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TOTAL MAGIC CORDIAL LABELING IN EXTENDED DUPLICATE GRAPHS

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Abstract: Total Magic Cordial labeling is one of the finest labeling in Graph theory. In this paper, we prove the extended duplicate of some known graphs is total magic cordial.

***Index Terms*–Graph Labeling, Duplicate graph.**

Introduction

Graph labeling is a captivating task that involves assigning integers to the vertices or edges, or both, of a graph while adhering to specific conditions. This form of labeling holds significant value in mathematical modeling and finds diverse applications in areas such as cryptography, data security, astronomy, coding theory problems, communications networks, bioinformatics, and X-ray crystallography. Since the inception of graph labeling, numerous types of labeling have been extensively explored and documented in more than 1900 research papers. These include graceful labeling, magic labeling, anti-magic labeling, prime labeling, cordial labeling, odd and even graceful labeling, and many more..

The concept of Total Cordial Labeling was initially introduced by Alex Rosa in approximately 1967 as a way to tackle the problem of cyclically decomposing the complete graph into trees. Since then, a substantial amount of literature has emerged in this field over the past four decades. Researchers have introduced various families of labeling, each with intriguing terms such as graceful, harmonious, magic, antimagic, bimagic, cordial, and prime, among others. Cordial labeling was first introduced by Cahit in 1987, and later in 2002, Cahit introduced a new labeling known as Total Magic cordial labeling. Graph labeling, as highlighted by Beineke and Hegde, plays a significant role in bridging number theory and graph structures.

The applications of graph labeling can be found not only in mathematics but also in various domains of computer science and communication networks. Kalantari, Khosrovshahi, and Mitchell explored the applications of magic labeling in optimization theory, particularly for the traveling salesmen problem. Baskoro et al. proposed a construction of a secret sharing scheme using edge magic labeling. More recently, Hartnell and Rall introduced a game based on vertex magic labeling. In this paper, we demonstrate the existence of a Total Magic cordial label.

Let f be a function from the vertices of G to {0,1} and for each xy assigns the label | f (x) – f (y)|. The function f is said to be a cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most one, the number of edges labeled 0 and the number of edges labeled 1 differ by atmost one. In 1990, Cahit has proved that every tree is cordial; Kn is cordial if and only if n < 3; Km,n is cordial for all m and n; all fans are cordial.

Preliminaries

Throughout this paper the letters v_i, e_j denote the name of the vertices and edges respectively where i and j are natural numbers.

Definition 1: The ladder graph L_m is a planar undirected graph (P_m) with 2m vertices and 3m– 2 edges. It is obtained as the Cartesian product of two path graphs, L_m=P_m x P_1, one of which has only one edge: where m is the number of rungs in the ladder.

Definition 2 (Duplicate graph) Let G (V, E) be a simple graph and the duplicate graph of G is DG = (V_1, E_1),where the vertex set V1 = V ∪V′ and V ∩V′ = ϕ and f : V →V′ is bijective (for v ∈V, we write f (v) = v′ for convenience) and the edge set E_1 of DG is defined as the edge *ab* is in E if and only if both *ab′* and *a′b* are edges in E_1.

Definition 3: Combgraph is a graph obtained by joining a single pendant edge to each vertex of a path.

Definition 4: Let EDG= (V_1, E_1) be a duplicate graph of the comb graph G(V, E). We add an edge between any one vertex from V to any other vertex in V' except the terminal vertices of V and V'. For convenience, we take v_iϵV and v_j'ϵV' and thus the edge v_iv_i is formed. We call this new graph as the extended duplicate graph of the Comb C_m and it is denoted by EDG (C_m). Clearly | Vm| = 2m and |Em| = 2m-1, where 'm' is the number of internal vertices of comb

Definition 5: The Helm graph H_n is the graph obtained from a $n - Wheel$ graph by adjoining a pendant edge at each node of the cycle.

Definition 6: The $n - Sunlet$ graph S_n is the graph on 2n vertices obtained by attaching n pendant edges to a cycle graph (C_n). i.e $C_n \odot K_n$

Definition 7: A graph G (V, E) is said to admit total magic cordial labeling if *f*: V∪E → {0, 1} such that

(i) {*f*(x) + *f* (y) + *f* (xy)} (mod 2) is a constant for all edges xyϵ E.

(ii) for all i, j ϵ{0,1} |{mi (*f*) + ni (*f*)} – {mj (*f*) + nj (*f*)}| ≤ 1, (i ≠ j)

Where mi (*f*) = {e ϵ E | *f* (*e*) = i} and ni (f) = {v ϵV | *f* (v) = i}

A graph which admits total magic cordial labeling is called total magic cordial.

MAIN RESULTS

In this section we present an algorithm and prove the existence of total magic cordial labeling for the extended duplicate of triangular Ladder graph EDG(TL_6).

Example 1. The following figures show the triangular Ladder graph (TL_6) and its duplicate graph.

Clearly the duplicate graph of the triangular ladder graph TL_6contains 2m vertices and 2m – 3edges.

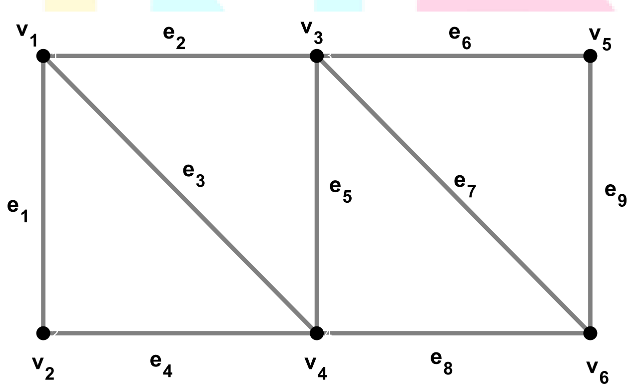

Figure 1 Triangular Ladder Graph TL6

Algorithm 1:

(Total magic cordial labeling for DG (TL_m))

V ← {v_1, v_2, …,v_m, v'$_1$, v'$_2$,… v'm}

E ← {e_1, e_2, …, e_{3m}, e_{4m-3}, e'$_1$,e'$_2$, …,e' $_{4m-3}$ }

Case (i) : when 2|m

$1 \leq i \leq \frac{m}{2}$

$v_{4i+1} \longleftarrow 1, \quad v_{4i+2} \longleftarrow 1, \quad v_{4i+1} \longleftarrow 0, \quad v_{4i} \longleftarrow 0,$

for i = 1to m-1 do

$v'_{2i+1} \longleftarrow 1, \quad v'_{2i} \longleftarrow 0$

For i=m

$v'_{2i+1} \longleftarrow 1, \quad v'_{2i} \longleftarrow 0$

end for

Case(ii): When $2 \nmid m$

for i = 1 to m do

$v_{2i-1} \longleftarrow 1, \quad v_{2i} \longleftarrow 0$

end for

$for\ 1 \leq i \leq \frac{m-1}{2}$

$v'_{4i-3} \longleftarrow 1, \ v'_{4i-2} \longleftarrow 1$

$for\ 1 \leq i \leq \frac{m-3}{2}$

$v'_{4i-1} \longleftarrow 0, \quad v'_{4i} \longleftarrow 0,$

$for\ i = \frac{m+1}{2}$

$v'_{4i-3} \longleftarrow 1, \quad v'_{4i-2} \longleftarrow 0$

end for

end for

Now to label the edges:

Label

$e_{4m-3} \leftarrow 1, \quad e'_{4m-3} \leftarrow 0$

$for\ 1 \leq i \leq \frac{m+1}{2}$

$e_{4i-3} \longleftarrow 1, \quad e_{4i-2} \longleftarrow 1$

$e_{4i-1} \longleftarrow 0, \quad e_{4i} \longleftarrow 0,$

end for

$for\ 1 \leq i \leq \frac{m+1}{2}$

$e'_{4i-3} \longleftarrow 0, \quad e'_{4i-2} \longleftarrow 0$

$e'_{4i-1} \longleftarrow 1, \quad e'_{4i} \longleftarrow 1,$

end for

end for

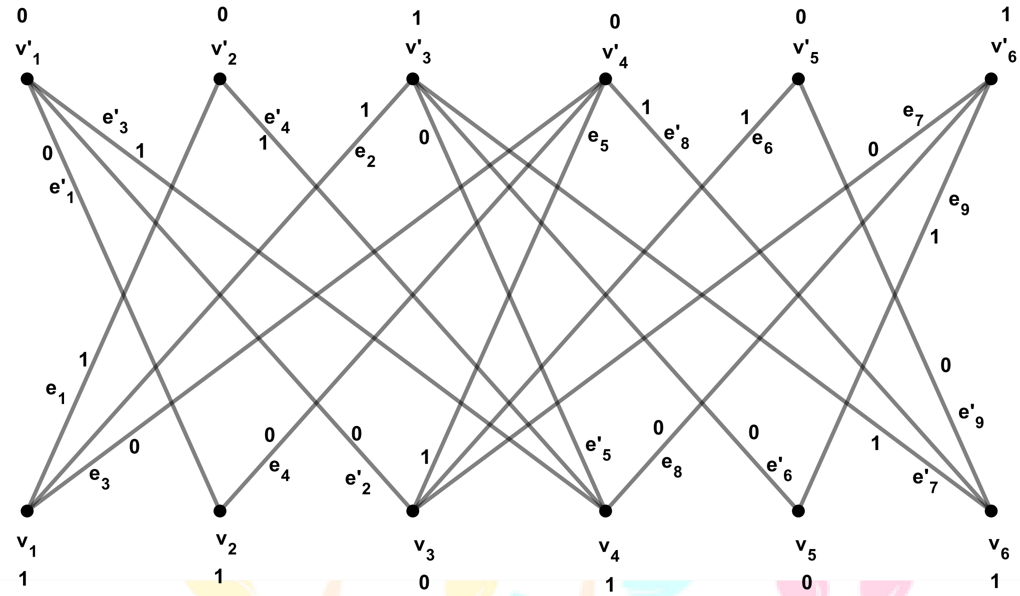

Figure 2 Total Cordial Labelling for DTL6

Theorem 1: The duplicate graph of the triangular ladder DG(TLm), m>1 admits total magic cordial labeling.

Proof:

Let V = $\{v_1, v_2, \ldots, v_m, v'_1, v'_2, \ldots v'_m\}$ and

E = $\{e_1, e_2, \ldots, e_{4m-3}, e'_1, e'_2, \ldots, e'_{4m-3}\}$ be the set of vertices and edges of the duplicate graph of triangular ladder DG (TLm). Using the algorithm (1) every single 2m vertices are labeled 0 or 1.

Case (i): when m is odd.

The vertices v_m and v'_m receive the labels 1 and 0 respectively.

The vertices $v_1, v_3, v_5, \ldots vm$ receives label 1 and $v_2, v_4, v_6, \ldots$ v2m receives label 0.

The vertices $v'_1, v'_3, v'_5, \ldots v'_m$ receives label 0 and $v'_2, v'_4, v'_6, \ldots v'_{2m}$ receives label 1.

The edges $e_1, e_2, e_5, e_6, \ldots e_{4m-3}, e_{4m-2}$ receive label 1

The edges $e_3, e_4, e_7, e_8, \ldots e_{4m-1}, e_{4m}$ receive label 0

The edges $e'_1, e'_2, e'_5, e'_6, \ldots e'_{4m-3}, e'_{4m-2}$ receive label 0

The edges $e'_3, e'_4, e'_7, e'_8, \ldots e'_{4m-1}$, e'4m receive label 1

Case (ii): when m is even.

The vertices $v_1, v_3, v_5, \ldots v_m$ receives label 0 and $v_2, v_4, v_6, \ldots v_{2m}$ receives label 1.

The vertices $v'_1, v'_3, v'_5, \ldots$ v'm receives label 1 and $v'_2, v'_4, v'_6, \ldots v'_{2m}$ receives label 0.

The edges $e_1, e_2, e_5, e_6, \ldots e_{4m-3}, e_{4m-2}$ receive label 0

The edges $e_3, e_4, e_7, e_8, \ldots e_{4m-1}, e_{4m}$ receive label 1

The edges $e'_1, e'_2, e'_5, e'_6, \ldots e'_{4m-3}, e'_{4m-2}$ receive label 0

The edges $e'_3, e'_4, e'_7, e'_8, \ldots e'_{4m-1}, e'_{4m}$ receive label 1.

This type of allocation of labels will make the triangular Ladder Graph to admit total magic cordial.

Comb Graph (C_m)

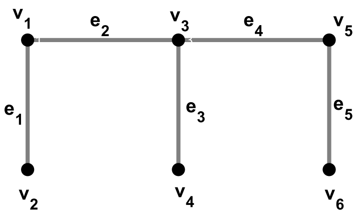

Figure 3 Comb Graph C6

Algorithm 2:

Procedure (Total Magic Cordial Labeling for EDG (C_m)

V ← {$v_1, v_2, \ldots, v2_m, v'_1, v'_2, \ldots v'2m$}

E ← {$e_1, e_2, \ldots, e_{2m-1}, e'_1, e'_2, \ldots, e'\ 2_{m-1}$ }

Case(i): When $2 \nmid m$

fori = 0 to m-1 do

$v_{2i} \longleftarrow 0, \quad v_{2i-1} \longleftarrow 1$

$v'_{2i} \longleftarrow 1, \quad v'_{2i-1} \longleftarrow 0$

end for

$for\ 1 \le i \le m-1$

$e_{4i-3} \longleftarrow 1,\ e_{4i-2} \longleftarrow 1,\ e_{4i-1} \longleftarrow 0,\ e_{4i} \longleftarrow 0$

$for\ 1 \le j \le m-1$

$e'_{4i-3} \longleftarrow 0, \quad e'_{4i-2} \longleftarrow 0,\ e'_{4i-1} \longleftarrow 1, \quad e'_{4i} \longleftarrow 1$

end for

end for

Case(ii): When m is even

$v_m \leftarrow 0,\ v'_m \leftarrow 1$

fori = 0 to m-1 do

$v_{2m} \longleftarrow 0, \quad v_{2m} \longleftarrow 0$

end for

$for\ 1 \le i \le \dfrac{m+1}{2}$

$v_{2i-1} \longleftarrow 1,\ v_{2i-1} \longleftarrow 1,$

$for\ 1 \le j \le m$

$v'_m \longleftarrow 1, \quad v'_m \longleftarrow 0,$

end for

end for

$e_1 \leftarrow 0,\ e'_1 \leftarrow 1;\ \ e_m \leftarrow 1,\ e'_m \leftarrow 0$

for i = 0 to m-1 do

$e'_{4i-2} \leftarrow 0,\ e'_{4i-2} \leftarrow 0$

$for\ 1 \le i \le \frac{m-1}{2}$

$e'_{4i} \leftarrow 1,\ e'_{4i+1} \leftarrow 1$

end for

end for

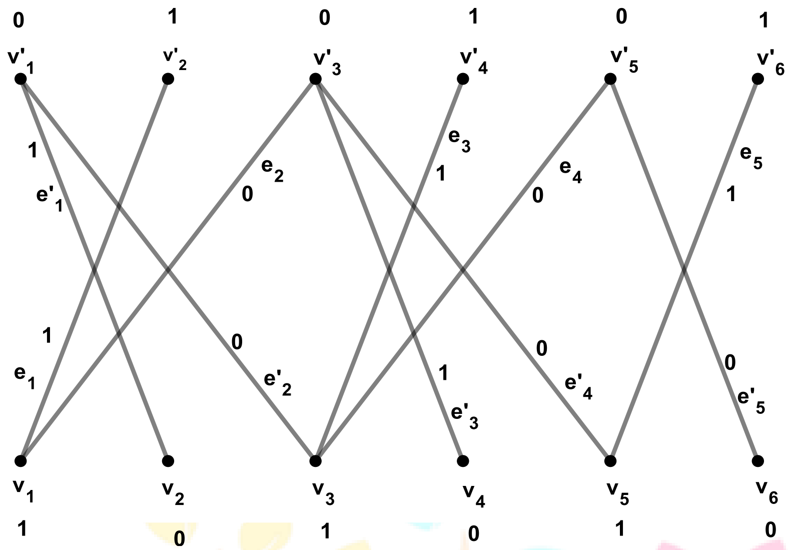

Figure 4 Total Magic Cordial Labeling of DC3

Theorem 2: The duplicate graph of the Comb Graph $DG(C_m)$ m>1 admits total magic cordial labeling.

Proof:

Let V ← {$v_1, v_2, \ldots, v_m, v'_1, v'_2, \ldots v'm$} and

E ← {$e_1, e_2, \ldots, e_{3m}, e_{2m-1}, e'_1, e'_2, \ldots, e'_{2m-1}$ } be the set of vertices and edges of the duplicate graph of the Comb Graph $DG(C_m)$.

Using the algorithm (1) every single 2m vertices are labeled 0 or 1.

Case (i): when m is odd.

The vertices v_m and v'_m receive the labels 0 and 1 respectively.

The vertices $v_1, v_3, v_5, \ldots v_{m-1}$ receives label 1 and $v_2, v_4, v_6, \ldots v_{m-2}$ receives label 1.

The vertices $v'_1, v'_3, v'_5, \ldots v'_{m-1}$ receives label 0 and $v'_2, v'_4, v'_6, \ldots v'_{m-2}$ receives label 0.

The edges $e_1, e_2, e_5, e_6, \ldots e_{2m-3}, e_{2m-2}$ receive label 0

The edges $e_3, e_4, e_7, e_8, \ldots e_{2m-1}, e_{2m}$ receive label 1

The edges $e'_1, e'_2, e'_5, e'_6, \ldots e'_{2m-3}, e'_{2m-2}$ receive label 1

The edges $e'_3, e'_4, e'_7, e'_8, \ldots e'_{2m-1}, e'_{2m}$ receive label 0

Case (ii): when m is even.

The vertices $v_1, v_3, v_5, \ldots v_{m-1}$ receives label 0 and $v_2, v_4, v_6, \ldots v_m$ receives label 0.

The vertices $v'_1, v'_3, v'_5, \ldots v'_{m-1}$ receives label 1 and $v'_2, v'_4, v'_6, \ldots v'_m$ receives label 1.

The edges $e_1, e_2, e_5, e_6, \ldots e_{2m-3}, e_{2m-2}$ receive label 1

The edges $e_3, e_4, e_7, e_8, \ldots e_{2m-1}, e_{2m}$ receive label 0

The edges $e'_1, e'_2, e'_5, e'_6, \ldots e'_{2m-3}, e'_{2m-2}$ receive label 1

The edges $e'_3, e'_4, e'_7, e'_8, \ldots e'_{2m-1}, e'_{2m}$ receive label 0.

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Thus it is proved that the number of vertices and edges labeled 1 and number of vertices and edges labeled 0 differ maximum by 1 satisfying the condition of total cordial labeling.

Theorem 3: Triangular Ladder graph is an extension of Comb graph

Let $|V_L|$ and $|E_L|$ denote the number of vertices and edges of Triangular ladder graph and let $|V_C|$ and $|E_C|$ denote the number of vertices and edges of Comb graph. Then $|V_L| = |V_C|$ and $|E_L|= 2|E_C|-1$.
Proof: It is obvious that $|V_L| = |V_C|$ for any m.
Now $|E_L|=4m-3= 2m-1+2m-1-1= |E_C|+|E_C|-1=2|E_C|-1$.
By theorem 3 the same labeling as triangular ladder can be used for comb graph.

Helm Graph (H_n)

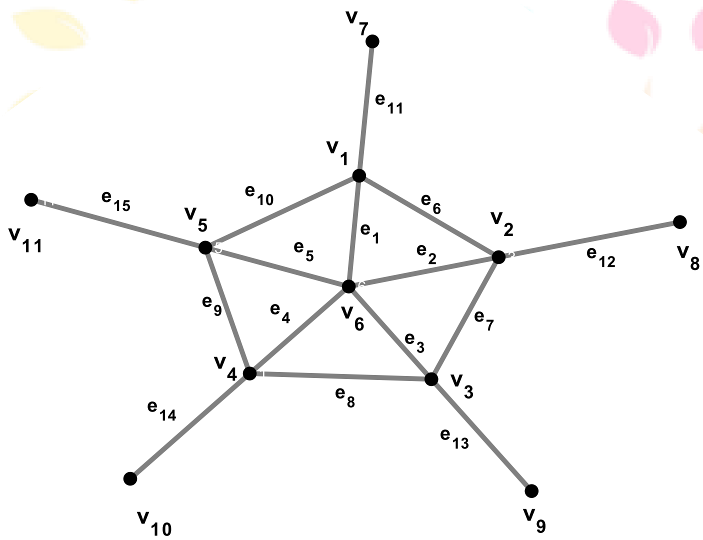

Figure 5 Helm Graph H5

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Algorithm

$V \leftarrow \{v_1, v_2, \quad v_3, \ldots, v_{2n+1}, v'_1, v'_2, v'_3, \ldots, v'_{2n+1}\}$

$E \leftarrow \{e_1, e_2, e_3, \ldots, e_{3n}, \quad e'_1, e'_2, e'_3, \ldots, e'_{3n}\}$

For $i = 1,2,3, \ldots 2n+1$

If $\deg(v_i) = n$, then $\quad i = 1,2,3, \ldots, 2n+1$

$v_i \leftarrow 1, v'_i \leftarrow 0$

If $\deg(v_i) = 4$, then $\quad i = 1,2,3, \ldots, 2n+1$

$v_i \leftarrow 1, v'_i \leftarrow 0$

If $\deg(v_i) = 1$, then $\quad i = 1,2,3, \ldots, 2n+1$

$v_i \leftarrow 0, v'_i \leftarrow 1$

Egde Labeling

Case i) When n is even

$e_k \leftarrow 0, \ e'_k \leftarrow 1 \quad k = 1,2,3, \ldots \frac{3n}{2}$

$e_k \leftarrow 1, \ e'_k \leftarrow 0 \quad k = \frac{3n+2}{2}, \frac{3n+4}{2}, \ldots 3n$

Case ii) When n is odd

$e_k \leftarrow 0, \ e'_k \leftarrow 1 \quad k = 1,2,3, \ldots \frac{3n+1}{2}$

$e_k \leftarrow 1, \ e'_k \leftarrow 0 \quad k = \frac{3n+3}{2}, \frac{3n+5}{2}, \ldots 3n$

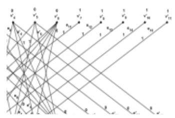

Figure 6 Total Cordial Labeling

Theorem:

The duplicate graph of the Helm graph H_n, $n \geq 3$admits total magic cordial labeling.

Proof:

Consider the set of vertices V = {v_1, v_2, v_3, ..., v_(2n+1), v_1', v_2', v_3', ..., v_(2n+1)'} and the set of edges E = {e_1, e_2, e_3, ..., e_(3n), e_1', e_2', e_3', ..., e_(3n)'} representing the vertices and edges of the duplicate graph of the Helm graph.

Using the provided algorithm, we assign a label of either 0 or 1 to each of the 2n+1 vertices. The vertex v_1 is adjacent to v_2,v_n,v_$_{n+1}$,v_$_{n+2}$. Vertex v_2 is adjacent to v_1, v_3, v_{n+1}, and v_{n+3}. Vertex v_3 is adjacent to v_2, v_4, v_{n+1}, and v_{n+4}, and so on. Hence, vertex vn is adjacent to v_1, v_{n-1}, v_{n+1}, and v_$_{2n+1}$. Vertex v_{n+1} is adjacent to v_1, v_2, v_3, ...,v_n.

The vertices v_{n+1} and v'_{n+1} receives label 1and 0.
The vertices $v_1, v_2, v_3, \dots . v_n$receive label 1
The vertices $v'_1, v'_2, \dots \dots v'_n$ receive label 0.
The vertices $v_{n+2}, v_{n+3}, \dots . v_{2n+1}$receive label 0
The vertices $v'_{n+2}, v'_{n+3} \dots \dots v'_{2n+1}$ receive label 1.

Case i) When n is even
The edges$e_1, e_2, e_3, \dots, e_{\frac{3n}{2}}$ receive label 0.

The edges $e'_1,\ e'_2, e'_3, \dots e'_{\frac{3n}{2}}$ receive label 1.

The edges $e_{\frac{3n+2}{2}}, e_{\frac{3n+4}{2}}, \dots, e_{3n}$ receive label 1.

The edges $e'_{\frac{3n+2}{2}}, e'_{\frac{3n+4}{2}}, \dots \dots, e'_{3n}$ receive label 0.

Case ii) When n is odd
The edges $e_1, e_2, e_3, \dots, e_{\frac{3n+1}{2}}$ receive label 0.

The edges $e'_1,\ e'_2, e'_3, \dots e'_{\frac{3n+1}{2}}$ receive label 1.

The edges $e_{\frac{3n+3}{2}}, e_{\frac{3n+5}{2}}, \dots, e_{3n}$ receive label 1.

The edges $e'_{\frac{3n+3}{2}}, e'_{\frac{3n+5}{2}}, \dots \dots, e'_{3n}$ receive label 0.

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Thus, all 4n+2 vertices and 6n edges are labeled in such a way that the number of vertices labeled '0' and '1' is equal to 2n+1. Additionally, the number of edges labeled '0' and '1' is equal to 3n, regardless of whether n is odd or even. Therefore, the duplicate graph of the Helm graph exhibits a total magic cordial labeling.

Sunlet graph

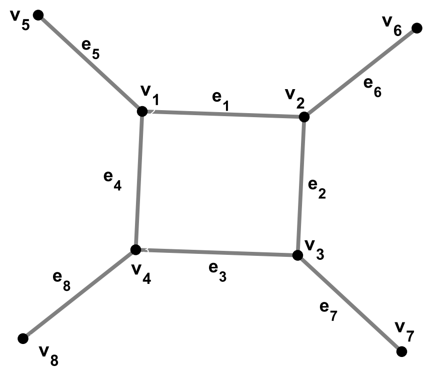

Figure 7Sun let Graph S4

Algorithm

$V \leftarrow \{v_1, v_2, v_3, \ldots., v_{2n}, v_1', v_2', v_3', \ldots., v_{2n}'\}$

$E \leftarrow \{e_1, e_2, e_3, \ldots, e_{2n}, e_1', e_2', e_3', \ldots\ldots, e_{2n}'\}$

If $\deg(v_i) = 3$

$v_i \leftarrow 1, v_i' \leftarrow 0$ $\quad i = 1, 2, 3, \ldots., 2n$

If $\deg(v_i) = 1$, then

$v_i \leftarrow 0, v_i' \leftarrow 1$ $\quad i = 1, 2, 3, \ldots., 2n$

Egde Labeling

$e_k \leftarrow 0, e_k' \leftarrow 1 \quad k = 1,2,3, \ldots n$

$e_k \leftarrow 1, e_k' \leftarrow 0 \quad k = n+1, n+2, n+3, \ldots., 2n$

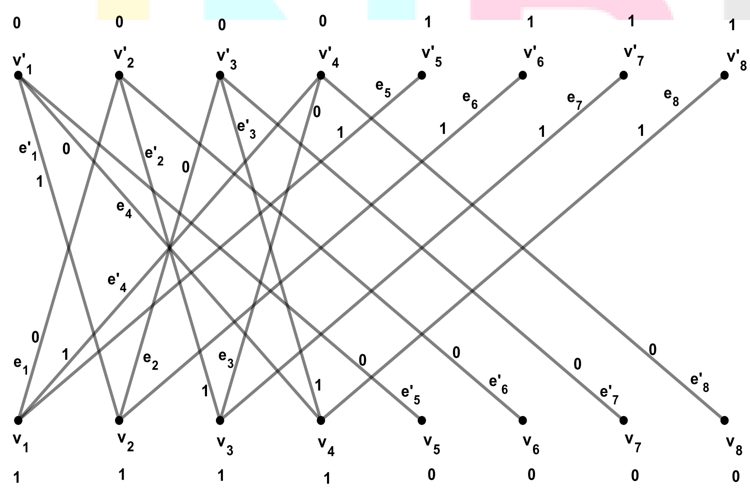

Figure 8 Total Magic Cordial Labeling DS4

Theorem:

The duplicate graph of the Sunlet graph admits total magic cordial labeling.

Proof:

Let $V = \{v_1, v_2, v_3, \ldots., v_{2n}, v'_1, v'_2, v'_3, \ldots., v'_{2n}\}$ and $E = \{e_1, e_2, e_3, \ldots, e_{2n}, e'_1, e'_2, e'_3, \ldots\ldots, e'_{2n}\}$ be the set of vertices and edges of the duplicate graph of the Sunlet graph.

Using the algorithm, every single $2n$ vertex is labeled with '0' or '1'.

The vertices $v_1, v_2, v_3, \ldots. v_n$ receive label 1 and the vertices $v'_1, v'_2, \ldots \ldots v'_n$ receive label 0.

The vertices $v_{n+1}, v_{n+2}, v_{n+3}, \ldots. v_{2n}$ receive label 0.

The vertices $v'_{n+1}, v'_{n+2}, \ldots \ldots v'_{2n}$ receive label 1.

The edges $e_1, e_2, e_3, \ldots, e_n$ receive label 0.

The edges $e'_1, e'_2, e'_3, \ldots e'_n$ receive label 1.

The edges $e_{n+1}, e_{n+2}, e_{n+3}, \ldots, e_{2n}$ receive label 1.

The edges $e'_{n+1}, e'_{n+2}, e'_{n+3}, \ldots e'_{2n}$ receive label 0.

Thus the entire $4n$ vertices and $4n$ edges are labeled in such a way that the number of vertices and edges labeled '0' and the number of vertices and edges labeled '1' are same as $2n$ and satisfied the required conditions.

Hence the duplicate graph of Sunlet graph is total magic cordial labeling.

References

[1] Selvam B, Thirusangu K &Ulaganathan P.P,(2011). (a,d)-Antimagic labelings in extended duplicate twig graphs, *Indian Journal of Science and Technology, 4* (2), 112-115.

[2] Thirusangu, K., Selvam, B., &Ulaganathan, P. P., (2010). Cordial labeling in extended duplicate twig graphs, *International Journal of Computer, Mathematical Sciences and Applications, 4* (3-4), 319-328

[3] M. Sundaram, R. Ponraj, and S. Somasundaram, (2006). "Total product cordial labeling of graphs," Bulletin of Pure & Applied Sciences Section E: Mathematics and Statistics, 25(1), 199–203,

[4] G.Divya Dharshini , U.Mary, (2019). "Cordial Labeling of Graphs" International Journal of Innovative Research in Technology, 5(9), 165-167.