\right\} \\
Xe^{-(\epsilon-Y_1)\xi-ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+Y_1\sqrt{\frac{\tau}{2}}\right] \\
-\left\{1+\alpha\xi\left(\frac{1}{X_1}\left((A_1+iA_2(1+b)+)+(1+b)^2\right)+A_3+iA_4(1+b)\right)\right\} \\
Xe^{-(\epsilon+X_3)\xi+ia\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-X_3\sqrt{\frac{\tau}{2}}\right] \\
-\left\{1-\alpha\xi\left(\frac{1}{X_1}\left((A_1+iA_2(1+b)+)+(1+b)^2\right)-A_3-iA_4(1+b)\right)\right\} \\
Xe^{-(\epsilon-X_3)\xi+ib\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+X_3\sqrt{\frac{\tau}{2}}\right] \\
-\left\{1+\alpha\xi\left(\frac{1}{Y_1}(A_1-A_2(b-1)+(1-b)^2)+A_3-iA_4(b-1)\right)\right\} \\
Xe^{-(\epsilon+Y_2)\xi-ib\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-Y_2\sqrt{\frac{\tau}{2}}\right] \\
-\left\{1-\alpha\xi\left(\frac{1}{Y_2}(A_1-A_2i(b-1)+(1-b)^2)-A_3+A_4i(b-1)\right)\right\} \\
Xe^{-(\epsilon-Y_2)\xi-ib\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+Y_2\sqrt{\frac{\tau}{2}}\right]
\end{array}\right]$$

$$+\frac{1}{2}\left(e^{ib\tau}+e^{-ib\tau}\right) \qquad (5.50)$$

For a = b = 1, $\frac{f}{\Omega l}+i\frac{g}{\Omega l}=$

$$\frac{1}{4}\left[\begin{array}{c}\left\{1+\alpha\xi\left(\frac{1}{Z_1}\left((A_1+A_2 2i+4)+(1+\alpha)^2\right)+A_3+A_4 2i\right)\right\} \\ Xe^{-(\epsilon+Z_1)\xi+i\tau}\mathrm{Erfc}\left[\frac{-\xi}{\sqrt{2\tau}}-Z_1\sqrt{\frac{\tau}{2}}\right] \\ +\left\{1-\alpha\xi\left(\frac{1}{Z_1}\left((A_1+A_2 2i+4\quad)\right)-A_3-A_4 2i\right)\right\} \\ Xe^{-(\epsilon-Z_1)\xi+i\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+Z_1\sqrt{\frac{\tau}{2}}\right] \\ +\left\{1+\alpha\xi\left(\frac{1}{Z_2}(A_1)+A_3\right)\right\} \\ Xe^{-(\epsilon+Z_2)\xi-i\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-Z_2\sqrt{\frac{\tau}{2}}\right] \\ \left\{1-\alpha\xi\left(\frac{1}{Z_2}(A_1)-A_3\right)\right\} \\ Xe^{-(\epsilon-Z_2)\xi-i\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+Z_2\sqrt{\frac{\tau}{2}}\right] \\ -\left\{1+\alpha\xi\left(\frac{1}{Z_1}(A_1+A_2 2i\quad+4)+A_3+A_4 i2\right)\right\} \\ Xe^{-(\epsilon+Z_1)\xi-i\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-Z_1\sqrt{\frac{\tau}{2}}\right] \\ \\ -\left\{1-\alpha\xi\left(\frac{1}{Z_1}(A_1+A_2 i2\quad+4)+A_3+A_4 i2\right)\right\} \\ Xe^{-(\epsilon-Z_1)\xi+i\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+Z_1\sqrt{\frac{\tau}{2}}\right] \\ -\left\{1+\alpha\xi\left(\frac{1}{Z_2}(A_1)+A_3\right)\right\} \\ Xe^{-(\epsilon+Z_2)\xi-i\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}-Z_2\sqrt{\frac{\tau}{2}}\right] \\ -\left\{1-\alpha\xi\left(\frac{1}{Z_2}(A_1)-A_3\right)\right\} \\ Xe^{-(\epsilon-Z_2)\xi+i\tau}\mathrm{Erfc}\left[\frac{\xi}{\sqrt{2\tau}}+Z_2\sqrt{\frac{\tau}{2}}\right]\end{array}\right]$$

$$\frac{1}{2}\left(e^{i\tau}+e^{-i\tau}\right) \qquad (5.85)$$

Replacing $\in = -\in$ in the suction solution, blowing solution can be obtained.

5.5 RESULTS AND DISCUSSION

- Figures 5.5.1 to 5.5.6 show the effects of second grade parameter α $(= 0.0,0.5,1.0)$ on the velocity profiles $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$. It is examined in figures 5.5.1 and 5.5.2 that $\frac{f}{\Omega l}$ first increases then decreases and $\frac{g}{\Omega l}$ increases. In figures 5.5.3 and 5.5.4 both $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ increases with increasing α. However in Figures 5.5.5 and 5.5.6 no variation occurs by increasing α in $\frac{f}{\Omega l}$ but in $\frac{g}{\Omega l}$ variation occurs. The layer thickness increases with the increase α in except in figures 5.5.5 and 5.5.6.
- To describe the variations of n (= 0.0, 1.0, 2.0) the figures 5.5.7 to 5.5.12. It is evident from figures 5.5.7 to 5.5.10 that there is an increase in $\frac{f}{\Omega l}$ and decreases in $\frac{g}{\Omega l}$ for large values of n. But no change has

been observed in figure 5.5.11 and changes observed in 5.5.12. Further it is deduced from these figures that except in figures 5.5.11 and 5.5.12 there is a reduction in layer thickness for large values of magnetic parameter n.

- Figures 5.5.13 to 5.5.18 depict the influence of porosity parameter $\in (= -1.0, 0.0, 1.0)$. From these Figures, it is found that by increasing $\in$, $\frac{f}{\Omega l}$ increases and $\frac{g}{\Omega l}$ decreases. But there is no effect on the velocity profiles in figures 5.5.17 and there is a small effects have been observed in 5.5.18. The layer thickness in suction shows the similar behaviour as of magnetic parameter. But in case of blowing the effects are reverse.
- The behaviour of velocity profiles $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for various values of $\tau(= 0.25, 0.5, 0.75)$ are given in figures 5.5.19 to 5.5.24. It is apparent from these Figures that an increase in τ leads to decrease in $\frac{f}{\Omega l}$. But for large τ $\frac{g}{\Omega l}$ increases except in Fig.5.5.24.

The variation of the velocity filed with distance from the disk for various values of magnetic parameter α , a = 1.75, b = 1.35, $\epsilon = 0$, $n = 0, and\ \tau = 2$

Real Part

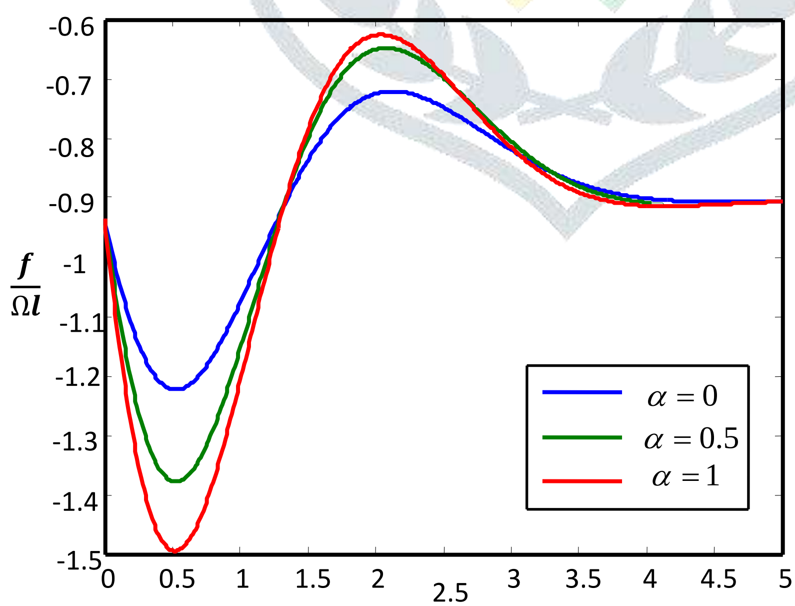

Figure 5.5.1

Imaginary Part

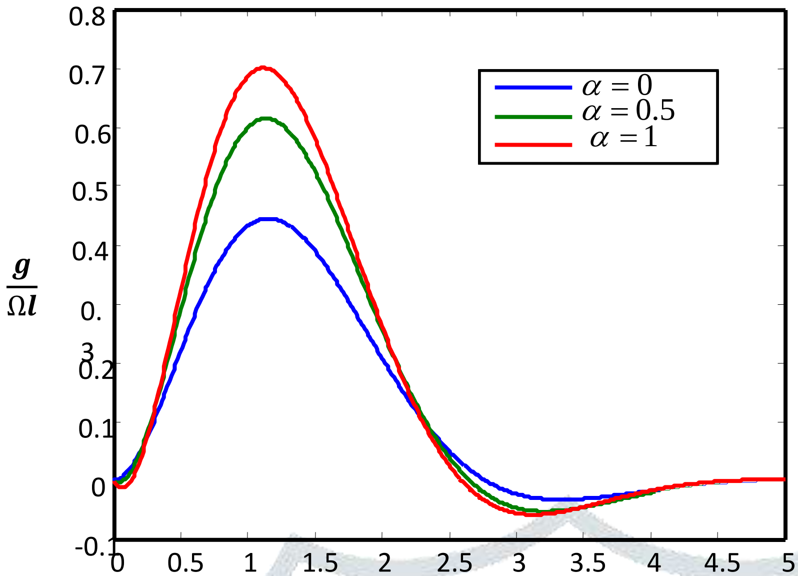

Figure 5.5.2

The variation of the velocity filed with distance from the disk for various values of magnetic parameter α , a = 0.75, b = 0. 35, $\epsilon = 0$, $n = 0$, $and\ \tau = 2$

Real Part

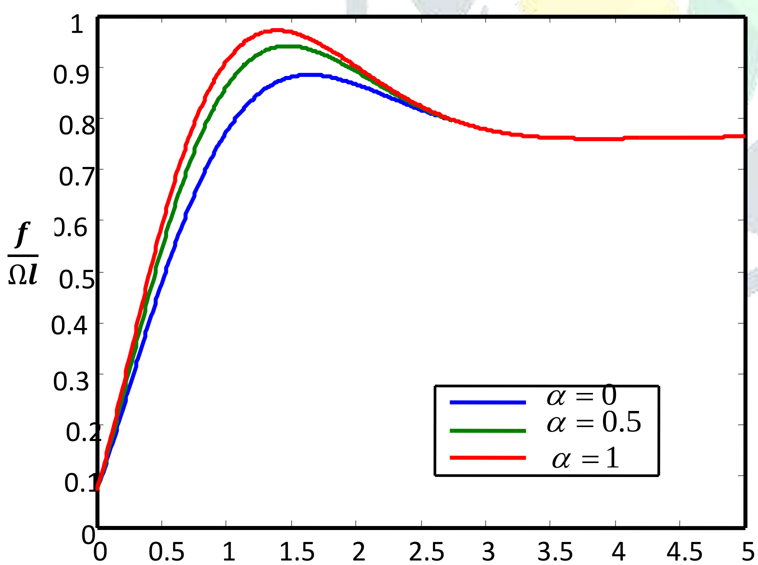

Figure 5.5.3

Imaginary Part

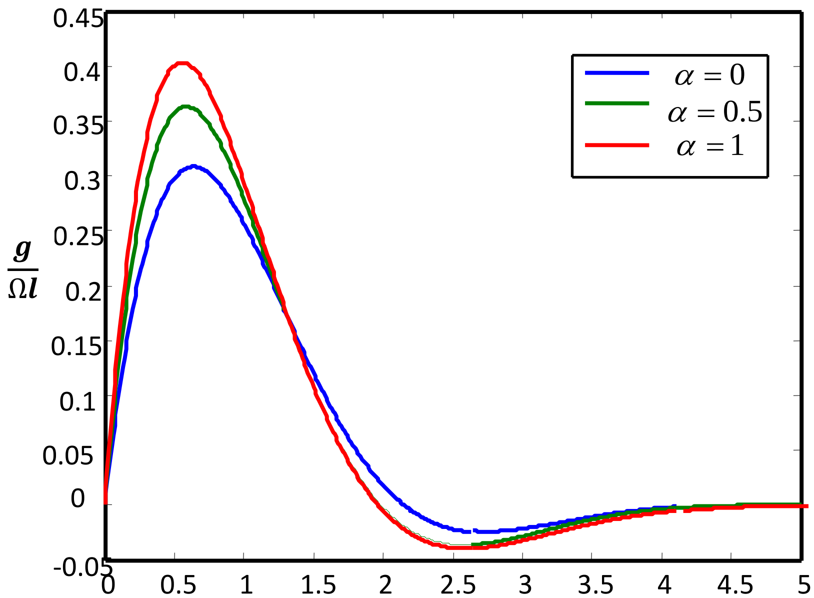

Figure 5.5.4

The variation of the velocity filed with distance from the disk for various values of magnetic parameter α , a = 1, b = 1, $\epsilon = 0$, $n = 0$, $and\ \tau = 2$

Real Part

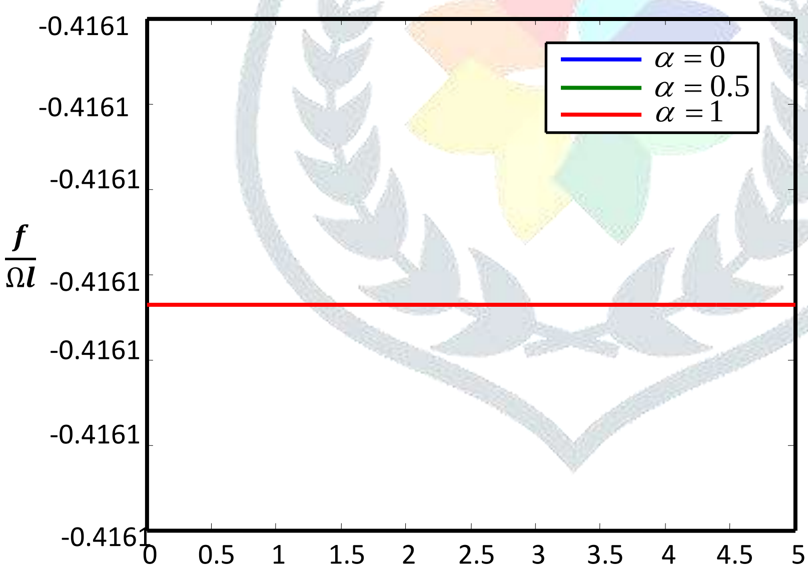

Figure 5.5.5

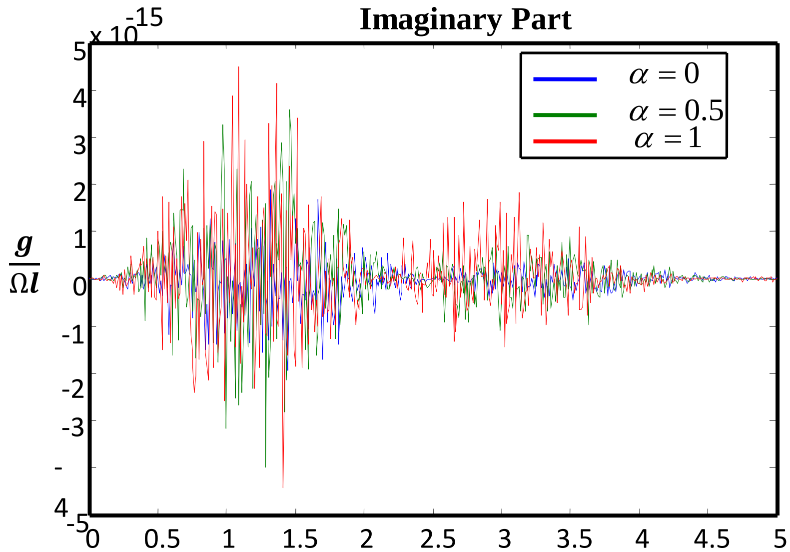

Figure 5.5.6

The variation of the velocity field with distance from the disk for various values of magnetic parameter n when a = 1.75, b= 1.25, $\epsilon = 0$, $\alpha = 0.08$, $and\ \tau = 2$

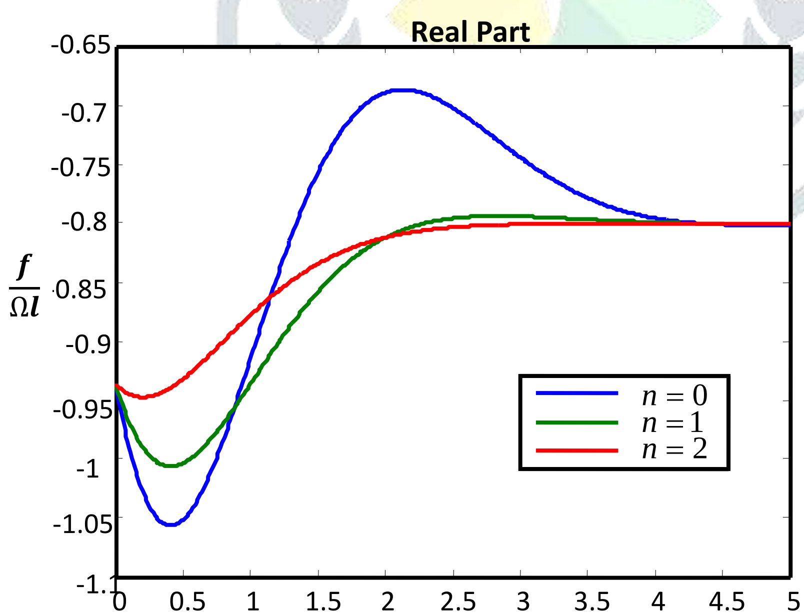

Figure 5.5.7

Imaginary Part

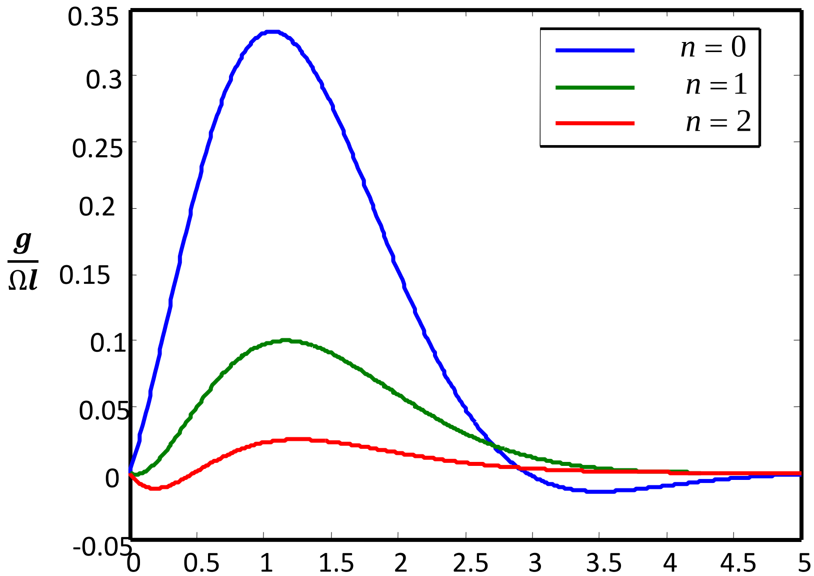

Figure 5.5.8

The variation of the velocity field with distance from the disk for various values of magnetic parameter n when a = 0.75, b = 0.25, $\epsilon = 0$, $\alpha = 0.08, and\ \tau = 2$

Real Part

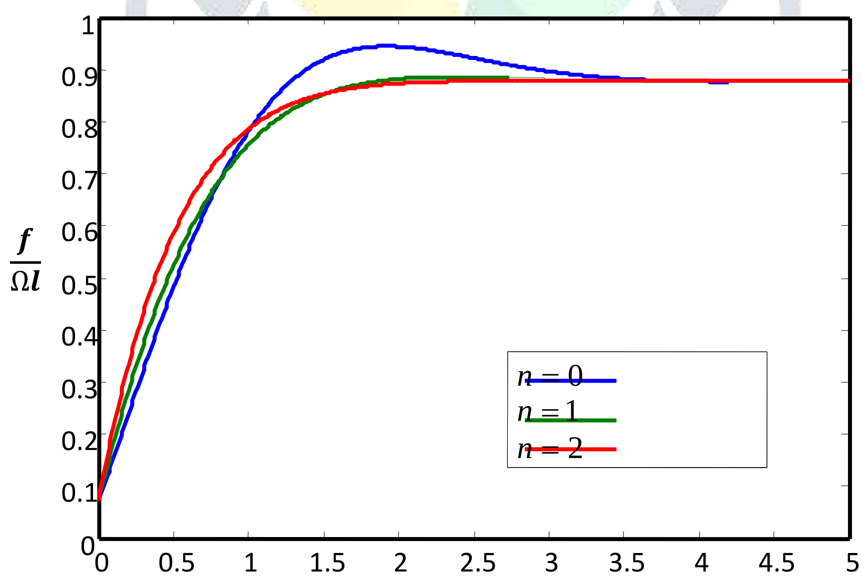

Figure 5.5.9

Imaginary Part

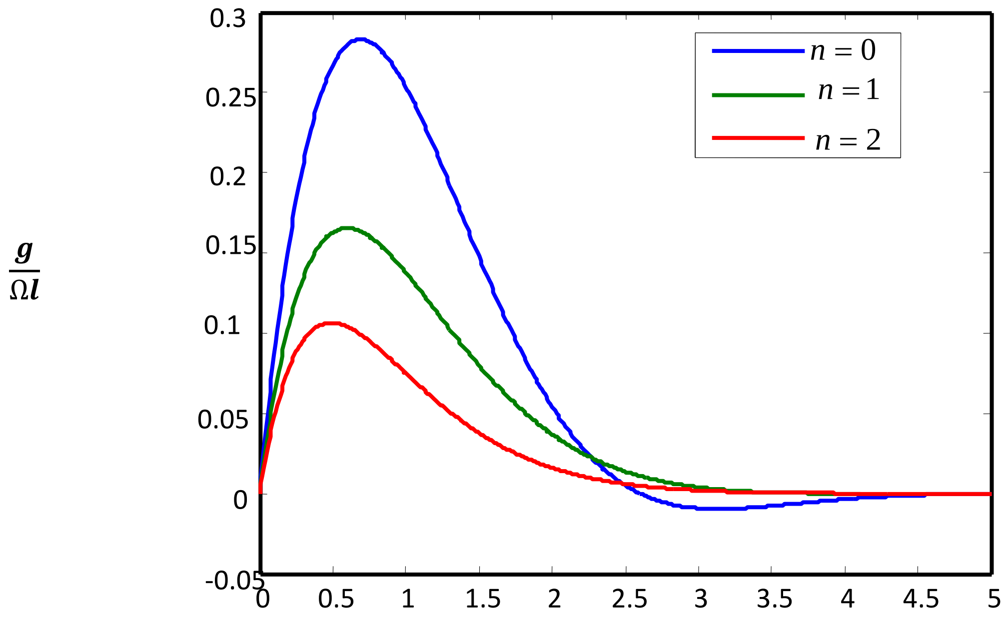

Figure 5.5.10

The variation of the velocity field with distance from the disk for various values of magnetic parameter n when a = 1, b= 1, $\epsilon = 0$, $\alpha = 0.08$, and $\tau = 2$

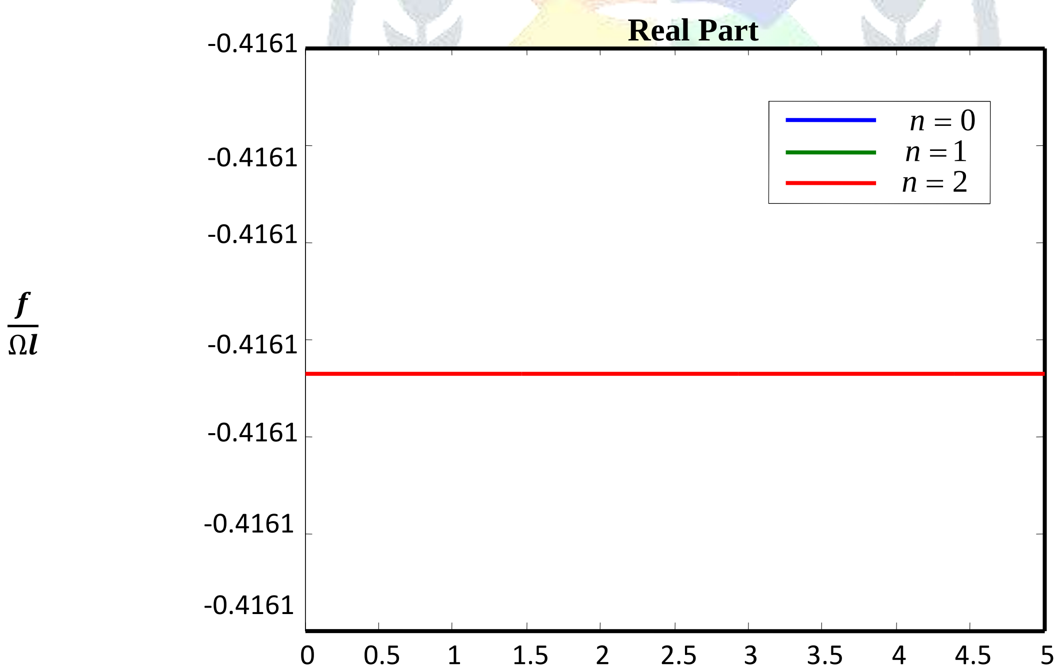

Figure 5.5.11

Imaginary Part

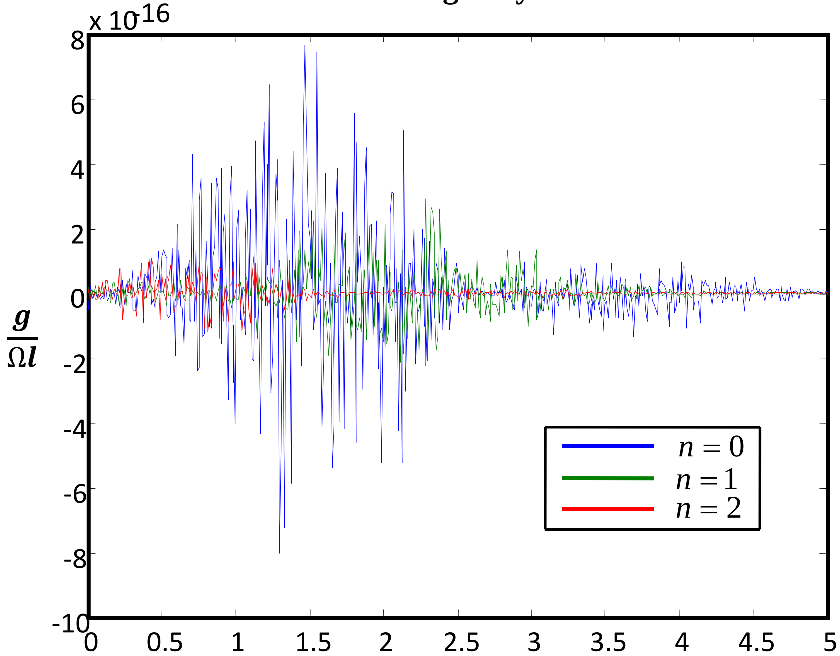

Figure 5.5.12

The variation of the velocity filed with distance from the disk for various values of magnetic parameter $\in$, a = 1.75, b = 1.25, $\epsilon = 0.5$, $n = 0$, $and\ \tau = 2$

Real Part

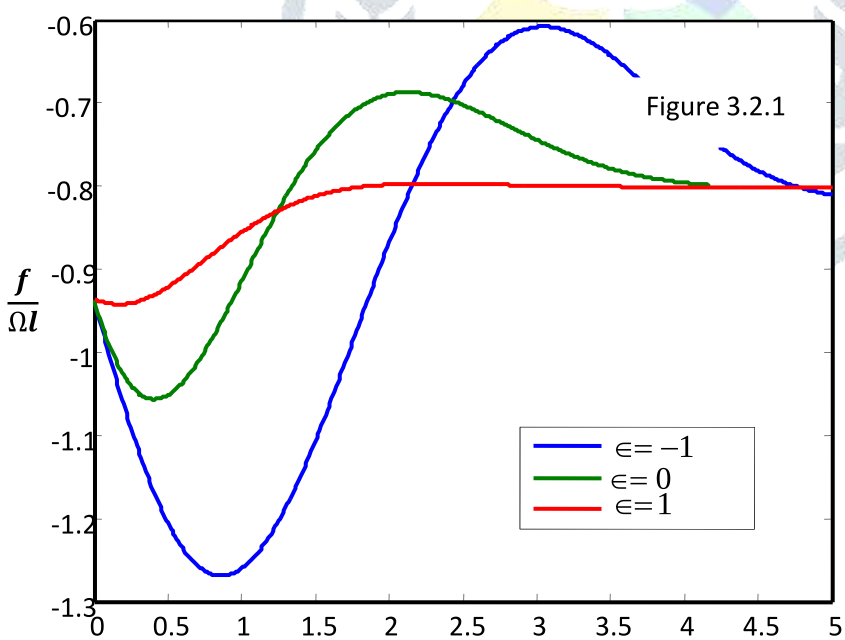

Figure 5.5.13

Imaginary Part

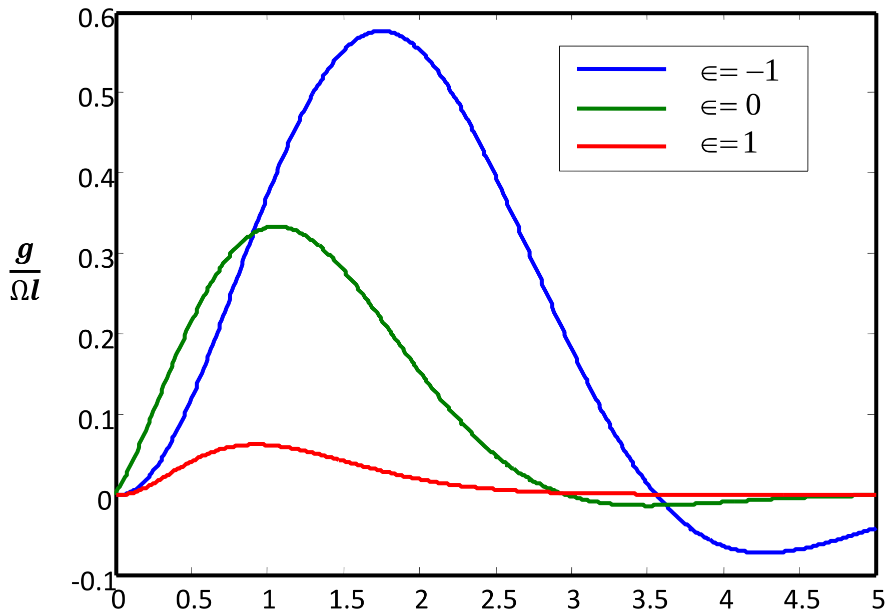

Figure 5.5.14

The variation of the velocity filed with distance from the disk for various values of magnetic parameter ∈ , a = 1.75, b = 1.25, $\epsilon = 0.5,\ n = 0,\ and\ \tau = 2$

Real Part

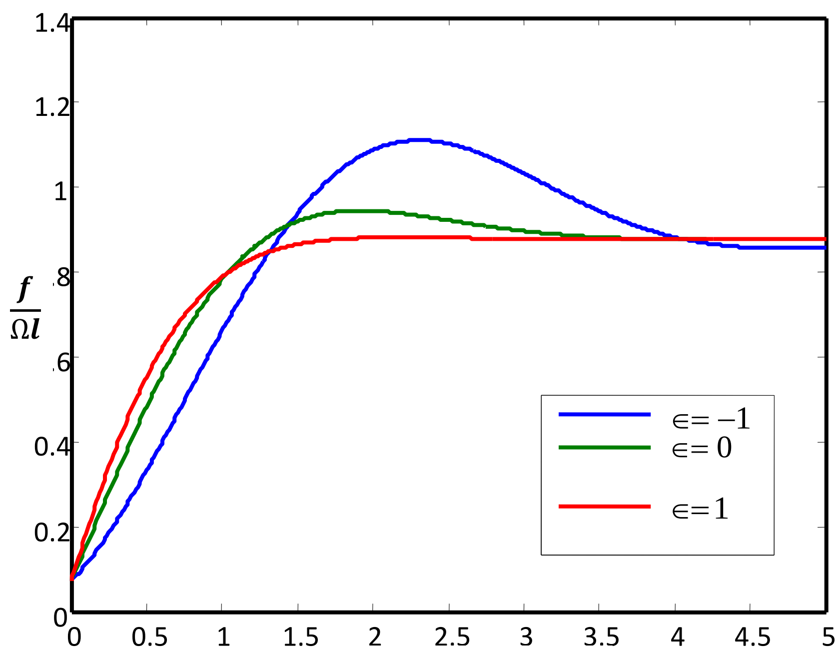

Figure 5.5.15

Imaginary Part

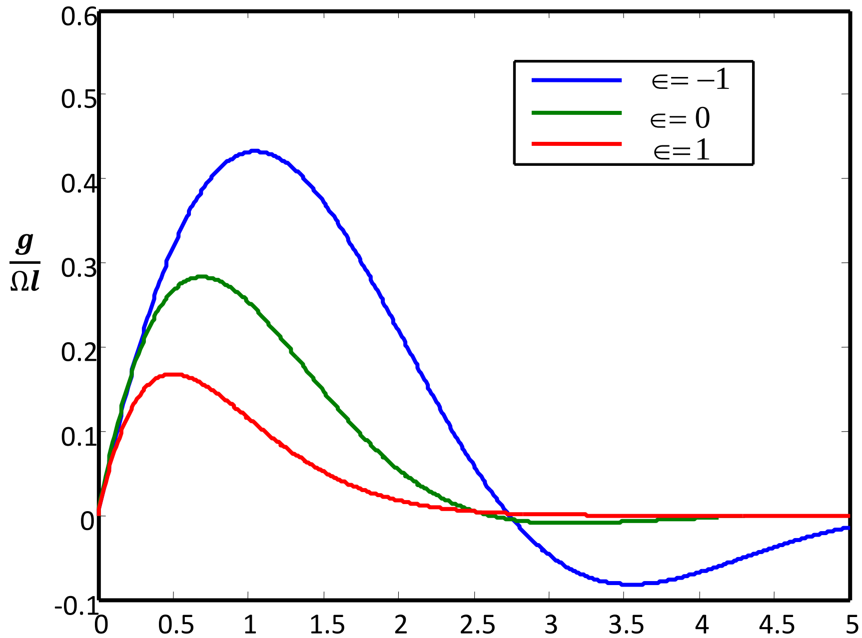

Figure 5.5.16

The variation of the velocity filed with distance from the disk for various values of magnetic parameter ∈ , a = 1, b = 1, $\epsilon = 0.5$, $n = 0$, $and\ \tau = 2$

Real Part

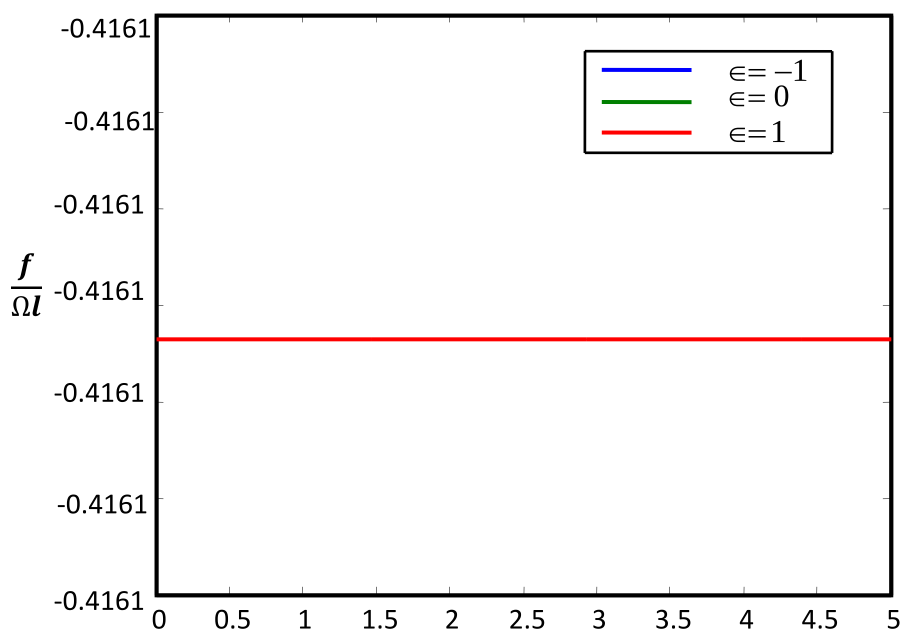

Figure 5.5.17

Imaginary Part

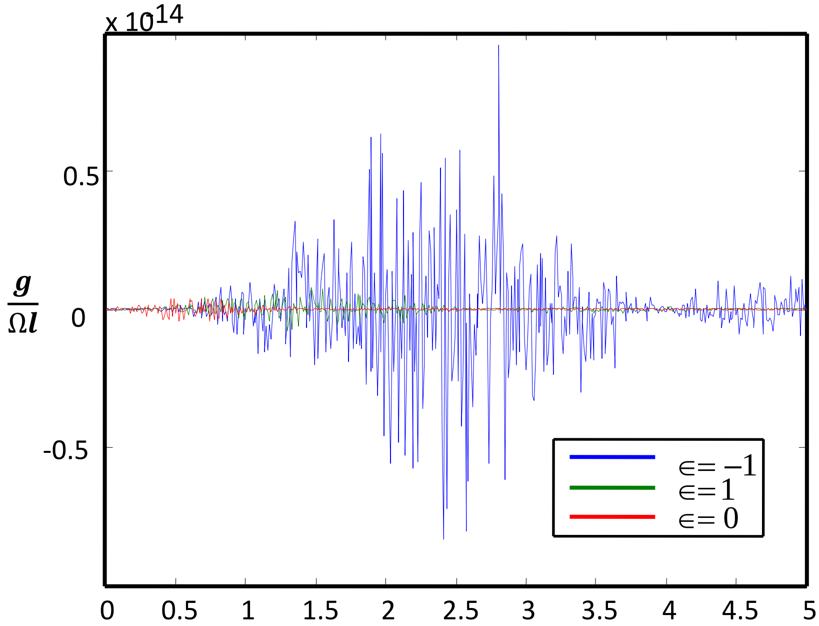

Figure 5.5.18

The variation of the velocity filed with distance from the disk for various values time τ when a = 1, b = $1,\ \epsilon = 0.5,\ n = 0,\ and\ \tau = 2$

Real Part

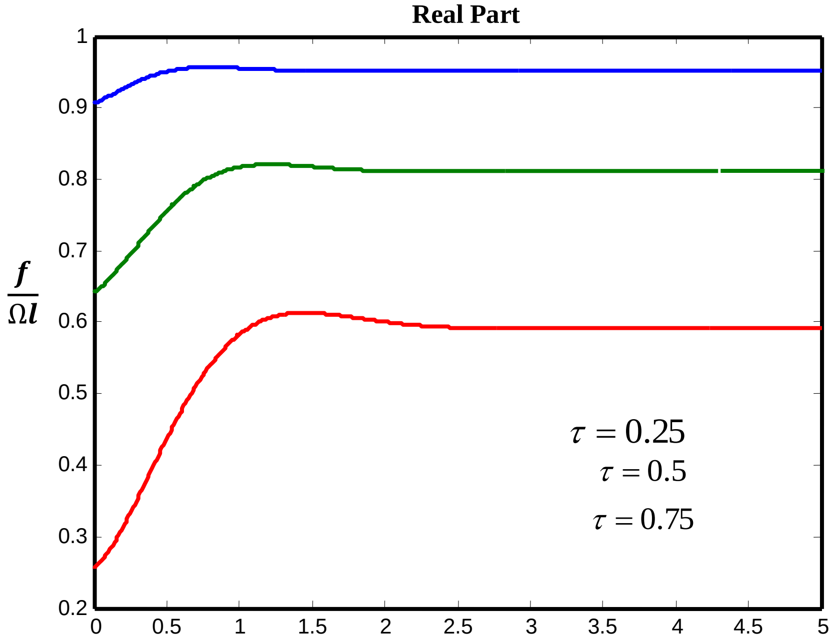

Figure 5.5.19

Imaginary Part

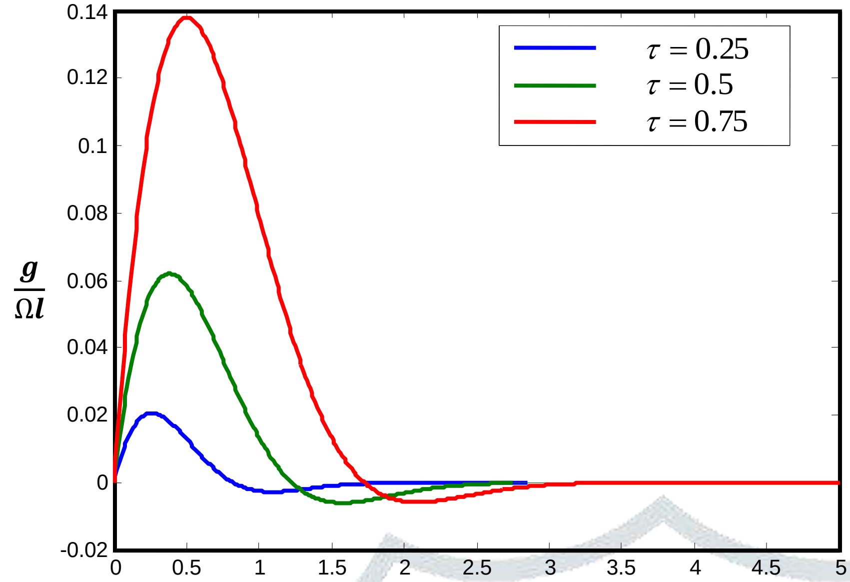

Figure 5.5.20

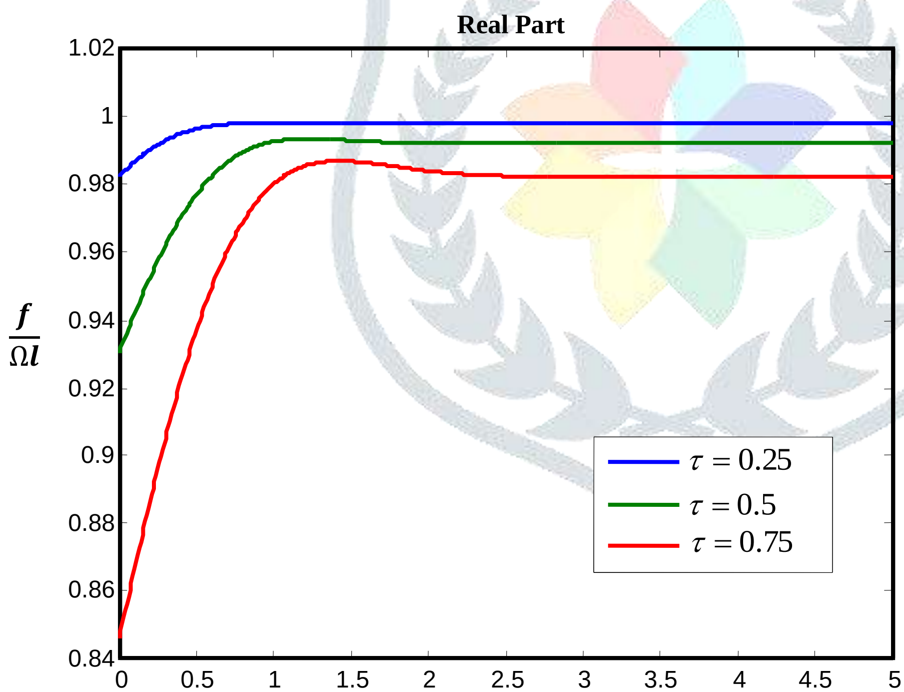

Figure 5.5.21

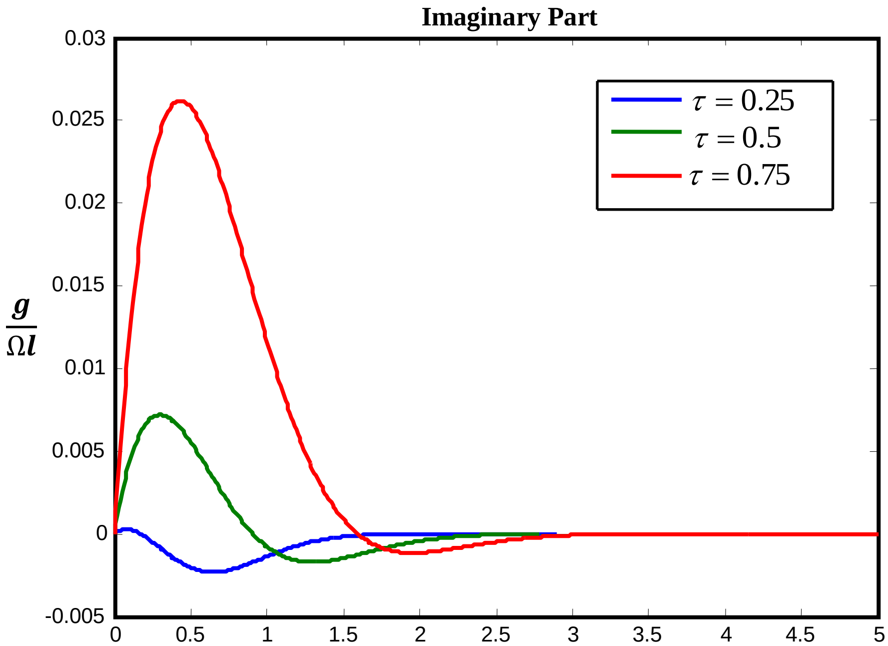

Figure 5.5.22

The variation of the velocity field with distance from the disk for various values of time when a = 1.75, b = 1.35, n = 0, $\alpha = 0.08$ and $\in = 0$

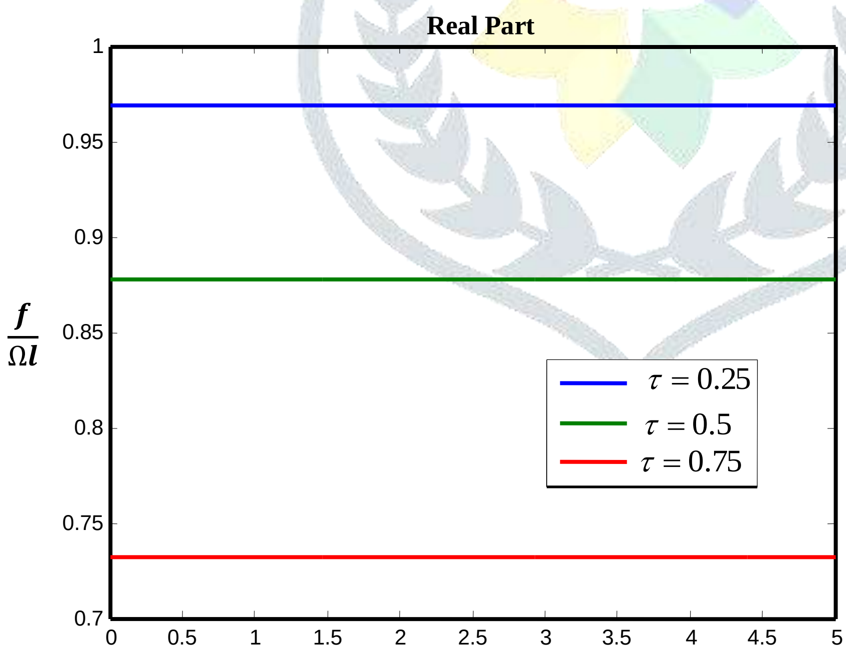

Figure 5.5.23

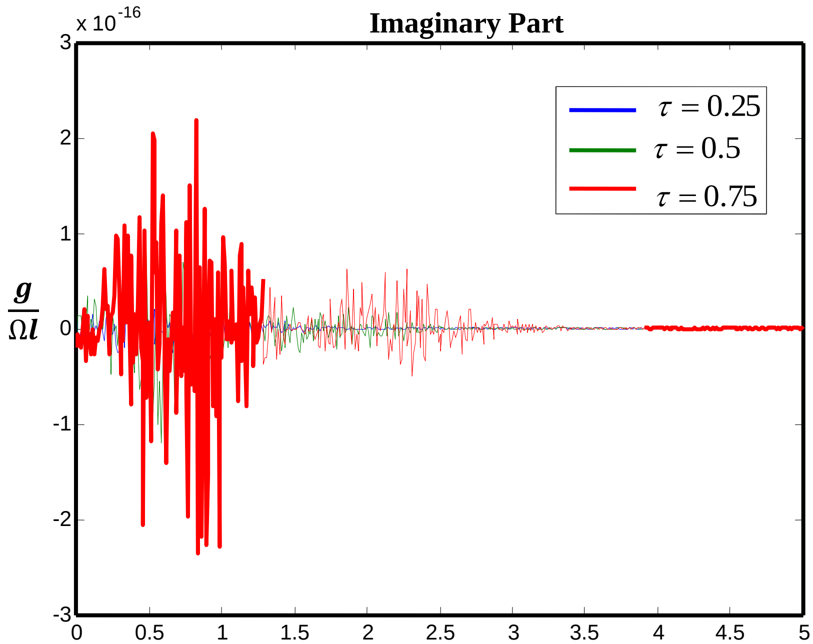

Figure 5.5.24

5.6 CONCLUSION

Coding are developed to get the graphical results which are in well agreement with the results given in the reference.

The following conclusions can be extracted from the analysis.

- $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ are first decreased and then increased by increasing the second grade parameter α
- It is observed that increase of α causes an increase in the boundary layer thickness
- It is clearly examined that an increase of the magnetic parameter n results the decrease in the boundary layer thickness.
- It is found that the boundary layer thicknesses are decreased by the increment of the suction parameter. But in the case of blowing the effects are reverse. Frome these the well known fact that suction and blowing has opposite characteristics on the boundary layer flows have been proved.
- It is further observed that the boundary layer thickness in suction shows the similar behaviour as in the case of magnetic parameter
- It is found that the increased time τ leads to decrement in $\frac{f}{\Omega l}$ but for large τ $\frac{g}{\Omega l}$ is increased
- The effect of magnetic field on any flow is an important problem related many practical application as in the case of boundary layer flow control. Since the blowing causes an increment in the boundary layer thickness, it is deduced that the boundary layer can be controlled by magnetic field.

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