JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

Statistical Techniques for Demographic Characteristics and Fertility Measures

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Abstract

One of the key causes of the population expansion is fertility. However, the majority of studies on the topic use heavily aggregated data. As a result, the studies do not adequately account for changes that occur at the micro level. This fact calls for deeper research into the matter. In order to do this, many models have been used by Brass (1958), as well as numerous other works (Singh 1963; Singh et al. 1971; and others). However, the majority of them have employed Univariate models, which in reality only partially achieve the desired results. These problems have prompted us to look for a good bivariate model. We have therefore tried to use the symmetrical bivariate negative binomial distribution (SBNBD) to research and predict the behaviour of fertility. This increases the SBNBD's usefulness in analysing fertility behaviours. The method aids in the determination of the correlation coefficient, the lines of regression, and the bivariate model for the observed bivariate birth distributions in human populations. The two completely separate sets of live birth data to demonstrate the technique. The first set is based on births that took place in India's Patna Medical College Hospital in 2018 while the second set is based on information on the French mothers mentioned by James in 1975. The results have significant implications for human biology, genetics, and demography.

INTRODUCTION

Fertility is one of the main factors of the population explosion. But most studies related to it are based on highly aggregated data. As such the analyses fail to reflect the changes that take place at the micro level. This fact, in turn necessitates its further investigation. With this end in view Brass (1958) and various other papers too (Singh 1963; Singh et al. 1971; and others) have employed various models for the purpose. But most of them have used the Univariate models which, in fact, serve the aim only to some extent. These predicaments have led us to search for a suitable bivariate model. Thus, we have made an attempt to employ the symmetrical bivariate negative binomial distribution SBNBD to investigate and predict the behaviour of fertility. The model has also been referred to by Lundberg (1940), Arbous and Kerrich (1951) and Arbous and Sichel (1954) in other areas of investigation. Rao et al. (1973) gave a formula to obtain the correlation coefficient between the two types of children (e.g. male and female) assuming the family size (sibship size) as a random variable with finite mean and variance. But Sinha (1985) established the equivalence between the formula and the one proposed by Arbous and Sichel (1954). This further strengthens the utility of the SBNBD to study the behaviour of fertility. Apart from providing a bivariate model to the observed bivariate distributions of births in human populations, the approach helps in the determination of the correlation coefficient, the lines of regression and the operating characteristics for the two characteristics under consideration. In order to illustrate the technique, we have used the two entirely different sets of data related to live births. The first set is based on the births that occurred at the Patna Medical College Hospital, India during 2018 and the second set is related to the family history of the French mothers referred to by James (1975). The results are of far-reaching consequence in human biology, genetics and demography.

THE MODEL

Let us suppose that the births in human population follow the Poisson distribution and λ is a fixed birth rate. Further, we assume that p and q denote the proportions of occurrences either of male and female births respectively or of births in the two consecutive intervals of a period of interest. On the assumption that the two types of births occur independently we have

$P(X = x/\lambda) = e^{-\lambda p} (\lambda p)^x/x!$; x=0,1,2,...and

$P(Y=y/\lambda) = e^{-\lambda q}(\lambda q)^y/y!$;y=0,1,2,...

Then, the joint probability mass function of the births is given by

$$P(X = x,Y=y/\lambda)= e^{-\lambda} \lambda^{x+y}p^x q^y/x!y! \quad (1)$$

since p+q=1

We, further, assume that λ follows the Pearson's type III distribution as mentioned below:

$$f(\lambda) = (k / \mu)^k \lambda^{k-1} e^{-k\lambda/\mu}/ \Gamma k \quad (2)$$

Where μ is the fixed average number of births per mother or the average number of births occurred in a period of interest and k is a parameter.

These assumptions result in a probability mass function of the bivariate compound Poisson distribution (generally known as the bivariate negative binomial distribution). It is given as follows:

$$P(X = x, Y= y) = (k/\mu)^{kpxqy} \int_0^\infty e^{-\lambda\{k/\mu+1\}} \lambda^{k+x+y-1} d\lambda/\Gamma k\Gamma(x+1)\Gamma(y+1)$$

Or

$$P(X=x,Y=y)= \Gamma(k+x+y)\{k/(k+\mu)\}^k\{\mu p/(k+\mu)\}^x.\{\mu q/(k+\mu)\}^y/\Gamma k\Gamma(x+1)\Gamma(y+1)$$

x,y=0,1,2,….. (3)

If we put p=q= ½ and μ=2m we obtain

$$P(X = x, Y= y) = \{k/ (k+2m)\}^k \{m/ (k+2m) \}^{x+y}\Gamma(k+x+y)/ \{\Gamma k\Gamma(y+1) \Gamma(x+1)\}$$

x, y=0,1,2... (4)

This is known as the probability mass function of the symmetrical bivariate negative binomial distribution (SBNBD). 'm' and 'k' are its parameters. The marginal distribution of (4), their means and variances are given by equations (5), (6) and (7) respectively.

$$P(X = x) = \{k/(k+m)\}^k \{m/(k+m)\}^x . \Gamma(k+x)/ \{\Gamma\ k\ \Gamma(x+1)\} \quad (5)$$

Mean = m (6)

Variance = m + (m^2/k) (7)

The conditional probability mass function of y is given by

$$P(Y= y/X= x) = \{k+m/ (k+2m)\}^{k+x} \{m/ (k+2m) \}^y \Gamma(k+x+y)/ \{\Gamma(k +x)\Gamma (y+1)\} \quad (8)$$

Similarly, P(X=x/Y=y) may be defined.

The product moment correlation coefficient of the SBNBD is obtained by

$$\rho=m/ (k+m) \quad (9)$$

The mean and the variance are given by

$$E(Y/X=x) = \rho(k+x) \quad (10)$$

Similarly,

$$E(X/Y=y) = \rho(k+y) \quad (10\ a)$$

$$V(Y/X=x) = \rho(1+\rho) (k+x) \quad (11)$$

$$V(X/Y=y) = \rho(1+\rho) (k+y) \quad (11a)$$

The operating characteristic of births of the second type (or in the second interval) is defined as the probability to have one or more births when x births of the first type (or in the first interval) has already taken place. It is obtained by

$$P(Y \geq \alpha /X = x) = 1 - \{(k+m)/(k+2m)\}^{x+k} .$$

$$.\{1/\Gamma(x+k)\} \sum_{y=0}^{\alpha-1} \{m/(k+\ 2m)\}^y \Gamma(x+y+k)/\Gamma(y+1) \quad (12)$$

Similarly, P (X≥β/Y=y) may be obtained.

In order to estimate the parameters of the SBNBD we obtain first the distribution of x+y (i.e., Univariate distribution). The moment estimators are computed of the distribution and then the maximum likelihood estimators are obtained as given by Johnson and Kotz (1969) and Sinha (1984). In fact, the value of 'm' is the half of the mean of the Univariate distribution.

EXAMPLES AND DISCUSSION

In order to illustrate the technique, we have considered (a) the live births that took place at the Patna Medical College Hospital during 2018 and (b) the bivariate distribution of births referred to by James (1975). The author has considered the births that occurred in France. The total births recorded in 2018 at the PMCH have been divided into two parts. The first part contains the births recorded each day during 6a.m. to 6 p.m. and the second part consists of births that occurred during 6 p.m. to 6a.m. First of all, the Univariate

Negative Binomial Distribution has been fitted to the distribution of total births. The fit is quite good. The adequacy of the model has been examined by the χ^2test. The maximum likelihood method has been used for the estimation of the parameters. Thus, for the fitting of the SBNBD we get

m=4.01095, k-9.82115, ρ=0.28997

Hence, 2m and k are the parameters of the Univariate distribution. The SBNBD provides good fit to the bivariate distribution. Table 1 gives the observed and the expected frequencies. The adequacy of the model has been judged by the method of goodness of fit test ofχ^2 .

Further, the 95 per cent confidence limits for the product moment correlation coefficient (0.22613) are 0.12641 (lower) and 0.32132 (upper).

These limits include the value of ρ (the correlation coefficient) obtained on the basis of equation (9). This illustrates the suitability of the theory with the observed data. The observed and the expected average number of births each day during 6 p.m. to 6 a.m. corresponding to each birth during 6 a.m. to 6 p.m. and vice versa are given in Table 2. The result has been obtained on the basis of the following equations:

E(Y/x) = 2.84780+0.28997x and

E(X/y) = 2.84780a + 0.28997y

Table 1: The observed and the expected number (within parentheses) of days for the births occurred during 6 a.m. to 6 p.m (X) and 6p.m. to 6 a.m. (Y) in 2018

X Y	0	1	2	3	4	5	6	7+	TOTAL
0	2 (1.0)	3 (2.3)	2 (2.8)	1 (2.5)	4 (1.8)	_ (1.1)	1 (0.6)	2 (0.5)	15 (12.6)
1	2 (2.3)	9 (5.6)	6 (7.4)	6 (7.1)	9 (5.5)	4 (3.7)	3 (2.2)	3 (3.9)	42 (37.7)
2	2 (2.8)	6 (7.4)	7 (8.6)	9 (11.9)	11 (9.2)	8 (6.6)	7 (4.9)	7 (6.2)	57 (57.6)
3	3 (2.5)	4 (7.1)	10 (11.9)	13 (11.2)	15 (10.9)	6 (8.2)	5 (5.5)	8 (7.1)	64 (64.4)
4	4 (1.8)	6 (5.5)	5 (9.2)	11 (10.9)	11 (9.3)	9 (8.2)	4 (5.8)	7 (8.9)	57 (59.6)
5	1 (1.1)	5 (3.7)	4 (6.6)	7 (8.2)	9 (8.2)	6 (7.0)	5 (5.2)	7 (7.0)	44 (47.0)
6	1 (0.6)	3 (2.2)	- (4.9)	4 (5.5)	6 (5.8)	4 (5.2)	4 (4.0)	8 (5.7)	30 (33.9)
7+	1 (0.5)	3 (3.9)	4 (6.2)	7 (7.1)	5 (8.9)	6 (7.0)	5 (5.7)	21 (12.9)	56 (52.2)
TOTAL	16 (12.6)	39 (37.7)	42 (57.6)	58 (64.4)	70 (59.6)	43 (47.0)	34 (33.9)	63 (52.2)	365 (365.0)

Table 2: The observed ($\bar{x}$)and the expected ($\bar{x}_e$) average number of births each day during 6 a.m. to 6 p.m. for each (Y) during 6 p.m. to 6 a.m. in 2018 and vice versa.

X or Y	$\bar{x}$	$\bar{y}$	$\bar{x}_e or \bar{y}_e$
0	3.267	3.125	2.847
1	3.238	3.102	3.138
2	3.982	3.571	3.428
3	3.328	3.793	3.718
4	3.895	3.414	4.008
5	4.227	4.046	4.297
6	4.867	4.088	4.588
7	4.348	4.807	4.877
8	4.667	4.945	5.167
9	6.571	6.125	5.457
10	8.125	6.182	5.747

It is evident that the correspondence between the observed and the expected frequencies is very good. This, in fact, justifies the application of the SBNBD. On substituting the values of k and m in equation (12),

We obtain

$$P(Y \geq \alpha/X = x) = 1 - (0.77521)^{9.82115-x} \ (1/\Gamma(9.82115 + x).$$

$$. \sum_{y=0}^{\alpha-1} (0.22479)^y \Gamma(x + y + 9.82115)/\Gamma(y + 1)$$

For each value of x we have obtained$P(Y \geq \alpha \ /X = x)$, and the expected number of days to record α(α=1, 2,. . .6) or more births between 6 p.m. and 6 a.m. in 2018. These values along with the observed frequencies are mentioned in Table 3.

Table 3: The observed and the expected number of days to record α (α=1, 2 ... 6) or more births during 6 p.m. to 6 a.m. when X (X=0, 1,..., 10) births have taken place during 6 a.m. to 6 p.m. in 2018

X	0	1	2	3	4	5	6	7	8	9	10
P(Y≥1/X)	0.92	0.94	0.95	0.96	0.97	0.98	0.98	0.99	0.99	0.99	0.99
Exp. No.	14.7	36.5	39.9	55.7	67.9	42.0	33.4	25.6	17.8	7.9	10.9
Obs. No.	14	36	40	57	66	43	33	25	18	8	10
P(Y≥2/X)	0.74	0.78	0.82	0.85	0.88	0.90	0.92	0.93	0.95	0.96	0.96
Exp. No.	11.8	30.5	34.4	49.4	61.5	38.7	31.2	24.3	17.0	7.6	10.6
Obs. No.	12	27	34	51	57	39	30	24	16	8	10
P(Y≥3/X)	0.52	0.58	0.63	0.68	0.72	0.76	0.74	0.83	0.86	0.88	0.90
Exp. No.	8.3	22.5	26.5	39.4	50.7	32.8	25.3	21.5	15.4	7.0	9.9
Obs. No.	10	21	27	42	46	31	23	21	13	7	10
P(Y≥4/X)	0.32	0.38	0.43	0.49	0.54	0.59	0.64	0.68	0.72	0.76	0.79
Exp. No.	5.1	14.7	18.3	28.4	38.0	25.5	21.7	17.7	13.0	6.0	8.7
Obs. No.	7	17	17	29	31	25	18	15	12	7	9
P(Y≥5/X)	0.18	0.22	0.27	0.32	0.37	0.42	0.47	0.52	0.56	0.61	0.65
Exp. No.	2.9	8.8	11.4	18.6	26.0	18.1	16.0	13.4	10.1	4.8	7.1
Obs. No.	3	11	12	18	20	16	14	14	9	6	7
P(Y≥6/X)	0.09	0.12	0.16	0.19	0.23	0.27	0.32	0.36	0.41	0.45	0.50
Exp. No.	1.5	4.8	6.5	11.2	16.3	11.8	10.8	9.5	7.3	3.6	5.5
Obs. No.	2	6	8	11	11	10	9	11	7	5	6

Likewise, we have computed the probabilities as well as the expected number of days to witness $\beta(\beta = 1,2,\ldots,6)$ or more births during 6 a.m. to 6 p.m. when Y(Y=0, 1... 10) births have occurred during 6 p.m. to 6 a.m. The following equation has been used for the purpose:

The findings are mentioned in Table 4.

The resemblance between the observed and the expected frequencies is quite good. This, in fact, reflects the adequacy of the model to describe the bivariate distribution.

The model describes the bivariate distribution of male and female births to the French mothers very fairly. It is obvious from the calculated value of chi-square (30.053) which is far below the table value of chi-square (42.557) at 29 degrees of freedom for 5 per cent level of significance. Table 5 provides the observed as well as the expected frequencies of the bivariate distribution.

Table 4: The observed and the expected number of days to record $\boldsymbol{\beta}(\boldsymbol{\beta}=\mathbf{1},\mathbf{2},..\mathbf{6})$ or more births during 6 a.m. to 6 p.m. when Y (Y=0,1,... ,10) births have taken place during 6 p.m. to 6 a.m.

Y	0	1	2	3	4	5	6	7	8	9	10
P(X≥1/Y)	0.92	0.94	0.95	0.96	0.97	0.98	0.98	0.99	0.99	0.99	0.99
Exp. No.	13.7	39.3	54.1	61.5	55.3	43.0	29.5	22.7	17.8	6.9	7.9
Obs. No.	13	40	55	61	53	43	29	23	17	7	8
P(X≥2/Y)	0.74	0.78	0.82	0.85	0.88	0.90	0.92	0.93	0.95	0.96	0.96
Exp. No.	11.0	32.8	46.7	54.5	50.0	39.6	27.5	21.5	17.0	6.7	7.7
Obs. No.	10	31	49	57	47	38	26	20	17	7	8
P(X≥3/Y)	0.52	0.58	0.63	0.68	0.72	0.76	0.80	0.83	0.86	0.88	0.90
Exp. No.	7.7	24.2	35.9	43.5	41.3	33.6	24.0	19.1	15.4	6.1	7.2
Obs. No.	8	25	42	47	42	34	26	16	13	7	8
P(X≥4/Y)	0.32	0.38	0.43	0.49	0.54	0.59	0.64	0.68	0.72	0.76	0.79
Exp. No.	4.8	15.9	24.8	31.4	30.9	26.0	19.2	15.7	13.0	5.3	6.3
Obs. No.	7	19	33	34	31	27	22	13	10	6	8
P(X≥5/Y)	0.18	0.22	0.27	0.32	0.37	0.42	0.47	0.52	0.56	0.61	0.65
Exp. No.	2.7	9.4	15.5	20.5	21.1	18.5	14.1	11.9	10.1	4.2	5.2
Obs. No.	3	10	22	19	20	18	16	10	9	5	8
P(X≥6/Y)	0.10	0.12	0.16	0.19	0.23	0.27	0.32	0.36	0.41	0.45	0.50
Exp. No.	1.5	5.2	8.9	12.3	13.3	12.1	9.6	8.4	7.3	3.2	4.0
Obs. No.	3	6	14	13	11	12	12	7	7	4	8

Table 5: The observed and the expected (under parentheses) number of mothers having X sons and Y daughters.

Y \ X	0	1	2	3	4	5	6+	TOTAL
0	1499 (1539.8)	552 (493.3)	163 (146.0)	45 (41.7)	13 (11.8)	4 (3.3)	2 (0.9)	2278 (2236.8)
1	506 (493.3)	290 (293.1)	114 (126.0)	43 (47.9)	16 (17.0)	7 (3.7)	3 (2.0)	979 (985.2)
2	142 (146.0)	107 (126.0)	60 (70.7)	30 (33.0)	15 (13.8)	7 (5.3)	3 (2.0)	364 (396.8)
3	37 (41.7)	38 (47.9)	29 (33.0)	19 (18.5)	11 (8.9)	6 (3.9)	3 (1.6)	143 (155.5)
4	10 (11.8)	14 (17.0)	13 (13.8)	11 (8.9)	8 (6.9)	5 (2.4)	3 (1.1)	64 (61.9)
5	3 (3.3)	6 (5.9)	6 (5.3)	6 (3.9)	5 (2.4)	3 (1.3)	0 (0.7)	29 (22.8)
6+	1 (0.9)	2 (2.0)	3 (2.0)	3 (1.6)	2 (1.1)	0 (0.7)	0 (0.7)	11 (9.0)
TOTAL	2198 (2236.8)	1009 (985.2)	388 (396.8)	157 (155.5)	70 (61.9)	32 (22.8)	14 (9.0)	3868 (3868.0)

The parameters have been estimated on the basis of the Univariate distribution. The negative binomial distribution describes the Univariate distribution of total births very well. We obtain

m=0.69584; k=1.18764; andρ =0.36944.

The 95 per cent confidence limits for the product moment correlation coefficient

(0.36063) are given by0.33289 (lower) and 0.38774 (upper).

The estimated value of ρlies in the interval. Further, we have obtained

E(Y/x) = 0.43876+0.36944xand E(X/y) =0.43876+0.36944y

On the basis of these expression we have computed the expected average number of daughters to a French mother having different number of sons and the vice versa. The findings are mentioned in Table 6.

Table 6: The observed (x) and the expected (x) average number of sons (or daughters) to a mother having Y daughters (or X sons)

X or Y	$\bar{x}$	$\bar{y}$	$\bar{x}_e or \bar{y}_e$
0	0.482	0.438	0.439
1	0.780	0.710	0.808
2	1.181	1.085	1.177
3	1.713	1.605	1.547
4	2.312	2.114	1.916
5	2.448	2.312	2.285
6	2.273	2.143	2.655

Further, on putting the values of k and m in equation (12) we get

$$P(Y \geq \alpha / X = x) = 1 - (0.73022)^{1.18764+x} \ (1/\Gamma(x + 1.18764).$$

$$. \sum_{y=0}^{\alpha-1} (0.26978)^y \Gamma(x + y + 1.18764)/\Gamma(y + 1)$$

$$P(X \geq \beta / Y = y) = 1 - (0.73022)^{1.18764+y} \ (1/\Gamma(y + 1.18764).$$

$$. \sum_{x=0}^{\beta-1} (0.26978)^x \Gamma(x + y + 1.18764)/\Gamma(x + 1)$$

Table 7 shows the probabilities, the observed and the expected number of mothers to have or more daughters when they had X(X=1, 2...6) sons and Table 8 shows the opposite.

Table 7: The observed and the expected number of mothers to have α (α= 1, 2, 6) or more daughters when they have X(X= 1, 2, 6) sons

X	0	1	2	3	4	5	6
P(Y≥1/X)	0.31	0.49	0.63	0.73	0.80	0.86	0.89
Exp. No.	683.4	497.4	245.2	114.8	56.3	27.4	12.5
Obs. No.	699	457	255	112	57	28	12
P(Y≥2/X)	0.09	0.20	0.32	0.43	0.53	0.62	0.69
Exp. No.	200.0	201.8	123.0	67.3	37.1	19.8	9.7
Obs. No.	193	167	111	69	41	21	9
P(Y≥3/X)	0.03	0.07	0.14	0.22	0.30	0.39	0.47
Exp. No.	57.1	73.7	53.5	34.1	21.1	12.4	6.6
Obs. No.	51	60	51	39	26	14	6
P(Y≥4/X)	0.01	0.02	0.06	0.10	0.15	0.22	0.28
Exp. No.	15.4	25.2	21.3	15.5	10.7	6.9	4.0
Obs. No.	14	22	22	20	15	8	3

P(Y≥5/X)	0.02	0.01	0.02	0.04	0.07	0.11	0.16
Exp. No.	4.51	8.33	8.18	6.62	5.06	3.56	2.21
Obs. No.	4	8	9	9	7	3	0
P(Y≥6/X)	0.001	0.003	0.01	0.02	0.03	0.05	0.08
Exp. No.	1.27	2.66	2.95	2.66	2.23	1.7	1.14
Obs. No.	1	2	3	3	2	0	0

Table 8: The observed and the expected number of mothers to have $\beta(\beta(1,2,\ldots,6)$ or more sons when they have Y(Y=0,1,2, ...6) daughters

Y	0	1	2	3	4	5	6
P(X≥1/Y)	0.31	0.49	0.63	0.73	0.80	0.86	0.89
Exp. No.	708.5	482.6	230.0	104.5	51.5	24.8	9.8
Obs. No.	779	473	222	106	54	26	10
P(X≥2/Y)	0.09	0.20	0.32	0.43	0.53	0.62	0.69
Exp. No.	207.3	195.8	115.4	61.3	33.9	17.9	7.6
Obs. No.	227	183	115	68	40	20	8
P(X≥3/Y)	0.03	0.07	0.14	0.22	0.30	0.39	0.47
Exp. No.	59.2	71.5	50.2	31.0	19.3	11.2	5.2
Obs. No.	64	69	55	39	27	14	5
P(X≥4/Y)	0.01	0.02	0.06	0.10	0.15	0.22	0.28
Exp. No.	15.9	24.5	20.0	14.2	9.8	6.3	3.1
Obs. No.	19	26	25	20	16	8	2
P(X≥5/Y)	0.002	0.01	0.02	0.04	0.07	0.11	0.16
Exp. No.	4.67	8.08	7.67	6.03	4.63	3.23	1.74
Obs. No.	6	10	10	9	8	3	0
P(X≥6/Y)	0.001	0.003	0.01	0.02	0.03	0.05	0.08
Exp. No.	1.32	2.59	2.77	2.42	2.04	1.54	0.89
Obs. No.	2	3	3	3	2	0	0

It is evident from these tables that the observed and the expected frequencies match fairly well.

These two examples explain the adequacy of the SBNBD very well to describe the births of human populations. The result so obtained could be successfully used by hospital administrators, policy and decision-makers for the study of births in human populations.

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